Decay of Quantum Correlations in Atom Optics Billiards with Chaotic and Mixed Dynamics

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We perform echo spectroscopy on ultracold atoms in atom-optics billiards to study their quantum dynamics. The detuning of the trapping laser is used to change the ''perturbation'', which causes a decay in the echo coherence. Two different regimes are observed: first, a perturbative regime in which the decay of echo coherence is nonmonotonic and partial revivals of coherence are observed in contrast with the predictions of random matrix theory. These revivals are more pronounced in traps with mixed dynamics as compared to traps where the dynamics is fully chaotic. Next, for stronger perturbations, the decay becomes monotonic and independent of the strength of the perturbation. In this regime no clear distinction can be made between chaotic traps and traps with mixed dynamics.

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The quantum manifestations of different types of classical dynamics is still an issue of unsettled debate, since the unitarity of quantum mechanics is inconsistent with the classical definition of chaos, i.e., an exponential sensitivity to initial conditions. Peres suggested that a signature of quantum chaos is the stability of the dynamics with respect to small changes in the Hamiltonian [1]. In this framework, the ''fidelity'' denotes the overlap between a state evolved by a Hamiltonian $H_\uparrow$ with the same state evolved by a slightly perturbed Hamiltonian $H_\downarrow$ [1]. Fidelity is often denoted as the Loschmidt Echo, since it is equivalent to the overlap between an initial state and the same state evolved forward in time in $H_\uparrow$ and then backwards in time in $H_\downarrow$ [2].

The decay of fidelity in chaotic systems and its dependence on the underlying classical dynamics and the strength and type of the perturbation have been the topic of intense theoretical studies in recent years (see Refs. [1–3] and references therein). However, experimental studies of chaotic systems are still lacking and so are both theoretical and experimental investigations of systems with mixed dynamics. Of special interest are ''intermediate'' regimes for which both quantum dynamics and classical models can be used. Understanding these regimes can shed light on the relation between classical and quantum properties of the system.

Ultracold atoms have been used in the past to experimentally study both quantum and classical dynamics. Quantum dynamics have been studied in driven 1D systems, where a broad variety of phenomena such as dynamical localization [4], dynamical tunneling [5], and quantum accelerator modes [6] have been demonstrated. Classical dynamics of cold atoms has been studied in gravitational wedge billiards [7] and in soft wall atom-optics billiards [8]. In these experiments regular, chaotic, and mixed dynamics were observed.

In this Letter we use microwave (MW) ''echo spectroscopy'' [9] to experimentally measure quantum dynamics of ultracold ^{85}Rb atoms trapped in atom-optics billiards, with underlying chaotic or mixed classical dynamics. Echo spectroscopy measures the overlap between two initially identical states evolved in a slightly different Hamiltonian (and is therefore closely related to the fidelity [10]), but it overcomes the need for narrow wave packet analysis and allows the use of thermal ensembles for the study of quantum dynamics of extremely high (up to $\sim 10^8$) excited states [9]. For such high lying states quantum dynamics can be essentially irreversible even for a closed system, such as ours. We demonstrate that for our trap the decay of the echo coherence displays qualitative distinct regimes, depending on perturbation strength. For weak perturbations, we observe a ''perturbative'' regime in which the decay depends on the perturbation strength and where partial coherence (wave packet) revivals are seen even when the underlying classical dynamics is fully chaotic. This observation is opposed to the monotonic decay of correlations, predicted by random matrix theory, reflecting a generic property of the perturbations associated with the traps.

For stronger perturbations we observe a crossover to a regime where the decay of the coherence is independent of the perturbation strength. In this case the decay is monotonic and no revivals are observed. This observation is explained in terms of a simple semiclassical model.

For traps where the classical motion exhibits a mixed phase space (i.e., stable ''islands'' in a chaotic ''sea'') we observe more pronounced revivals in the perturbative regime, due to the periodic classical orbits in the islands. However, a perturbation-independent regime exists also for these systems, similar to that of traps with fully chaotic dynamics.

We use ^{85}Rb atoms in a coherent superposition of their two magnetic-insensitive Zeeman substates of the ground state. These two levels, $|5S_{1/2}, F = 2, m_F = 0\rangle$ denoted $|\downarrow\rangle$, and $|5S_{1/2}, F = 3, m_F = 0\rangle$ denoted $|\uparrow\rangle$, are separated by the hyperfine (HF) energy splitting $E_{\mathrm{HF}} = \hbar\omega_{\mathrm{HF}}$ with $\omega_{\mathrm{HF}} = 2\pi \times 3.036 \times 10^9\ \mathrm{s}^{-1}$. The atoms are trapped in a dipole potential formed by a linearly polarized laser. The dipole potential is inversely proportional to the trap laser detuning Δ_L from the $5S_{1/2} \to 5P_{3/2}$ line, hence it is slightly different for atoms in $|\uparrow\rangle$ and $|\downarrow\rangle$. The external (center of mass) potential depends on the internal (spin) state, hence the internal and external degrees of freedom cannot be separated, and the entire Hamiltonian (neglecting interactions between the atoms) is written as

$$H = H_\downarrow|\downarrow\rangle\langle\downarrow| + (H_\uparrow + E_{\mathrm{HF}})|\uparrow\rangle\langle\uparrow| = \left[\frac{p^2}{2m} + V_\downarrow(\mathbf{x})\right]|\downarrow\rangle\langle\downarrow| + \left[\frac{p^2}{2m} + V_\uparrow(\mathbf{x}) + E_{\mathrm{HF}}\right]|\uparrow\rangle\langle\uparrow|, \quad (1)$$

where $V_\downarrow$ and $V_\uparrow$ are the external potentials for atoms in states $|\downarrow\rangle$ and $|\uparrow\rangle$, respectively. These potentials include the gravitational potential, equal for both states, and the dipole potential, which can be written as $V_{d,\downarrow}$ and $V_{d,\uparrow} = (1+\epsilon)V_{d,\downarrow}$, where $\epsilon \equiv \omega_{\mathrm{HF}}/\Delta_L$ is the perturbation strength typically 10^{-3}–10^{-2} in our experiments [9]. For such weak perturbations the transition from quantum (perturbative) to classical behavior can be studied for very high lying exited states.

The eigenenergies of this Hamiltonian consist of two manifolds (belonging to $|\downarrow\rangle$ and $|\uparrow\rangle$) separated in energy by E_{HF}. The atoms are initially prepared in their internal ground state $|\downarrow\rangle$ and their total wave function can be written as $\Psi = |\downarrow\rangle \otimes \psi$, where ψ represents the center of mass part of their wave function. The echo sequence consists of three short and strong MW pulses, each of which changes only the internal state of the atoms [9]. First a $\pi/2$ pulse puts the atoms into a coherent superposition of $|\downarrow\rangle$ and $|\uparrow\rangle$. After a time τ a π pulse inverts the populations and after another time τ the atoms are irradiated by a second $\pi/2$ pulse. The populations of $|\downarrow\rangle$ and $|\uparrow\rangle$ are then measured. If we start with an eigenstate $|n_\downarrow\rangle$ of $H_\downarrow$ then the population of $|\uparrow\rangle$ after the echo pulse sequence is [9] $P_\uparrow = \frac{1}{2}[1 - \mathrm{Re}(F_{\mathrm{echo}})]$, where $F_{\mathrm{echo}} = \langle n_\downarrow|e^{iH_\downarrow\tau/\hbar}e^{iH_\uparrow\tau/\hbar}e^{-iH_\downarrow\tau/\hbar}e^{-iH_\uparrow\tau/\hbar}|n_\downarrow\rangle$ is denoted the "echo amplitude". $F_{\mathrm{echo}} = 1$ indicates perfect coherence and yields $P_\uparrow = 0$ and $F_{\mathrm{echo}} = 0$, yielding $P_\uparrow = 0.5$ indicates complete loss of coherence. If $\epsilon = 0$ then internal and center of mass motion degrees of freedom decouple and $F_{\mathrm{echo}} = 1$ for all times. When considering eigenstates the echo amplitude can be written as a time correlation function $F_{\mathrm{echo}} = e^{i\omega_{n,\downarrow}\tau}\langle\varphi_n(t=0) \mid \varphi_n(t=\tau)\rangle$, where $|\varphi_n(t=0)\rangle \equiv e^{-iH_\uparrow\tau/\hbar}|n_\downarrow\rangle$ and $|\varphi_n(t=\tau/\hbar)\rangle \equiv e^{-iH_\downarrow\tau}|\varphi_n(t=0)\rangle$. Therefore, the decay of the echo coherence corresponds to a decay of quantum correlations due to dynamics in the trap, while it is insensitive to summation over the thermal ensemble.

In our experiment ^{85}Rb atoms are loaded into a far off resonance optical trap, cooled to a temperature of 20 μK, and optically pumped into the $F = 2$ hyperfine state. By changing the detuning of the trap laser, and simultaneously adjusting its power, the perturbation strength is controlled, without changing the trapping potential. After the MW echo pulses $P_\uparrow$, the population of state $|\uparrow\rangle$, is measured using fluorescence detection, and the signal is normalized to $P_\uparrow$ after a short π pulse.

The trap is a light-sheet wedge billiard [7], made from two crossed blue detuned light sheets defining the billiard walls and where gravity confines the atoms in the vertical direction [11]. The light sheets have $(1/e^2)$ dimensions of 20×250 μm, and by the use of cylindrical lenses mounted on rotational stages, the wedge angle is adjusted in order to control the classical dynamics. The very elongated shape of the trap allows us to neglect the longitudinal motion, which has a time scale much longer than the experiment time. In this experiment the atoms typically occupy many (up to $\sim 10^8$) states in the trap. The measured echo signal is the ensemble average of all of them.

As predicted [12] and previously measured [7] the classical dynamics in the wedge billiard is determined by its vertex half-angle α. Figures 1(b) and 1(d) present the Poincaré surface of section for the two wedge billiards of Fig. 1(a) and 1(c), respectively, used in our experiments. As seen, the classical dynamics is indeed almost fully chaotic and mixed for $\alpha = 52.5°$ and $\alpha = 31°$, respectively.

In Fig. 2, the decay of the echo signal for different perturbation strength ϵ is presented for a wedge with $\alpha = 52.5°$, where the dynamics is almost fully chaotic. For small perturbations a nonmonotonic decay is seen with a faint partial revival of correlations for $\tau = 15$ ms. The echo (ECH) amplitude of an eigenstate in the perturbative regime can be rewritten as [10]

$$F_{\mathrm{ECH}}(|m_\downarrow\rangle, \tau) \simeq 4F_{\mathrm{SRV}}(|m_\downarrow\rangle, \tau) - F_{\mathrm{SRV}}(|m_\downarrow\rangle, 2\tau) - 2 \quad (2)$$

Where $F_{\mathrm{SRV}} = \langle m_\downarrow|\exp(iH_\uparrow\tau/\hbar)\exp(-iH_\downarrow\tau/\hbar)|m_\downarrow\rangle$ is the survival amplitude [9]. The survival amplitude in the base

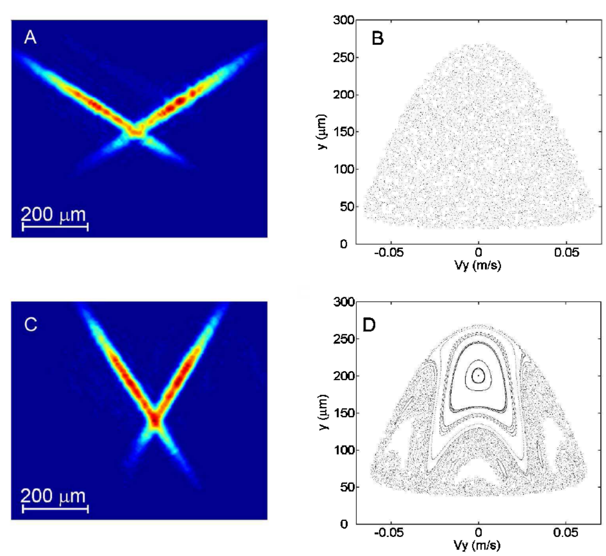

FIG. 1 (color online). CCD images of wedge billiards, with $\alpha = 52.5°$ (a) and $\alpha = 31°$ (c), used in our experiments and their corresponding calculated Poincaré surface of sections, respectively, indicating chaotic dynamics (b) and mixed dynamics with large islands of stability (d).

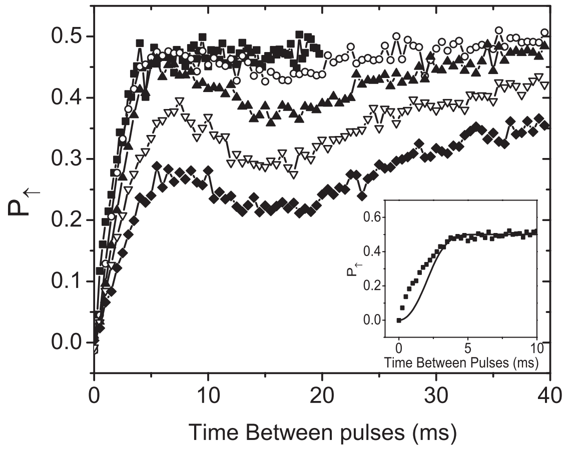

FIG. 2. Echo signal for a light-sheet wedge with chaotic classical dynamics, for different perturbation strengths (trap laser wavelength tuned from $\lambda = 775.9$ to $\lambda = 779.7$). \diamondsuit: $\epsilon = 1.44 \times 10^{-3}$, ∇: $\epsilon = 1.90 \times 10^{-3}$, $\blacktriangle$: $\epsilon = 2.43 \times 10^{-3}$, $\bigcirc$: $\epsilon = 3.80 \times 10^{-3}$, and $\blacksquare$: $\epsilon = 1.52 \times 10^{-2}$. For small perturbations a nonmonotonic decay with revivals around $\tau = \tau_{\mathrm{bl}}$ is seen, whereas for large ones a monotonic decay is observed. Inset: Echo signal for short times. Solid line: $P_{\uparrow}$ calculated by classical model. $\blacksquare$: $P_{\uparrow}$ measured for a perturbation of $\epsilon = 1.52 \times 10^{-2}$.

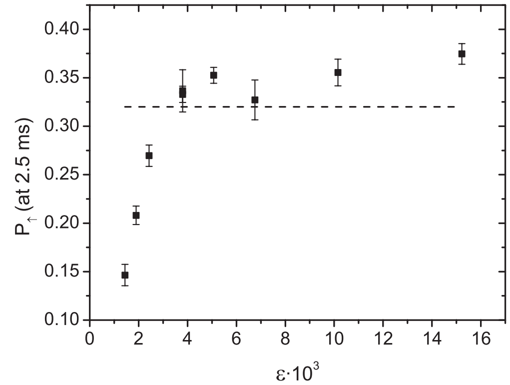

FIG. 3. Echo signal for a time between pulses of 2.5 ms, as a function of perturbation strength. For a small perturbation the decay depends strongly on the perturbation strength, whereas for $\epsilon > 4 \times 10^{-3}$ the decay is almost independent of perturbation strength. Dashed line: $P_{\uparrow}$ (at 2.5 ms) predicted by the simple classical model described in the text.

of the eigenfunctions of $H_{\uparrow}$, gives $F_{\mathrm{SRV}} = \sum_{m \neq n} \exp(-i(\omega_n - \omega_m)\tau)|\langle m_{\uparrow} \mid n_{\downarrow}\rangle|^2$. The expression $|\langle m_{\uparrow} \mid n_{\downarrow}\rangle|^2$ as a function of $E_{m\uparrow} - E_{n\downarrow}$ averaged over an ensemble of $n_{\downarrow}$ with approximately the same energy is known as the smooth part of the local density of states (LDOS) or the band profile of the overlap matrix between the perturbed ($\uparrow$) basis and the unperturbed ($\downarrow$) basis [13–15]. The smooth part of the LDOS can be computed using classical trajectories [10,13,14], revealing that classical periodic motion leads to distinct sidebands in the LDOS, leading to partial revivals in the survival probability and echo amplitude [10]. The faint revivals of Fig. 2 originate from the fact that even when the classical motion is fully chaotic, there exists a characteristic time τ_{bl} between encounters with the billiard walls, where the perturbation is located. This again leads to (now broadened) peaks in the LDOS, and thereby to partial revivals for $\tau = \tau_{\mathrm{bl}}$ [10].

For larger perturbations, Fig. 2 reveals a crossover to a regime where the echo decay is monotonic and independent of perturbation strength. This is evident from Fig. 3, in which $P_{\uparrow}(\tau = 2.5\ \mathrm{ms})$ is plotted as a function of perturbation strength. It is seen that $P_{\uparrow}$ initially grows with perturbation strength but for $\epsilon > 0.004$, $P_{\uparrow}$ is roughly constant [16]. Previously, for sufficiently small perturbations, a perturbation-independent regime was observed in NMR polarization echo decay experiments [17,18]. In the perturbation-independent regime observed here the perturbation is large enough so the overlap of equivalent eigenstates is small, indicating that the effects of quantization of the trap levels should not play a role and a classical description might be possible [2]. Since in our system the thermal de Broglie wavelength is much smaller than the billiard's dimensions, it is possible to approximate the echo amplitude by a semiclassical propagator [19]. The classical trajectories contributing to the ensemble average of the echo amplitude are those that after evolving forward in time in $H_{\uparrow}$ and $H_{\downarrow}$, and then backwards in time in $H_{\uparrow}$ and $H_{\downarrow}$, return to the vicinity of their initial position. Since $H_{\uparrow}$ and $H_{\downarrow}$ are highly different mainly in the vicinity of the wall, then the action integral along trajectories that hit the wall during the propagation time yields a very large phase and their contribution to the ensemble average of the echo amplitude averages out. Alternatively, classical trajectories that do not hit the wall do not feel the difference between the potentials and retrace their forward propagation backwards in time causing the action integral to vanish. These trajectories thus give a perfect contribution to the echo signal. Therefore, the echo amplitude in the perturbation-independent regime measures the classical probability that the particles have not yet hit the wall. The dashed line in Fig. 3 is a classical estimation of $P_{\uparrow}(\tau = 2.5\ \mathrm{ms})$ calculated assuming an idealized hard wall wedge populated with a thermal ensemble of atoms at a temperature of 20 μK, clipped at an energy equal to the depth of the trap. In the inset of Fig. 2 the classical calculation is shown together with the measured echo decay for a perturbation of $\epsilon = 1.52 \times 10^{-2}$, and reasonable agreement is seen.

To isolate the role of the LDOS we consider next a similar billiard, but with a wedge angle of $\alpha = 31°$ that has mixed dynamics (see Fig. 1). A perturbation-independent regime is also observed here ($\square$ in Fig. 4), in which the decay is essentially indistinguishable from that of the chaotic billiard, as expected from the classical model given above. The revival for a small perturbation is more pronounced than for the chaotic motion of Fig. 2, and the reminiscence of a second revival for $\tau \simeq 2\tau_{\mathrm{bl}}$ can be seen indicating that the peak in the LDOS is narrower, due

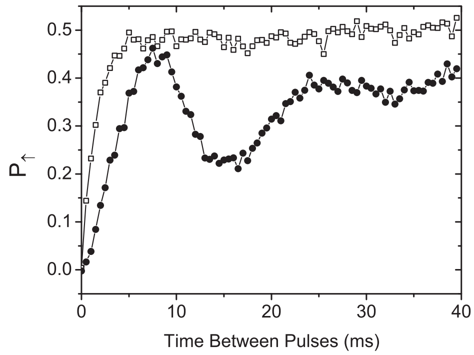

FIG. 4. Echo signal for a light-sheet wedge where the classical phase space is mixed. •: $\epsilon = 1.44 \times 10^{-3}$. □: $\epsilon = 1.52 \times 10^{-2}$. For the small perturbation a revival that is more pronounced than the one in the trap with classical chaotic dynamics is observed, whereas for large perturbations the decay cannot be distinguished from the decay in a trap with chaotic dynamics.

to the contributions from the periodic classical motion within the island.

In previous studies a perturbation-independent decay rate of fidelity above a certain perturbation strength has been predicted in a Hamiltonian system [2]. The effect predicted in [2] is, however, unrelated to what is observed in this work, since it happens for much longer time scales, and depends on the classical Lyapunov exponent, whereas the effect observed here happens at short time scales and is related to the special nature of the perturbation associated with atom-optics billiards.

Our results indicate that fingerprints of the classical dynamics appear in fidelity decays even when the perturbation is distributed throughout the entire billiard wall. The revivals we observe in the echo signal are sensitive to the regularity of time intervals between interactions with the perturbation, which is determined by the type of classical dynamics governing the system. Since the echo signal is an ensemble averaged quantity, the full information from the entire phase space about this regularity is achieved even at short times. Quantitatively, our results are sensitive to the existence of elliptical islands around periodic orbits with a period shorter than the experiment time. We also investigated the decay of quantum correlations as a result of more general perturbations, not localized on the billiards walls and generated with an additional laser field. The results (to be published in detail elsewhere) confirm that whenever the perturbation is associated with a short periodic orbit we observe partial revivals of coherence and a perturbation-independent regime.

In summary, we studied the decay of quantum correlations in atom-optics billiards in which the classical dynamics is chaotic or mixed. We observed two distinct regimes for the perturbation strength in which the decay was qualitatively different. In the perturbative regime the decay rate increased with perturbation strength and was nonmonotonic, with revivals at times corresponding to the typical time between bounces from the wall. The revivals were more pronounced in traps with mixed phase space as compared with traps where the dynamics is almost fully chaotic.

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