

## Origin of Increase of Damping in Transition Metals with Rare-Earth-Metal Impurities

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(Received 18 May 2006; published 11 September 2006)

The damping due to rare-earth-metal impurities in transition metals is discussed in the low concentration limit. It is shown that all established damping mechanisms based on spin-orbit and/or spin-spin interactions cannot explain experimental observations even qualitatively. We introduce a different relaxation channel due to the coupling of the orbital moments of the rare-earth-metal impurities and the conduction  $p$  electrons that leads to good agreement with experiment. Using an itinerant picture for the host ions, i.e., write their magnetization in terms of the electronic degrees of freedom, is key to the success of our model.

DOI: 10.1103/PhysRevLett.97.117601

PACS numbers: 76.30.Kg, 72.25.Rb, 76.60.Es

Magnetization dynamics has become one of the most important issues of modern magnetism. This development is driven by the technological demand to tailor magnetic responses on ever smaller length and shorter time scales. The importance of this issue manifests itself in a completely new area of research, spintronics, and a huge literature that cannot be cited here. Selected highlights include precessional switching by tailored field pulses [1,2], spin-torque [3,4], and laser-induced magnetization dynamics [5,6].

In general, magnetization dynamics is described via the Landau-Lifshitz-Gilbert equation (LLG) [7] including additional terms to incorporate spin-torque effects [8] or those due to pulsed optical excitations [9]. All these descriptions account for energy dissipation via a phenomenological damping parameter  $\alpha$  which governs the time needed for a nonequilibrium magnetic state to return to equilibrium. Recently it has even been suggested that  $\alpha$  determines the magnetic response to ultrafast thermal agitations [10].

Technological applications call for the ability to tailor  $\alpha$  [11]. The most systematic experimental investigation on this topic was published by Bailey *et al.* [12] who studied the effect of rare-earth-metal doping on the damping in permalloy. Most rare-earth-metal ions induced a large increase of  $\alpha$ , but neither Eu nor Gd altered the damping of permalloy (cf. Fig. 2). Since  $\text{Gd}^{3+}$  and  $\text{Eu}^{2+}$  have no orbital momentum, this points immediately to the importance of the angular momentum in the damping process. Bailey *et al.* determined damping by reproducing their data via the LLG equation using  $\alpha$  as a fit parameter. This widely used procedure points to a fundamental problem of this phenomenological approach. Though the LLG equation describes data well, a more microscopic approach is needed to understand the origin of damping.

It was Elliott [13] who first studied damping in semiconductors due to spin-orbit coupling. Later Kambersky [14] argued that the Elliot-Yafet mechanism should be also operable in magnetic conductors. Korenman and Prange [15] developed a more microscopic treatment and found

that spin-orbit coupling should be important at low temperature in transition metals. Recent measurements of damping in magnetic multilayers at room temperature [16] suggest that the  $s$ - $d$  interaction might also be at the origin of damping [17,18]. However, all of the present models fail to reproduce the data of Ref. [12].

In this Letter, we explain the increase of damping in rare-earth-metal-doped transition metals via a novel orbit-orbit coupling between the conduction electrons and the impurities. The well-known  $s$ - $f$  interaction [19] gives rise to a  $(g_J - 1)^2$  dependence of the damping that is in contradiction to experimental observations [12]. In contrast, the orbit-orbit coupling considered here reproduces the measured  $(g_J - 2)^4$  dependence of the damping. Both dependencies on the Lande  $g$  factor  $g_J$  follow directly from the fact that the rare-earth-metal ions are in their ground state. Hence, their angular momentum  $\mathbf{L}_f$ , spin  $\mathbf{S}_f$ , and total angular momentum  $\mathbf{J}_f$  are related by the Wigner Eckard theorem:  $\mathbf{L}_f = (2 - g_J)\mathbf{J}_f$  and  $\mathbf{S}_f = (g_J - 1)\mathbf{J}_f$ . Deriving the magnetic moments of the transition-metal ions from the electronic degrees of freedom is essential to capture the correct behavior of damping as a function of  $\mathbf{J}_f$ . For the uniform mode, the damping due to orbit-orbit coupling is of Gilbert form in the low frequency limit.

Taking the wave functions of the  $d$ -,  $f$ -, and conduction electrons orthogonal, the Hamiltonian for the rare-earth-metal-doped transition metal in an external field  $\mathbf{H}$  is

$$\mathcal{H} = \mathcal{H}_e + \mathcal{H}_f + \mathcal{H}_d. \quad (1)$$

This approximation should be valid for the heavy rare-earth metals but probably fails for elements like cerium where valence fluctuations are important. The conduction electron Hamiltonian  $\mathcal{H}_e$  is the usual one,  $\mathcal{H}_e = \sum_{k,\sigma} \epsilon_{k,\sigma} a_{k,\sigma}^\dagger a_{k,\sigma}$ , where  $a_{k,\sigma}^\dagger$  and  $a_{k,\sigma}$  are the creation and annihilation operators of a conduction electron with momentum  $\mathbf{k}$  and spin  $\sigma$ .  $\epsilon_{k,\sigma}$  is the energy of the conduction electrons including a Zeeman term.

$\mathcal{H}_f$  is the Kondo Hamiltonian [20] of the localized rare-earth-metal moment

$$\mathcal{H}_f = \Gamma \mathbf{S}_e \cdot \mathbf{S}_f + \lambda \mathbf{L}_e \cdot \mathbf{L}_f - \mu_f \cdot \mathbf{H}. \quad (2)$$

$\mathbf{S}_{e/f}$  and  $\mathbf{L}_{e/f}$  are the spin and angular momentum of conduction and  $f$  electrons, respectively.  $\mathbf{L}_{e/f}$  are taken with respect to the position of the impurity. The spin-spin term is the well-known  $s$ - $f$  coupling used by de Gennes to reproduce the Curie temperatures in rare-earth metals with  $\Gamma$  being of the order 0.1 eV [19]. The last term is again a Zeeman term. The middle term is the essential orbit-orbit interaction needed in our discussion. To get a nonzero orbit-orbit term due to a single impurity at the center, it is essential to include higher terms of the partial wave expansion for the wave functions of the conduction electrons:  $\psi_k(\mathbf{r}) = \frac{4\pi}{\sqrt{V}} \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} i^l f(r) j_l(kr) Y_{lm}(\theta_k, \phi_k) \times Y_{lm}^*(\theta, \phi)$ . The first nontrivial contribution for  $l = 1$  is [20]

$$\mathcal{H}_{LL} = i(2 - g_j) \sum_{k,k'} \lambda(\mathbf{k}, \mathbf{k}') \hat{\mathbf{k}} \times \hat{\mathbf{k}}' \cdot \mathbf{J}_f a_k^\dagger a_{k'}, \quad (3)$$

where the orbit-orbit coupling  $\lambda$  will be assumed to be a function of the relative angles of the  $\mathbf{k}$  vectors and is almost everywhere zero except for  $k$  close to the Fermi level  $k_F$ . The magnitude of  $\lambda$  is not known but is expected to be of the same order as the spin-spin coupling constant  $\Gamma$  [21,22]. The crystalline electric field effect in transition metals is less than 0.1 meV which is small and hence the spin-orbit term  $\mathbf{S}_e \cdot \mathbf{L}_f$  is neglected. At room temperature all the rare-earth-metal ions studied in Ref. [12] are in their ground state making the term  $\mathbf{S}_f \cdot \mathbf{L}_f$  ineffective as damping mechanism. This follows immediately from the Wigner-Eckart theorem.

The Hamiltonian for the host transition-metal ions is based on the Anderson Hamiltonian with explicit spin rotational invariance in the absence of a Zeeman term [15,23,24]. It is

$$\mathcal{H}_d = \epsilon_d d_\sigma^\dagger d_\sigma + \sum_k V_{kd} (a_{k,\sigma}^\dagger d_\sigma + d_\sigma^\dagger a_{k,\sigma}) + \frac{U}{8} \rho^2 - \frac{U}{2} \mathbf{S}_d \cdot \mathbf{S}_d - \mu_d \cdot \mathbf{H}, \quad (4)$$

where  $\mathbf{S}_d$  is the spin operator of the local  $d$  electrons while their orbital angular momentum is assumed quenched.  $\rho$  is the charge density operator of the  $d$  electrons. In transition-metal ions such as Ni,  $V_{kd} \approx 1.0$ – $10.0$  eV is comparable to the Coulomb potential  $U$ . The hybridization term between the conduction and  $d$  electrons is essential to establish a *spin-independent* orbit-orbit coupling between the  $d$  and the  $f$  ions. The degree of localization of the magnetic moments increases with decreasing  $V_{kd}$  [25] and controls the extent to which rare-earth-metal impurities enhance damping.

The orbit-orbit coupling [cf. Eq. (3)] gives no contribution for  $\text{Gd}^{3+}(4f^7)$  as observed in the experiment [12]. As for the element Eu, it is believed from measurements of the paramagnetic susceptibilities that the ionic state is  $\text{Eu}^{2+}(4f^7)$  and not  $\text{Eu}^{3+}(4f^6)$  [19,26]. If this is the case

then clearly this is a state with  $\mathbf{L}_f = 0$  and it is the same as that of  $\text{Gd}^{3+}$ . Yb is also present in a double-ionized state [27] and therefore doping with  $\text{Yb}^{2+}(4f^{14})$  should not increase damping. This result remains to be confirmed by experiment. For Eu there is an additional reason why its angular momentum is quenched. The first excited state of this latter element lies only about 400 K above the ground state [27] and this can lift the degeneracy of the ground state. The average orbital angular momentum will therefore be zero even though  $L^2$  remains a good quantum number [28]. Hence our Hamiltonian from the outset reproduces the experimental results for Eu and Gd and predicts that doping with Yb should not change the damping. We next address the remaining rare-earth elements.

First, we outline the steps to derive the damping due to the orbit-orbit coupling term. We are only interested in the damping of the  $d$  moments of the transition metal; therefore, it is advantageous to adopt a functional integral approach. Since our system is near equilibrium and far from the Curie point, we use the spin wave approximation and expand the spin operators of the  $f$  moments in terms of Boson operators  $f^\pm$ , where  $f^\pm = S_f^y \pm iS_f^x$ . We keep only the first nontrivial terms. The integration of the conduction electrons is carried out exactly. Afterward we integrate the impurity variables,  $f$  and  $f^\dagger$ , also exactly but keep only quartic terms in  $d$  and  $d^\dagger$ . The remaining effective action has now only the fields  $d$  and  $d^\dagger$  and from their equations of motion the spin propagator  $\langle m^-(\tau) m^+(\tau') \rangle$  of the  $d$  moments,  $m^\pm = S_d^x \pm iS_d^y$ , can be determined. We use a Stratonovich-Hubbard transformation to write this effective Lagrangian in terms of  $m^\pm$ . Then a stationary phase approximation of the functional generator allows us to determine the desired propagator and hence the damping. We finally compare the functional form of this result to that of LLG and discuss why the electronic (itinerant) picture of the host transition-metal ions is essential.

The fundamental quantity in our calculation is the generating functional

$$\mathcal{Z}[\eta^*, \eta] = \text{Tr} e^{-\int_0^\beta d\tau \{\mathcal{H} - \eta^*(\tau) m^-(\tau) - \eta(\tau) m^+(\tau)\}}, \quad (5)$$

where  $\eta$  and  $\eta^*$  are external sources and  $\beta$  is inverse temperature. The propagator, i.e., the connected two-point Green's function, of the volume mode of the transition-metal ions is found by functional differentiations with respect to the external sources  $\eta^*$  and  $\eta$ ,  $\langle m^+(\tau) m^-(\tau') \rangle_c = \delta^2 \ln \mathcal{Z}[\eta^*, \eta] / \delta \eta(\tau) \delta \eta^*(\tau')$ . It is calculated within a double random phase approximation (RPA2) method. The *true* single particle propagator of the  $d$  bands is first found within a RPA in the presence of an effective field due to the conduction electrons and the impurities. In turn, the effect of the  $f$  impurities on the conduction electrons is calculated within RPA. The resulting effective Lagrangian is now written in terms of  $\mathbf{m}$  only

$$\mathcal{L} = -\frac{1}{2} m_{ij} \mathcal{K}_{ijkl} m_{kl} - \text{Tr} \ln [G_d^{-1} + \mathcal{K}m], \quad (6)$$

where  $G_d^{-1}(\sigma_1, \sigma_2) = \partial_\tau - \bar{\epsilon}_d + V^2 G_c + \text{Tr}_k \{ G_f G_c B G_c A \}$

is the propagator of the  $d$  electrons in the presence of the conduction electrons and the rare-earth-metal impurity ( $\sigma_i = 1, 2$  for spin-up and spin-down, respectively). The quadratic term in  $\mathbf{m}$  represents effective anisotropy and spin-charge interactions and is given by

$$\begin{aligned} \mathcal{K}_{\sigma_1\sigma_2\sigma_3\sigma_4} = & \frac{-U}{4}(\delta_{\sigma_1\sigma_2}\delta_{\sigma_3\sigma_4} - 2\delta_{1\sigma_1}\delta_{2\sigma_2}\delta_{1\sigma_3}\delta_{2\sigma_4}) - 2^2V^4G_f(G_cBG_cAG_c)_{\sigma_1\sigma_2}G_f(G_cBG_cAG_c)_{\sigma_3\sigma_4}\delta_{\sigma_1\sigma_4}\delta_{\sigma_2\sigma_3} \\ & - V^4G_cAG_cG_fG_cBG_c. \end{aligned} \quad (7)$$

Integrations over momentum and spin are implied in all these expressions. The different terms that appear in  $\mathcal{K}$  are as follows:  $G_c$  is the Green's function of the conduction electrons in the mean field approximation

$$G_c^{-1}(\mathbf{k}, \sigma_1, \mathbf{k}', \sigma_2, \tau) = (\partial_\tau + \bar{\epsilon}_{\mathbf{k}\sigma_1} - \mu_f)\delta_{\mathbf{k}\mathbf{k}'}\delta_{\sigma_1\sigma_2} + i\lambda(k, k')(2 - g_J)\langle J_f^z \rangle (k'_x k_y - k'_y k_x)\delta_{\sigma_1\sigma_2}, \quad (8)$$

which is off diagonal in momentum due to the orbit-orbit coupling.  $\bar{\epsilon}_{\mathbf{k},\sigma}$  now includes Zeeman terms due to the external field and the  $z$  component of the field due to impurity. The propagator  $G_f$  is that of the  $f$  ions in the presence of both the conduction electrons and the transition-metal ions,  $G_f^{-1}(\tau) = \partial_\tau + \mu_f H + \text{Tr}_{\mathbf{k},\sigma}\{G_cAG_cB\}$ . The  $A$  and  $B$  matrices are solely due to the presence of the impurity and represent the indirect coupling between the transition-metal ions and the  $f$  ions

$$A(\mathbf{k}', \sigma_1; \mathbf{k}, \sigma_2) = B(\mathbf{k}, \sigma_1; \mathbf{k}', \sigma_2)^* = \Gamma_0\sigma_{\sigma_1\sigma_2}^+ - i\lambda_0\Delta_{\mathbf{k}'\mathbf{k}}^+, \quad (9)$$

where we have set  $\Gamma_0 = \frac{\Gamma\sqrt{2J_f}}{4}(g_J - 1)$ ,  $\lambda_0 = \frac{\lambda\sqrt{2J_f}}{2}(2 - g_J)$ , and  $\Delta_{\mathbf{k}'\mathbf{k}}^\pm = (\hat{\mathbf{k}}' \times \hat{\mathbf{k}})_\pm$ . In the trace log term of the effective Lagrangian, the first nontrivial contribution is of order  $V^4$  and is given by Fig. 1. The diagram with a single insertion of an  $f$  propagator does not contribute due to the antisymmetry of the orbit-orbit coupling in the momentum space. Varying the effective action with respect to  $m_{ij}$  gives four equations which can be averaged and differentiated with respect to the external sources to get the  $\mathbf{m}$  propagators. We are only interested in  $\mathcal{C}(1221) = \langle m_{12}m_{21} \rangle$  which is given by

$$\{G_{d11}^{-1} + \mathcal{K}_{11ij}\langle m_{ij} \rangle\}\mathcal{C}(1221) + \mathcal{K}_{11ij}\mathcal{C}(ij21)\langle m_{12} \rangle = -\langle m_{22} \rangle - \mathcal{K}_{21ij}\mathcal{C}(ij21)\langle m_{22} \rangle - \mathcal{K}_{21ij}\langle m_{ij} \rangle\mathcal{C}(1221). \quad (10)$$

In the absence of impurities, these equations are to lowest order the time-dependent generalization of the Hartree-Fock equations derived by Anderson [23]. Using the RPA2 method, we solve for  $\mathcal{C}(1221)$

$$\mathcal{C}_{1221}(\omega_l) = \sum_n m_{11}(\omega_n)m_{22}(\omega_n + \omega_l) \left/ \left[ 1 + \sum_{n,m} \mathcal{K}_{2112}(\omega_m)m_{11}(\omega_n + \omega_m)m_{22}(\omega_n + \omega_m + \omega_l) \right] \right., \quad (11)$$

where  $\omega_l = (2l + 1)\pi/\beta$  for integer  $l$ . If we ignore the impurity interaction and replace the average values of the  $m_{ij}$  by the Anderson solution, we recover the RPA result for the propagator of the magnetization. To include the impurities, we evaluate the  $d$  propagators,  $m_{ij}$ , within RPA. In the low frequency limit,  $\omega \ll \Delta \ll \omega_c$ , we find that the (retarded) propagator  $C^R$  of the theory is proportional to  $(\omega - \omega_0 + i\alpha\omega)^{-1}$ . Here,  $\Delta^{-1}$  is the lifetime of the virtual  $d$  states [23],  $\omega_c$  denotes the frequency of the conduction electrons, and  $\omega_0$  is the ferromagnetic resonance frequency of the transition metal. This low frequency limit for the damping is similar to that of the LLG result [15]. The damping  $\alpha$  in the spin-conserving channel is proportional to  $J_f(J_f + 1)[(g_J - 2)|V|]^4$  and is given by

$$\begin{aligned} \alpha = & c|\lambda V|^4 J_f(J_f + 1)(2 - g_J)^4 \\ & \times \left[ \frac{U\Delta E}{2^5\pi^3(E - \Delta E)^2(E + \Delta E)^2} \frac{(nmk_F)^2}{18\omega_c^4} + Q(\omega_f) \right]. \end{aligned} \quad (12)$$

Here  $n$  is the density of conduction electrons,  $c$  is the concentration of the  $f$  impurities, and  $E \pm \Delta E$  is the energy of the up-down  $d$  states. These latter energies can be determined self-consistently as in the Anderson solution [23] and hence their form is not expected to depend

strongly on the atomic number of the rare-earth-metal impurity at low concentrations. The explicit form of the function  $Q$  is not needed here but it represents contributions beyond the ‘‘mean’’ field approximation of the  $f$  impurities and is given by Fig. 1. In Fig. 2, we show that the leading coefficient of the damping due to non-spin-flip scattering (solid curve) is in very good agreement with the experimental results of Bailey *et al.* [12].

Finally we point out the reasons behind insisting on using the itinerant electrons explicitly instead of the simpler  $s$ - $d$  exchange interaction which accounts well for damping in permalloy [16]. Using a localized-type Hamiltonian for the  $d$  moments

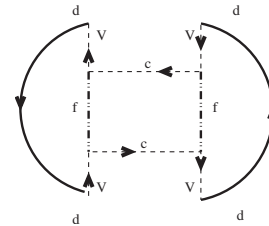


FIG. 1. The first diagram that is contributing to the damping of the  $d$  electrons due to the  $f$  impurities through the conduction electrons.

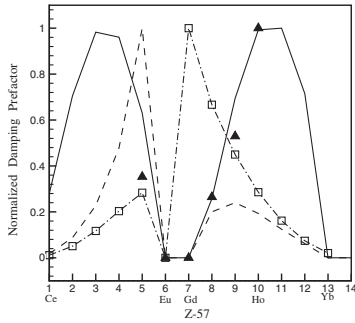


FIG. 2. Comparison of the normalized leading factor in the damping as a function of the rare-earth-metal impurity in Eq. (12) (solid line) and Eq. (13) (dashed line) to the data of Ref. [12]. The squares represent damping due to  $s$ - $f$  coupling only, Eq. (2), without the orbit-orbit coupling.

$$\mathcal{H}_d = -JS_e \cdot \mathbf{S}_d - \mu_d \cdot \mathbf{S}_d \quad (13)$$

instead of Eq. (4), leads to a damping which differs significantly from experiment (dashed curve in Fig. 2). This localized moment Hamiltonian, however, appears to describe well damping in insulators such as heavy rare-earth-metal-doped garnets [29]. In garnets, the hybridization coupling is smaller than in metals. Hence our result also explains why the damping in rare-earth-metal-doped garnets is not as strong as in the rare-earth-metal-doped transition metals. The experimental measurements (triangles) clearly show that at room temperature non-spin-flip scattering is more important than spin-flip scattering which only becomes important close to the critical temperature. Again, the data are well reproduced by the orbit-orbit coupling and the relatively large increase in damping is due to the large virtual mixing parameter  $V_{kd}$ . In contrast, the  $s$ - $f$  coupling (squares in Fig. 2) is in conflict with experiment.

In summary, we have shown that the damping in rare-earth-metal-doped transition metals is mainly due to an orbit-orbit coupling between the conduction electrons and the impurity ions. For near equilibrium conditions and in the low frequency regime this leads to damping for the uniform mode that is of Gilbert form. The orbit-orbit mechanism introduced here is much stronger than the spin-orbit based Elliott-Yafet-Kambersky mechanism since the charge-spin coupling at the host ion is of the order of 1–10 eV compared to 0.01 eV for spin-orbit coupling. The predicted increase of damping is proportional to  $V^4$  which in transition-metal ions is of the same order as  $U$  the Coulomb potential. A localized model for the  $d$  moments based on the  $s$ - $d$  exchange is unable to account for the increase in damping in these doped systems as a function of the orbital moment of the rare-earth-metal impurities. An additional test of this damping theory would be to measure the effect of a single rare-earth element on the damping in various transition metals. Such experiments will provide further insight into the dependence of damp-

ing on  $V$  and will improve our understanding of the itinerant versus localized pictures of magnetism.

We acknowledge fruitful discussions with P. Asselin, W. Bailey, O. Heinonen, P. Jones, O. Myarosov, and Y. Tserkovnyak.

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- [1] Th. Gerrits, H. A. M. van den Berg, J. Hohlfield, L. Bär, and Th. Rasing, *Nature (London)* **418**, 509 (2002).
- [2] H. W. Schumacher, C. Chappert, R. C. Sousa, P. P. Freitas, and J. Miltat, *Phys. Rev. Lett.* **90**, 017204 (2003).
- [3] S. I. Kiselev, J. C. Sankey, I. N. Krivorotov, N. C. Emley, R. J. Schoelkopf, R. A. Buhrman, and D. C. Ralph, *Nature (London)* **425**, 380 (2003).
- [4] S. Kaka, M. R. Pufall, W. H. Rippard, T. J. Silva, S. E. Russek, and J. A. Katine, *Nature (London)* **437**, 389 (2005).
- [5] E. Beaurepaire, J. C. Merle, A. Daunois, and J. Y. Bigot, *Phys. Rev. Lett.* **76**, 4250 (1996).
- [6] J. Hohlfield, E. Matthias, R. Knorren, and K. H. Bennemann, *Phys. Rev. Lett.* **78**, 4861 (1997).
- [7] L. D. Landau and E. M. Lifshitz, *Phys. Z. Sowjetunion* **8**, 153 (1935); T. L. Gilbert, *Phys. Rev.* **100**, 1243 (1955).
- [8] Z. Li and S. Zhang, *Phys. Rev. B* **69**, 134416 (2004).
- [9] M. Vomir, L. H. F. Andrade, L. Guidoni, E. Beaurepaire, and J. Y. Bigot, *Phys. Rev. Lett.* **94**, 237601 (2005).
- [10] B. Koopmans, J. J. M. Ruigrok, F. D. Longa, and W. J. M. de Jonge, *Phys. Rev. Lett.* **95**, 267207 (2005).
- [11] E. M. Ryan *et al.*, B22.00003, March APS Meeting, Baltimore, Maryland, 2006.
- [12] S. G. Reidy, L. Cheng, and W. E. Bailey, *Appl. Phys. Lett.* **82**, 1254 (2003).
- [13] R. J. Elliott, *Phys. Rev.* **96**, 266 (1954).
- [14] V. Kambersky, *Can. J. Phys.* **48**, 2906 (1970).
- [15] R. E. Prange and V. Korenman, *Phys. Rev. B* **19**, 4691 (1979); **19**, 4698 (1979).
- [16] S. Ingvarsson *et al.*, *Phys. Rev. B* **66**, 214416 (2002).
- [17] B. Heinrich, D. Fraitova, and V. Kambersky, *Phys. Status Solidi* **23**, 501 (1967).
- [18] A. Rebei and M. Simionato, *Phys. Rev. B* **71**, 174415 (2005).
- [19] P.-G. de Gennes, *Compt. Rend.* **247**, 1836 (1958); *J. Phys. Radium* **23**, 510 (1962); P.-G. de Gennes, C. Kittel, and A. M. Portis, *Phys. Rev.* **116**, 323 (1959).
- [20] J. Kondo, *Prog. Theor. Phys.* **27**, 772 (1962).
- [21] J. H. Van Vleck and R. Orbach, *Phys. Rev. Lett.* **11**, 65 (1963); R. C. LeCraw, W. G. Nilsen, and J. P. Remeika, and J. H. Van Vleck, *Phys. Rev. Lett.* **11**, 490 (1963).
- [22] P. Levy, *Phys. Rev. Lett.* **20**, 1366 (1968).
- [23] P. W. Anderson, *Phys. Rev.* **124**, 41 (1961).
- [24] C. A. Macedo, M. D. Coutinho-Filho, and M. A. de Moura, *Phys. Rev. B* **25**, 5965 (1982).
- [25] J. R. Schrieffer, *J. Appl. Phys.* **38**, 1143 (1967).
- [26] B. T. Thole *et al.*, *Phys. Rev. B* **32**, 5107 (1985).
- [27] R. J. Elliott, *Magnetic Properties of Rare Earth Metals* (Plenum, New York, 1972).
- [28] C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 2005).
- [29] P. E. Seiden, *Phys. Rev.* **133**, A728 (1964).