Negative S Parameter from Holographic Technicolor

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We present a new class of 5D models, Holographic Technicolor, which fulfills the basic requirements for a candidate of comprehensible 4D strong dynamics at the electroweak scale. It is the first Technicolor-like model able to provide a vanishing or even negative tree-level S parameter, avoiding any *no-go theorem* on its sign. The model is described in the large-N regime. S is therefore computable: possible corrections coming from boundary terms follow the $1/N$ suppression, and generation of fermion masses and the S parameter issue do split up. We investigate the model's 4D dual, probably walking Technicolor-like with a large anomalous dimension.

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Introduction.—The idea that electroweak symmetry breaking (EWSB) could be due to the onset of the strong-coupling regime in an asymptotically free gauge theory was first put forward to solve the hierarchy problem in Ref. [1]. Technicolor was based on the example of massless QCD with two flavors, where the global SU(2) × SU(2) symmetry is spontaneously broken to the diagonal subgroup. A similar theory with a mass scale of order 3000 larger would feed its three Goldstone bosons to the standard model (SM) $SU(2)_L \times U(1)_Y$ gauge fields, yielding masses for the $W^\pm$ and Z, without an associated Higgs boson. It was, however, shown that a simple rescaled version of QCD fails, since it leads to the famous S parameter being too large and positive as compared to the value extracted from experiments [2], unless the number of Technicolors is small. This last possibility is, however, undesirable, as it signifies the loss of our last nonperturbative handle, namely, the large-N expansion.

The recent developments in holographic QCD [3–5] give us a computable way of departing from rescaled QCD. The models of Holographic QCD aim to describe the dynamics of the QCD bound states in terms of a 5D gauge theory: the input parameters in such a description can be identified with the number of colors, the confinement scale, and the *condensates*. The present class of models for dynamical EWSB works in a similar spirit. For the first time, the tree-level S parameter is negative. This has consequences on the resonance spectrum.

Holographic Technicolor.—Our starting point is a model in five dimensions (5D) describing electroweak symmetry breaking via boundary conditions (BCs). The extra dimension we consider here is an interval. The two ends of the space are located at l_0 (the *UV brane*) and l_1 (the *IR brane*). We focus on metrics that can be recast as $ds^2 = w(z)^2 \times (\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$. We only consider the dynamics of the bulk 5D symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry. As in Higgs-less models [6], the BCs are chosen to break the LR symmetry to the diagonal $SU(2)_D$ on the IR brane, while the breaking on the UV brane reduces $SU(2)_R \times U(1)_{B-L}$ to the hypercharge subgroup. The remaining 4D gauge symmetry is thus $U(1)_Q$.

An important ingredient of holographic Technicolor comes from the lessons learned in holographic QCD: breaking on the brane is too soft to account for all phenomena found in QCD, in particular, power corrections at high energies due to condensates. Besides this breaking by BCs, we therefore introduce breaking in the bulk in the form of a crossed kinetic term between L and R gauge fields, just as in [7]. (The z dependence of this term could be obtained from the profile of a scalar.) At the quadratic level, this well-defined procedure may *effectively* be summarized as yielding different metrics, $w_A(z) \neq w_V(z)$ [7]. This bulk breaking will play an important role in our description of strong dynamics at the TeV.

The spectrum.—In terms of physical states, no massless mode survives except for the photon. The remainder will pick up masses via the compactification. For the class of metrics that decrease away from the UV as anti—de Sitter (AdS) or faster (*gap metrics*), the massive modes can be separated into two groups: *ultralight excitations* [6] and Kaluza-Klein (KK) *modes*. If we interpret the ultralight modes as the W and Z, the gap suppresses the KK contributions to the electroweak observables [6]: this can be seen clearly using sum rules (SRs).

For any gap metric, the KK modes are repelled from the UV brane, and the massive modes approximately split into separate towers of axial and vector fields (and B fields). Thus, W', the first KK mode above the W would *a priori* be a vector (the techni-rho), while the next one, W'', would be an axial resonance (techni-a_1), etc. One can extract SRs involving KK-mode masses (excluding ultralight modes)

$$\sum_{n=1}^{\infty} \frac{1}{M_{X_n}^2} \simeq \int_{l_0}^{l_1} dz w_X(z) \alpha_X(z) \int_{l_0}^{z} \frac{dz'}{w_X(z')}, \qquad (1)$$

where $X = V,\ A,\ B$ and $\alpha_{V,B}(z) = 1$ and $\alpha_A(z) = \int_z^{l_1} \frac{dz'}{w_A(z')} / \int_{l_0}^{l_1} \frac{dz''}{w_A(z'')}$. The SR in Eq. (1) is exact at order

$\mathcal{O}(G^0)$, where G is the gap between the ultralight mode and the heavy modes: in AdS, $w(z) = l_0/z$ and the gap is $G = \log(l_1/l_0)$. As in holographic QCD, the function $\alpha_A(z)$ [5] is the wave function of the "would-be" Goldstone boson matrix, $D_\mu U(x)$: it is monotonously decreasing with BCs $\alpha_A(l_0) = 1$ and $\alpha_A(l_1) = 0$.

On the other hand, another exact SR can be obtained, involving both heavy *and* ultralight modes. For gap metrics, it can be expanded to obtain the mass of the ultralight mode: at order $\mathcal{O}(1/G)$, we get

$$M_W^2 \simeq 1\bigg/\left(\int_{l_0}^{l_1} dz(w_V(z) + w_A(z)) \int_{l_0}^{l_1} \frac{dz'}{w_A(z')}\right), \qquad (2)$$

which can be shown to agree with the expression involving the 4D gauge coupling g and techni-pion decay constant f, $M_W^2 \simeq g^2 f^2/4 \simeq 1/(G l_1^2)$ as expected from Technicolor. At leading order $\mathcal{O}(1/G)$, $M_{W,Z}^2$ do not feel any breaking of conformality in the bulk: their mass is dominated by the UV physics. On the other hand, Eq. (1) showed that the KK masses do feel the effect of this bulk breaking at leading order $\mathcal{O}(G^0)$. Their mass is dictated by the position of the IR brane, $m \propto 1/l_1$, and does, in addition, depend on the condensates, as shown in Fig. 1 for the metrics of Eq. (5): the ratio m_{A_1}/m_{V_1} tends to be lowered as negative condensates are switched on. Many other results can be shown in terms of SRs [8]: the Fermi constant, $\rho_*(0)$, $T \simeq 0$, and the behavior of $W_L W_L$ scattering.

The S parameter.—The tree-level contribution to the S parameter can be expressed [2] in terms of the L_{10} coupling of chiral Lagrangians $S_{\rm tree} = -16\pi L_{10}$. The value extracted from LEP physics is [9] $S = -0.13(0.07) \pm 0.10$ with reference Higgs mass $m_H = 117(150)$ GeV, where the value in parentheses is the most recent analysis of data at the Z pole. A sizeable negative L_{10} would easily upset the experimental constraint (note that in $N_c = 3$ QCD, $-16\pi L_{10} \sim 0.3$). On the other hand, large-N models of strong dynamics predict the value of L_{10} in terms of

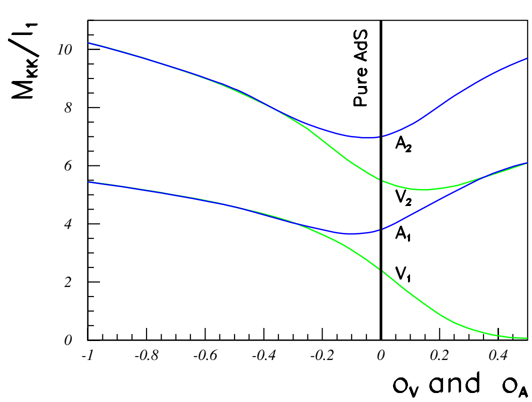

FIG. 1 (color online). Masses at $\mathcal{O}(G^0)$ divided by l_1 for the lightest vector and axial KK modes of the W, as a function of the condensate in their respective channel $o_{V,A}$, for $d = 2$.

contribution of spin-1 resonances via their decay constants f_{X_n} [10].

$L_{10} = -1/4 \sum_{n=1}^{\infty} f_{V_n}^2 - f_{A_n}^2$ Other contributions are down by $1/N$. Higgs-less models thus face a serious challenge, a *no-go theorem* [11]: L_{10} is bounded to be negative. This is readily understood by using a SR: one can translate the sum over resonance contributions into a purely geometric factor

$$L_{10} = -\frac{N}{48\pi^2} \int_{l_0}^{l_1} \frac{dz}{l_0} w(z)[1 - \alpha(z)^2], \qquad (3)$$

where we have defined $N/12\pi^2 \equiv l_0/g_5^2$. The bound $\alpha(z) \leq 1$ implies that L_{10} is negative and proportional to the loop expansion parameter, N. The most natural value for L_{10} thus drives a large positive S parameter, excluding the simplest realization of the model. For example, pure AdS yields $S_{\rm tree} = N/4\pi$.

One possibility would be to consider these models in the low-N regime. This situation is most unwelcome, as has been stressed by many authors [12]. The main reason is that N sets the range of computability of the model. Low N implies strong coupling of the gauge KK modes. A way of putting it is via the *position-dependent cutoff* [13]: a cutoff Λ at the position where $w(z)$ is normalized to unity will be redshifted for processes located near a position z as $\Lambda(z) = \Lambda\sqrt{g_{00}} = \Lambda w(z)$. For example, in pure AdS, the 5D loop expansion breaks down when $\Lambda(z) z \sim 24\pi^3 l_0/g_5^2 = 2\pi N$. The other parameter playing an important role is the gap G. Reproducing the Fermi constant and the W mass implies $NG \sim 500$. Pushing to low values of N would conflict with the premise of perturbativity.

Returning to the large-N regime, one is then cornered into hoping for miraculous cancellations. Efficient possibilities would be introducing IR localized kinetic terms proportional to $SU(2)_D$ or hoping for cancellations against fermion contributions [6]. Both possibilities face again new challenges, difficult to resolve. Trying to add large localized kinetic terms with the "wrong" sign, which are of order $1/N$ directs again towards the low-N problem. Besides, it leads to a tachyon instability [11]. The way out with bulk fermions poses a problem of naturalness and dangerously ties the S parameter problem with the fermion mass hierarchy, and therefore with nonoblique corrections [6,14].

Here we propose a different point of view, which arises naturally in holographic QCD and should therefore appear in a Technicolor-like model. Local order parameters of the symmetry-breaking imply a *different* behavior for the V and A combinations of bulk fields [3,5]. In the simplest realization of this IR behavior [7], L_{10} is modified from Eq. (3) to read

$$L_{10} = -\frac{N}{48\pi^2} \int_{l_0}^{l_1} \frac{dz}{l_0}[w_V(z) - w_A(z)\alpha_A(z)^2], \qquad (4)$$

where $w_{V,A}$ are the metrics felt by the axial and vector combinations of fields.

L_{10} is still *proportional to* N, but its sign is not fixed. A large-N scenario is then preferred, extending the pertubativity regime. In particular, the bulk value of S will *not* receive sizeable corrections from the localized kinetic terms, since these are still suppressed by $1/N$. The S parameter is therefore computable. Contrary to the spectrum, contributions to S come mainly from the bulk far from the branes [8].

We now assume that the metrics behave as AdS near the UV brane and deviate from conformality in the bulk according to

$$w_X(z) = \frac{l_0}{z} \exp\left[\nu_X \left(\frac{z - l_0}{l_1 - l_0}\right)^{2d} \right]. \qquad (5)$$

As explained at the beginning of the Letter, this parametrically simple form encodes effects of couplings with other background fields, whose dynamics we neglect here. At order $\mathcal{O}(G^0)$, one can obtain an analytic expression for S in the case $\nu_A = 0$ and $\nu_V < 0$

$$S_{\rm tree} = \frac{N}{4\pi}\left[1 - \frac{2}{3d}(\Gamma(-\nu_V) + \log(-\nu_V) + \gamma_E)\right]. \qquad (6)$$

In $\nu_V \equiv \frac{12\pi^{3/2}\Gamma(d+1/2)}{d^2\Gamma(d)^3} o_V$, NDA sets $o_V \sim \mathcal{O}(1)$ [7].

In Fig. 2, one sees that a negative vector condensate can lead to vanishing or negative $S_{\rm tree}$, even more so if it is accompanied by an axial condensate of the same sign: a direction not explored by the authors of Refs. [4,15]. Also, assuming $o_X \sim \mathcal{O}(1)$, the effect disappears if the dimension of the condensate is increased. Our results thus extend those of Ref. [16], which indicated that increasing o_A-o_V, preferably with a low d, could decrease the S parameter, in connection with a lowering of the ratio m_{A_1}/m_{V_1}.

A refinement in the computation of the S parameter comes from taking into account the pion loop effects [2] and subtracting the SM value with a reference Higgs mass. This additional contribution is positive and of order 0.1,

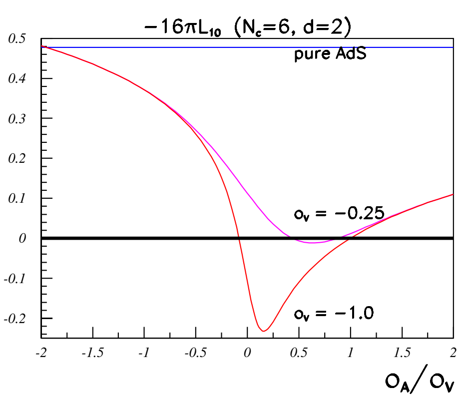

FIG. 2 (color online). Value of $S_{\rm tree}/N$—for $d = 2$ and for different values of o_V—as a function of the ratio of condensates in the two channels o_A/o_V, and for the pure AdS case.

requiring a vanishing or slightly negative $S_{\rm tree}$, as provided by the present model.

Four-dimensional dual.—Holographic models are inspired from the correspondence between the conformal field theory and anti–de Sitter space [17]. The precise form of this conjecture relates two highly symmetric theories and is, unfortunately, far from being of direct phenomenological relevance. After a pioneering work by Pomarol [18], authors in Ref. [19] explored the audacious conjecture that more realistic models like Randall-Sundrum [20] would somehow inherit properties of the duality. Since then, more evidence has been gathered towards a 5D-4D duality, the latest being bottom-up models of holographic QCD [3–5,7]. The success of these models in capturing the behavior of a strongly coupled theory like QCD provides an incentive for applications to Technicolor. In this case, one starts off on a firmer footing: in the presence of condensates, the number of (Techni)-colors can be made large since it is no longer in conflict with the S parameter.

The effect in the 4D two-point correlator of the current $X = V, A, B$ of a metric of the form given by Eq. (5) is to add a term $\langle \mathcal{O}_{2d_X}\rangle/Q^{2d_X}$ where the parameter $o_X \equiv \langle \mathcal{O}_{2d_X}\rangle/(N l_1^{-2d_X}) \sim \mathcal{O}(1)$. To have a chance of obtaining a positive value for L_{10}, we need $\langle O_{2d}\rangle_V < \langle O_{2d}\rangle_A$. This is in agreement with Witten's positivity condition for $\Pi_A - \Pi_V$ [21], ensuring the stability of the selected vacuum [22]. Holography tells us that this bulk field X is dual to some operator $\mathcal{O}$ on the 4D side with the same quantum numbers: the correlators generated by X and by $\mathcal{O}$ are the same. In this particular case we see that deviations from conformality with a given power of z^{2d} in Eq. (5) mimic the effects of a condensate of dimension $2d$ in the 4D dual.

Generally speaking, nonperturbative effects in QCD-like Technicolor models make them unreliable. The same goes for the case of a flat extra dimension, the cutoff of the theory is quite low, $\Lambda \sim 2\pi N/l_1$ and quantities like the S parameter are no longer computable. On the other hand, extra-dimensional models in AdS behave in a similar fashion to walking Technicolor. The warping suppresses convolutions of wave functions, as walking kills unwanted operators. But in pure AdS, one cannot choose which operators will be suppressed: their scaling is dictated by the warping, whereas gap metrics with violations of conformality like Eq. (5) do change the scaling.

If the 4D dual of holographic Technicolor is going to yield small or negative S parameter, the net effect of condensates must go in the direction of $w_A\alpha^2 > w_V$. For example, imagine that strong dynamics generate a technicondensate $\langle Q\bar{Q}\rangle$ responsible of breaking the Technicolor gauge group SU(N): this condensate is represented in the 5D dual as the rescaled vacuum expectation value of $\langle\Phi\rangle$. Assume now that the anomalous dimensions is large, for example, due to the running mass in the 5D picture. Then, there will be a difference between the canonical dimension of $\langle \bar{Q}Q \rangle$ and the running dimension of the operator. A way of modelling this anomalous dimension would be that the vector and axial fields couple to a scalar representing the techni-quark condensate, Φ, via a *running mass*, such that $m_\Phi(l_0)^2 = -3/l_0^2$ and $m_\Phi(l_1)^2 = d(d-4)/l_0^2$ with $d < 3$ ($d = 2$ for extreme walking).

Conclusions.—In this Letter we have shown quantitatively how Technicolor models which depart from rescaled QCD can exhibit a negative tree-level S parameter. This was done using a holographic model (i.e., using a 5D gauge theory) for the resonances created by a strongly interacting theory such as Technicolor. It is based on the recent successes of similar 5D models for the resonances of QCD. These successes themselves validated the idea of the duality between 4D strongly coupled theories and 5D weakly coupled ones at the quantitative level.

We have presented the first Technicolor-like model able to provide a small S parameter, and to remain computable since it is defined in the large-N limit. The 5D picture shows generic features of this class of models: (1) the metric has to fall off fast near the UV to generate a gap, (2) deviations from conformality must be introduced in the bulk, describing condensates, (3) a condensate of natural size can produce the desired effect if it has dimension close to 4 (as would happen for $\alpha_{TC}\langle \bar{Q}Q \rangle^2$ in walking Technicolor), (4) W' and Z' (vector resonances) then tend to become degenerate with the W'' and Z'' (axial) resonances.

In this Letter, the fermions were located for simplicity on the UV brane. As soon as we let them live in the bulk, much more interesting phenomena should arise: one big advantage of the present models is that the fermion profiles are not constrained by the requirement of canceling the S parameter contributions. For example, topcolor assisted models would be implemented as in Ref. [23].

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