Luttinger Liquid at the Edge of Undoped Graphene in a Strong Magnetic Field

H. A. Fertig[1] and L. Brey[2]
[1]Department of Physics, Indiana University, Bloomington, Indiana 47405, USA
[2]Instituto de Ciencia de Materiales de Madrid (CSIC), Cantoblanco, 28049 Madrid, Spain
(Received 8 April 2006; published 13 September 2006)

We demonstrate that an undoped two-dimensional carbon plane (graphene) whose bulk is in the integer quantum Hall regime supports a nonchiral Luttinger liquid at an armchair edge. This behavior arises due to the unusual dispersion of the noninteracting edge states, causing a crossing of bands with different valley and spin indices at the edge. We demonstrate that this stabilizes a domain wall structure with a spontaneously ordered phase degree of freedom. This coherent domain wall supports gapless charged excitations, and has a power law tunneling *I-V* with a nonintegral exponent. In proximity to a bulk lead, the edge may undergo a quantum phase transition between the Luttinger liquid phase and a metallic state.

DOI: 10.1103/PhysRevLett.97.116805 PACS numbers: 71.10.Pm, 73.20.−r, 73.22.Gk, 73.43.Jn

Introduction.—Two-dimensional carbon sheets [1], known as graphene, are emerging as one of the most exciting new systems supporting the quantum Hall effect. This material is different than more standard two-dimensional electron gases (2DEGs) because in the absence of a magnetic field the single particle spectrum is linear in the vicinity of two inequivalent points in the Brillouin zone. The low-energy states near these points are described by the Dirac equation [2], and in a strong magnetic field the quantum Hall steps that emerge are shifted relative to standard 2DEGs [3,4]. This effect arises because the spectrum of the Dirac equation with a magnetic field has doubly degenerate Landau levels (LLs) for each spin, with one pair at zero energy, half of which are filled in the nominally undoped situation, yielding a shifted step pattern in the Hall conductance.

In an undoped standard 2DEG system, there is little interesting electron physics because the filled valence states are far below the chemical potential. By contrast, the partially filled LLs at zero energy in graphene allow for interesting low-temperature physics even in this nominal ‘‘vacuum’’. When interactions are included, the half-filled zero energy states represent a multicomponent system, which in the absence of spin or valley splitting potentials, and ignoring small symmetry-breaking terms due to the lattice structure [5], spontaneously polarizes due to exchange [6,7]. In this situation, the vacuum is a quantum Hall ferromagnet, with an associated low-energy spin wave.

In addition to these surprising bulk properties, graphene also has an unusual edge structure even in a noninteracting picture [8]. This is illustrated in Fig. 1, which shows the tight-binding energy levels for the p_z orbitals of a narrow graphene ribbon with ‘‘armchair edges,’’ illustrated in the inset, and a perpendicular magnetic field. The electronic states with momenta k_y are approximately localized around guiding center positions $X = k_y \ell^2$, with $\ell = \sqrt{\hbar c/eB}$, and B the magnetic field. The degenerate levels near the center of the figure may be identified as Landau level states. Besides X, these states have two internal quantum numbers, a ‘‘valley’’ index with two distinct values, and the spin index. When X approaches an edge, the levels disperse as is apparent in Fig. 1, with the lowest Landau levels supporting both upward (particlelike) dispersing states and downward (holelike) dispersing states [8]. Because the Fermi level for undoped graphene—the ‘‘graphene vacuum’’—lies precisely at energy $\varepsilon = 0$, the character of the filled states change as X approaches the edge [9], as may be explicitly seen in Fig. 1.

In this work, we demonstrate a remarkable effect when this edge structure and the quantum Hall ferromagnetism are both taken into account. Under appropriate circum-

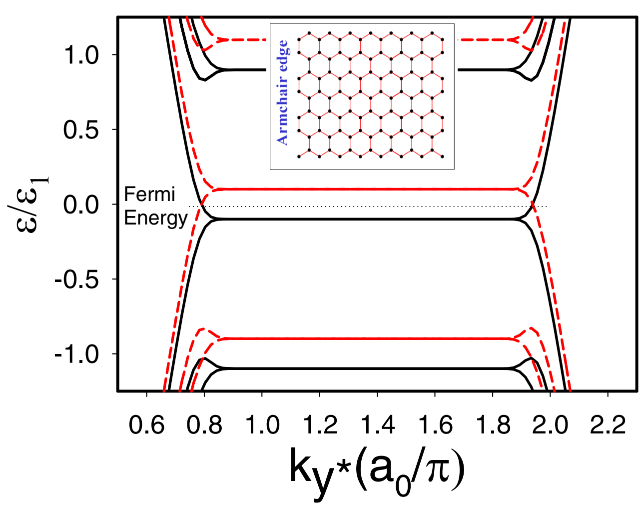

FIG. 1 (color online). Energy bands for electrons in a graphene ribbon with armchair edges in a magnetic field from tight-binding calculations. a_0 = lattice constant, unit of energy $\varepsilon_1 = \sqrt{2}\gamma a_0/\ell$. $k_y = X/\ell^2$ with X the guiding center coordinate. $B = 100$ T; ribbon width is 460 Å. Solid black lines are spin up states, red dashed lines are spin down. Inset: Graphene ribbon with armchair edges.

stances undoped graphene forms a coherent domain wall (DW) between the spin-polarized state in the bulk and an unpolarized region at the edge. The low-energy theory of this DW has a U(1) symmetry with a Luttinger liquid Hamiltonian [10]. More specifically, the DW may be described by a variational wave function of the form

$$|\Psi\rangle = \prod_{X<L}\left(\cos\frac{\theta(X)}{2} + \sin\frac{\theta(X)}{2}e^{i\phi}C^{\dagger}_{-\downarrow X}C_{+\uparrow X}\right)|\mathrm{Vac}\rangle, \quad (1)$$

where $C^{\dagger}_{+\uparrow X}$ and $C^{\dagger}_{-\downarrow X}$ create electrons in the two levels closest to the Fermi energy, $X < L$ for an edge located at L, $|\mathrm{Vac}\rangle$ denotes the bulk undoped graphene state (i.e., vacuum) which is partially polarized since the two spin up lowest LLs are fully occupied, and $\theta(X)$ and ϕ are variational parameters. An example of $\theta(X)$ found by minimizing the effective energy functional is illustrated in Fig. 2. The energy of the state is independent of ϕ, indicating a spontaneously broken symmetry in the DW ground state, with an associated gapless collective mode which may be understood as states in which ϕ has a spatial gradient [11–13]. Gradients in ϕ carry a charge density, and a full 2π rotation contains a single electron above the vacuum [14,15]. Thus this system supports *gapless* charged excitations.

This coherent DW may be probed by tunneling into it from a bulk metallic lead [16]. For a standard 2DEG, in the undoped case the system trivially behaves as an insulator, and for integer quantum Hall edge states, one finds a metallic response [16]. For the coherent DW, we expect a power law tunneling I-V, with exponent determined by the pseudospin stiffness and the strength of the confinement potential. This is quite different from standard quantum Hall edges, where the tunneling exponent is thought to be set by the bulk filling factor [17]. By varying the strength of the electron-electron interaction (for example, by a screening gate) or the edge confinement potential, one can vary the tunneling exponent, and in principle may drive the system through a quantum phase transition in which the tunneling perturbation becomes relevant in the renormalization group sense. This presumably drives the system into a metallic state with a linear I-V. We now provide details of these results.

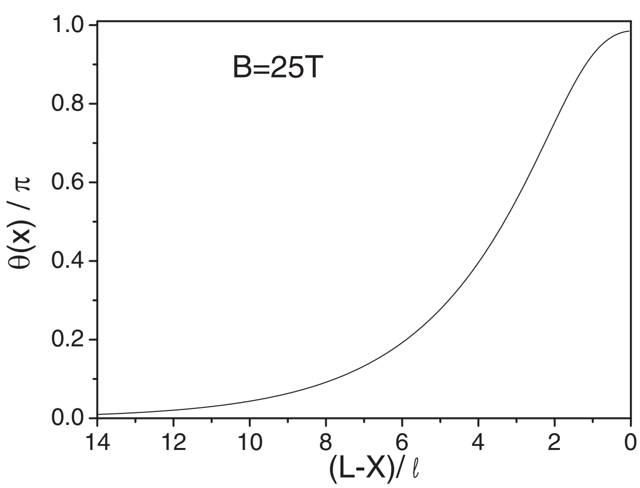

FIG. 2. Example of a domain wall configuration. $\cos[\theta(X)/2]$ ($\sin[\theta(X)/2]$) denotes the amplitude for an electron to occupy a spin up electronlike (spin down holelike) single particle state near the edge.

Pseudospin ferromagnetism in graphene.—We begin with the noninteracting spectrum of the graphene ribbon illustrated in Fig. 1. We denote the spin and valley degrees of freedom, respectively, by $\sigma = \pm\frac{1}{2}$ and $\tau = \pm 1$. In the undoped system, all the negative energy states and two of the zero energy states [18] are filled at zero temperature. In what follows, we will ignore the LLs well away from zero energy, since these are either completely filled or empty.

Retaining just the four lowest Landau levels (LLLs) near zero energy, apart from constant terms the Hamiltonian may be written as

$$H = \sum_{\tau,\sigma,X}[-E_z\sigma C^{\dagger}_{\tau\sigma X}C_{\tau\sigma X} + \Delta(X)\tau C^{\dagger}_{\tau\sigma X}C_{\tau\sigma X}] + \frac{N_\phi^2}{2S}\sum_{\sigma,\sigma',\tau,\tau',\mathbf{q}} e^{-q^2\ell^2/2}V_{\mathbf{q}}\rho_{\tau,\sigma}(-\mathbf{q})\rho_{\tau',\sigma'}(\mathbf{q}). \quad (2)$$

In this expression, $E_Z = g\mu_B B$ is the Zeeman coupling, $\Delta(X)$ is the energy splitting produced by the edge [8], N_ϕ/S is the number of flux quanta per unit area passing through the system, and $V_{\mathbf{q}} = \frac{2\pi e^2}{\epsilon_0 q}$ is the Coulomb interaction. Note that we have assumed an SU(4) symmetric form for the interaction. Although not exact this should be an excellent approximation for LLL states provided the magnetic length is much larger than the lattice constant [2]. Finally, the LLL density operators have the form $\rho_{\tau,\sigma}(\mathbf{q}) = \frac{1}{N_\phi}\sum_X e^{-(i/2)q_x(2X+q_y)}C^{+}_{\tau\sigma X}C_{\tau\sigma X+q_y}$.

The vacuum state (i.e., undoped ground state) involves filling two of the four LLLs. For $\Delta = E_z = 0$, any two orthogonal states will yield the same energy. In the bulk (where $\Delta = 0$) the Zeeman coupling breaks this symmetry, and in the Hartree-Fock approximation the vacuum state may be written in the form

$$|\mathrm{Vac}\rangle = \prod_X C^{\dagger}_{+\uparrow X}C^{\dagger}_{-\uparrow X}|n<0\rangle, \quad (3)$$

where $|n < 0\rangle$ denotes the state with all the negative energy levels filled. Our vacuum is thus ferromagnetic [19]. The energy per particle of the ground state is $-E_z + \frac{1}{2}\Sigma$ with $\Sigma = -\sqrt{\frac{\pi}{2}}\frac{e^2}{\epsilon_0\ell}$ the electron self-energy. Note that although this energy scale is much larger than E_z, within our model E_z picks out which two states are occupied.

The ferromagnetic vacuum supports four collective neutral excitations, which may be approximately generated by applying the operators

$$\rho_{\tau,\tau'\downarrow,\uparrow}(\mathbf{q}) = \frac{1}{N_\phi}\sum_X e^{-(i/2)q_x(2X+q_y)} C^+_{\tau\downarrow X} C_{\tau'\uparrow X+q_y} \quad (4)$$

to $|\text{Vac}\rangle$. Each of these involves a spin flip, and two of them also include a valley density wave. Because of the SU(4) symmetry of the interactions, all four excitations are degenerate with energy at long wavelengths $\omega_0(q) \approx 2E_z + 4\pi\rho_s q^2\ell^2$, with $\rho_s = 1/16\sqrt{2\pi}$ in units of $e^2/\epsilon_0\ell$. This is precisely the form one expects for a Heisenberg ferromagnet.

Coherent domain wall at the edge.—As discussed above, near the edge of the system an electronlike edge state crosses a holelike edge state at the Fermi level. In a noninteracting system this would lead to a sharp change in the internal state of the electrons as one approaches the edge. However, since the system bears ferromagnetic properties such a change would induce a large exchange energy cost. Thus the ground state does not precisely adhere to the lowest energy single particle state, and changes in a more continuous fashion. If we regard the amplitudes of the two states closest to the Fermi energy as components of an effective spin-1/2 pseudospin, we can regard the transition region as a DW in the effective magnetization of this pseudospin. Equation (1) describes this DW when $\theta(X)$ passes from 0 to π. The energy of this state can be shown to have the form [20,21]

$$E \simeq \pi\ell^2\rho_s \sum_{X<L}\left(\frac{d\theta}{dx}\right)^2 + \sum_{X<L}[\Delta(X) - E_z]\cos\theta(X),$$

provided $\theta(X)$ does not vary rapidly on the scale of ℓ. This sine-Gordon-like energy functional may be straightforwardly minimized numerically [21], yielding results such as those illustrated in Fig. 2.

The appearance of a DW in this system may be regarded as a kind of edge reconstruction. We note that for an appropriately chosen edge potential, a standard 2DEG can support other types of edge reconstruction [22], driven by the Hartree interaction which favors a slowly varying edge density. The DW by contrast is driven by the electronic structure and will be present for a sharply defined graphene edge, and as we shall see has different tunneling characteristics.

Because the energy of $|\Psi\rangle$ is independent of ϕ, it is immediately clear that this represents a broken symmetry state which must support a gapless mode. This may be described by an effective action

$$S_0 = \int d\tau dy\left[\frac{\Gamma}{2}m(y,\tau)^2 + \frac{\tilde\rho}{2}\left(\frac{\partial\phi}{\partial y}\right)^2 + im(y,\tau)\left(\frac{\partial\phi}{\partial\tau}\right)\right] \quad (5)$$

where τ is imaginary time. The dependence of S_0 only on gradients of ϕ guarantees that there will be a gapless mode dispersing linearly with k_y, the wave vector along the domain wall. The field m is the conjugate variable to ϕ and is a measure of fluctuations in the difference of particle number in the two levels of the effective pseudospin, $(+\uparrow)$ and $(-\downarrow)$. This is controlled by the position of the DW relative to the edge of the system, so m may be qualitatively viewed as a displacement variable. The constants Γ and $\tilde\rho$ may be estimated from $\theta(X)$ and $E_z - \Delta(X)$ using spin-wave theory [13,21].

It is well-established that spin-textures in quantum Hall ferromagnets carry real physical charge densities [14]. If we use the variables $\theta(\mathbf{r})$ and $\phi(\mathbf{r})$ to specify the direction of the local pseudospin, charge density fluctuations may be written as $\delta\rho(\mathbf{r}) = \frac{e}{4\pi}[\partial_y\phi(\mathbf{r})\partial_x\cos\theta(\mathbf{r}) - \partial_x\phi(\mathbf{r})\partial_y\cos\theta(\mathbf{r})]$. In the region of the DW [15], $\int dx\partial_x\cos\theta(\mathbf{r}) = \int dx\partial_x\cos\theta(x) = 2$, so $\delta\rho(\mathbf{y}) = \int dx\delta\rho(\mathbf{r}) = \frac{e}{2\pi}\partial_y\phi(y)$ [23]. Thus if $\phi(y)$ overturns by $\pm 2\pi$ along the DW, charge $\pm e$ accumulates. This phase overturn can be spread over the entire length of the DW, leading to an arbitrarily small energy $\frac{\tilde\rho}{2}\int dy(\partial_y\phi)^2$, so that charge can be introduced into the DW at arbitrarily low energy. This has important implications for tunneling.

Tunneling from a metallic electrode.—The geometry for tunneling into the DW is illustrated in Fig. 3. We assume the electrons in the lead are in a Fermi liquid state and can be modeled as noninteracting electrons in a magnetic field, and support field operators $\Phi^\dagger_\sigma(x,y,z)$ which create electrons of spin σ at (x, y, z). Choosing our origin of coordinates such that the DW is located at $x = 0$, and the surface of the lead at $z = 0$, the imaginary time action for tunneling between the lead and the DW may be written as

$$S_{\text{tun}} = \frac{t}{\beta}\sum_{\omega_n,\sigma}\int dy\Phi^*_\sigma(x=0,y,z=0;\omega_n)\psi_\sigma(y,\omega_n) + \text{H.c.},$$

where ψ_σ is the annihilation operator for an electron of spin σ in the domain wall and t is the tunneling amplitude. Using standard bosonization [17], one may write the fermion operator in the form $\psi_{(\uparrow,\downarrow)}(y,\tau) \sim e^{(+/-)i\phi(y,\tau)/2}e^{i2\pi\int_{-\infty}^x dy' m(y',\tau)}$. After tracing out the lead degrees of freedom (details will be presented elsewhere

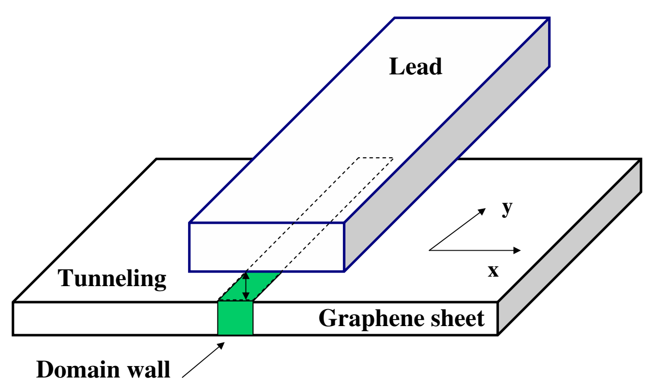

FIG. 3 (color online). Geometry for tunneling into the coherent domain wall.

[21]), one finds the partition function may be written in the form $Z \propto \int \mathcal{D}\phi \mathcal{D}m e^{-S_0-\tilde{S}}$, with

$$\tilde{S} \approx -t^2 \sum_\sigma \int_0^\beta d\tau_1 d\tau_2 \int dy \psi^*_\sigma(y, \tau_1) K(\tau_1 - \tau_2) \times \psi_\sigma(y, \tau_2). \quad (6)$$

Taking the zero temperature limit, one may show $K \sim 1/(\tau_1 - \tau_2)$ for large $|\tau_1 - \tau_2|$.

Our first question is whether $\tilde{S}$ qualitatively affects the state of the system; i.e., is it a relevant operator? A perturbative renormalization group analysis may be applied to $\tilde{S}$ [21], leading to the result

$$\frac{dt^2}{d\ell} = -(\kappa - 2)t^2$$

with the anomalous dimension $\kappa = (x + 1/x)/2$, and $x = 4\pi\sqrt{\tilde{\rho}/\Gamma}$. Estimates of $\tilde{\rho}$ and Γ using a spin-wave approach [21] yield $\kappa \approx 6.8$, 6.0, and 5.3 for $B = 15$, 25, and 45 T, respectively. This indicates that under usual conditions, $\tilde{S}$ is irrelevant, and the DW remains in a Luttinger liquid phase. The physical reason for this is that the confinement energy (Γ) of the DW is small compared to the stiffness of the phase angle ($\tilde{\rho}$), because the Zeeman energy which sets the scale of Γ is small compared to the electron-electron energy scale. This suggests that enhancing Γ can drive the system into a state in which $\tilde{S}$ is relevant, perhaps by judicious use of a gating geometry. In this situation the coupling to the metallic lead becomes important in the low-energy physics, and presumably a current injected into the domain wall will behave metallically.

The irrelevance of $\tilde{S}$ under ambient conditions indicates that we can compute the tunneling conductance perturbatively [10,24]. The resulting expression involves the domain wall correlation function $\langle\psi(y, \tau)\psi^\dagger(y, 0)\rangle \sim 1/\tau^\kappa$, whose form leads to a power law *I-V* with an exponent set by κ. It should be emphasized that this differs considerably from edge state tunneling in standard 2DEGs, where the exponent is set by the bulk filling factor, even for reconstructed edges when backscattering due to disorder is taken into account [17]. Such backscattering is not possible with nonmagnetic impurities in the DW, because scattering from an electronlike to a holelike state requires a spin flip. The anomalous exponent of the DW is robust, and is sensitive to the edge potential through Γ. In these ways the Luttinger liquid properties of the DW are distinct from those of standard quantum Hall edge states.

The authors thank F. Guinea and C. Tejedor for useful discussions. This work was supported by MAT2005-07369-C03-03 (Spain) (L. B.) and by the NSF through Grant No. DMR-0454699 (H. A. F.).

[1] K. S. Novoselov *et al.*, Science **306**, 666 (2004).
[2] For recent reviews, see T. Ando, J. Phys. Soc. Jpn. **74**, 777 (2005); N. M. R. Peres, F. Guinea, and A. H. Castro Neto, Phys. Rev. B **73**, 125411 (2006), and references therein.
[3] Y. Zheng and T. Ando, Phys. Rev. B **65**, 245420 (2002); V. P. Gusynin and S. G. Shaparov, Phys. Rev. Lett. **95**, 146801 (2005); A. H. Castro Neto, F. Guinea, and N. M. R. Peres, Phys. Rev. B **73**, 205408 (2006).
[4] K. S. Novoselov *et al.*, Nature (London) **438**, 197 (2005); Y. Zhang *et al.*, Nature (London) **438**, 201 (2005).
[5] See J. Alicea and M. P. A. Fisher, cond-mat/0604601. Note that the domain wall described here will exist even with moderate symmetry-breaking interactions, provided the bulk of the system is in a spin-polarized state. This is always possible for a sufficiently large Zeeman energy.
[6] For a review, see article by S. M. Girvin and A. H. MacDonald in *Perspectives in Quantum Hall Effects*, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1997).
[7] Spontaneous polarization at higher fillings may recently have been observed. See Y. Zhang *et al.*, Phys. Rev. Lett. **96**, 136806 (2006).
[8] L. Brey and H. A. Fertig, Phys. Rev. B **73**, 195408 (2006).
[9] L. Brey, Bull. Am. Phys. Soc. **51**, 459 (2006); D. A. Abanin *et al.*, Phys. Rev. Lett. **96**, 176803 (2006).
[10] G. Mahan, *Many-Particle Physics* (Plenum, New York, 2000).
[11] H. A. Fertig, Phys. Rev. B **40**, 1087 (1989).
[12] L. Brey and H. A. Fertig, Phys. Rev. B **62**, 10 268 (2000);
[13] R. Côté and H. A. Fertig, Phys. Rev. B **65**, 085321 (2002).
[14] D. H. Lee and C. L. Kane, Phys. Rev. Lett. **64**, 1313 (1990).
[15] V. Fal'ko and S. V. Iordanskii, Phys. Rev. Lett. **82**, 402 (1999).
[16] F. P. Milliken, C. P. Umbach, and R. A. Webb, Solid State Commun. **97**, 309 (1996); A. M. Chang, L. Pfeiffer, and K. W. West, Phys. Rev. Lett. **77**, 2538 (1996).
[17] For a review, see article by C. L. Kane and M. P. A. Fisher in Ref. [6].
[18] The states are precisely at zero energy only for vanishing Zeeman coupling.
[19] Quantum Hall ferromagnetism in doped, disordered graphene has recently been examined in K. Nomura and A. H. MacDonald, Phys. Rev. Lett. **96**, 256602 (2006).
[20] R. Côté, L. Brey, H. Fertig, and A. H. MacDonald, Phys. Rev. B **51**, 13 475 (1995).
[21] L. Brey and H. A. Fertig (unpublished).
[22] C. Chamon and X. G. Wen, Phys. Rev. B **49**, 8227 (1994); A. Karlhede *et al.*, Phys. Rev. Lett. **77**, 2061 (1996).
[23] Because of this one might add to Eq. (5) a term of the form $\int dy dy' \delta\rho(y)\delta\rho(y')/|y - y'|$. We assume, however, that this long-range potential is screened, for example, by the lead in Fig. 3.
[24] J. Palacios and H. A. Fertig, Phys. Rev. Lett. **79**, 471 (1997).