Implications of the Measurement of the $B_s^0 \bar{B}_s^0$ Mass Difference

Zoltan Ligeti,[1,2] Michele Papucci,[1,3] and Gilad Perez[1]

[1]*Ernest Orlando Lawrence Berkeley National Laboratory, University of California, Berkeley, California 94720, USA*

[2]*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

[3]*Department of Physics, University of California, Berkeley, California 94720, USA*

(Received 21 April 2006; published 6 September 2006)

We analyze the significant new model independent constraints on extensions of the standard model (SM) that follow from the recent measurements of the $B_s^0 \bar{B}_s^0$ mass difference. The time-dependent *CP* asymmetry in $B_s \to \psi\phi$, $S_{\psi\phi}$, will be measured with good precision in the first year of CERN Large Hadron Collider (LHC) data taking, which will further constrain the parameter space of many extensions of the SM, in particular, next-to-minimal flavor violation. The *CP* asymmetry in semileptonic B_s decay, $A_{\rm SL}^s$, is also important to constrain these frameworks, and could give further clues to our understanding the flavor sector in the LHC era. We point out a strong correlation between $S_{\psi\phi}$ and $A_{\rm SL}^s$ in a very broad class of new physics models.

DOI: 10.1103/PhysRevLett.97.101801 PACS numbers: 12.60.−i, 11.30.Hv, 12.15.Ff

Recently the D0 [1] and CDF [2] Collaborations reported measurements of the $B_s^0 \bar{B}_s^0$ mass difference

$$17\ {\rm ps}^{-1} < \Delta m_s < 21\ {\rm ps}^{-1} (90\%\ {\rm C.L.,\ D0}), \quad \Delta m_s = (17.31^{+0.33}_{-0.18} \pm 0.07)\ {\rm ps}^{-1} ({\rm CDF}). \tag{1}$$

The probability of the signal being a background fluctuation is 0.2% (5%) for CDF (D0). More important than the (moderate) improvements in the standard model (SM) global fit for the Cabibbo-Kobayashi-Maskawa (CKM) elements is that these measurements provide the first direct constraint on new physics (NP) contributions to the $B_s \bar{B}_s$ mixing amplitude.

We focus below on a large class of NP models with the following features [3]: (I) the 3×3 CKM matrix is unitary; (II) tree-level decays are dominated by the SM contributions. These assumptions are rather mild and allow for large deviations from the SM predictions. It is therefore important to examine how present and near future experimental data constrain the parameter space of such models.

We expect NP contributions to modify the predictions for observables that are related to flavor-changing neutral current (FCNC) processes. *A priori*, we have no knowledge of the expected size of these contributions. However, due to the hierarchy problem, new degrees of freedom should be present near the electroweak symmetry breaking (EWSB) scale. Allowing for 10% fine-tuning in the SM Higgs potential, the new degrees of freedom which regularize the Higgs quadratic divergence from the top-loop should have masses $m_X \sim 3$ TeV (see, e.g., [4]). In a most generic natural theory, such a particle can have tree-level nonuniversal couplings to the SM quarks. Thus, after integrating out X, four-fermion operators of the form $(\bar{d}^i d^j)^2/m_X^2$ ($i, j = 1, \dots, 3$) are expected to be generated with order one complex coefficients. This would contribute to many well-measured processes in the B_q^0 ($q = d, s$) and K^0 systems. For instance, in $K^0 \bar{K}^0$ and $B_q^0 \bar{B}_q^0$ mixing we can parametrize the ratio between the NP and the SM short distance contributions by $h_{K,q} e^{2i\sigma_{K,q}}$. Assuming arbitrary *CP* violating phases, we expect the following orders of magnitudes for these parameters in the general case:

$$h_{K,d,s}^{\rm gen} \sim \left(\frac{4\pi v}{m_X \lambda^{5,3,2}}\right)^2 \sim \mathcal{O}(10^5, 10^3, 10^2), \tag{2}$$

where v is the EWSB scale. Clearly, such huge values are excluded by many other observables, but this way of presenting the NP expectation will be useful in the following discussion. The smaller the ratio between the experimental bounds on $h_{K,d,s}$ and $h_{K,d,s}^{\rm gen}$, the more disfavored this framework is.

The bounds on the above parameters prior to the Δm_s measurement were given in [5], $h_{K,d} \lesssim 0.6, 0.4$, which are $\mathcal{O}(10^{-6}, 10^{-4})$ times smaller than Eq. (2), while no significant bound was found on h_s. The smallness of these ratios demonstrates that generic models which address the SM fine-tuning problem are in great tension with indirect bounds from FCNC processes. These require that the scale of m_X should be orders of magnitude above the TeV scale.

The SM quark flavor sector is far from being generic as well. Most of the SM flavor parameters are small and hierarchical, and the flavor sector possesses an approximate $U(3)_d \times U(2)_u \times U(2)_Q$ flavor symmetry (here d, u, Q correspond to the down- and up-type singlet and doublet quarks, respectively). Roughly speaking, only the top Yukawa coupling violates these approximate symmetries. Thus it is not inconceivable that the NP at m_X will share the same flavor symmetries. In this case its contributions to FCNC processes will be suppressed and Eq. (2) overestimates their size. Below we therefore assume that this is the case, and the new non-flavor-universal higher dimensional operators are invariant under these symmetries.

The special case in which these new operators are fully aligned with the SM Yukawa matrices corresponds to the minimal flavor violation (MFV) framework. Then the only sources of flavor and CP violation are due to the SM [6]. A more general case is when the new operators are only quasialigned with the SM Yukawa matrices; that is, in the basis where the new operators are flavor diagonal, the diagonalizing matrices of the Yukawa couplings are at least as hierarchical as the CKM matrix. This constitutes next-to-minimal minimal flavor violation (NMFV) [5]. In this case there are new flavor and CP violating parameters, so NMFV is almost as generic as the class of models defined above by conditions (I) and (II). However, our assumption of quasialignment provides a useful way for "power counting" and to estimate the size of the expected NP contributions. Moreover, it is also realized by many supersymmetric and nonsupersymmetric models (see [5] for more details), providing a powerful framework for model independent analysis.

What is the expected size of the NP contributions? Four-fermion operators are generated when the NP is integrated out at a scale of order $\Lambda_{\rm NMFV} \sim m_X \sim 3$ TeV. Consider, for example, the operator $(\bar{Q}_3 Q_3/\Lambda_{\rm NMFV})^2$ defined in the interaction basis [gauge, Lorentz indices, and $\mathcal{O}(1)$ coefficients are omitted]. In the mass basis, this operator contributes to $\Delta F = 2$ processes as $[(D_L^*)_{3i}(D_L)_{3j}\bar{Q}_i Q_j/\Lambda_{\rm NMFV}]^2 \sim [(V_{\rm CKM}^*)_{3i}(V_{\rm CKM})_{3j}\bar{Q}_i Q_j/\Lambda_{\rm NMFV}]^2$, where D_L is the rotation matrix of the down-type doublet quarks. Comparing the NP contributions to the SM ones, we find that within the NMFV we expect

$$h_{K,d,s}^{\rm NMFV} \sim \mathcal{O}(1). \tag{3}$$

The magnitudes of $h_{K,d,s}$ are inversely proportional to the cutoff of the theory and provide a measure of the tuning in the model. Moreover, a connection between $\Lambda_{\rm NMFV}$ and m_X relates this fine-tuning to the one in the Higgs sector. Consequently, just as in the case of electroweak precision tests, any model of this class will be disfavored if the constraints on the $h_{K,d,s}$ drop below the 0.1 level.

Below we focus on NP in $\Delta F = 2$ processes, which are in general theoretically cleaner and have simpler operator structures. To constrain deviations from the SM in these processes, the tree-level observables $|V_{ub}/V_{cb}|$ and γ extracted from the CP asymmetry in $B^\pm \to DK^\pm$ modes are crucial, because they are unaffected by NP. We consider in addition the following observables: the $B_q^0\bar{B}_q^0$ ($q = d, s$) mass differences, Δm_q; CP violation in B_q^0 mixing, $A_{\rm SL}^q$ [7]; the time-dependent CP asymmetries in B_d^0 decays, $S_{\psi K}$ and $S_{\rho\rho,\pi\pi,\rho\pi}$; the time-dependent CP asymmetry in B_s^0 decay, $S_{\psi\phi}$ [by $S_{\psi\phi}$ we mean the CP asymmetry divided by $(1\text{–}2f_{\psi\phi}^{\rm odd})$ to correct for the CP-odd $\psi\phi$ fraction, which also equals $-S_{\psi\eta^{(\prime)}}$]; and the lifetime difference between the CP-even and CP-odd B_s states, $\Delta\Gamma_s^{CP}$ [8]. (Of these, $A_{\rm SL}^s$ and $S_{\psi\phi}$ have not been measured; however, they will be important in the discussion below.)

The NP contributions to B_d^0 and B_s^0 mixing can be expressed in terms of four parameters, h_q and σ_q defined by $M_{12}^q = (1 + h_q e^{2i\sigma_q})M_{12}^{q,\rm SM}$, where $M_{12}^{q,\rm SM}$ is the dispersive part of the $B_q^0\bar{B}_q^0$ mixing amplitude in the SM. (For a similar parametrization of NP in the K^0 system, see [5].) Then the predictions for the above observables are modified compared to the SM as follows:

$$\begin{aligned} \Delta m_q &= \Delta m_q^{\rm SM}|1 + h_q e^{2i\sigma_q}|, \\ S_{\psi K} &= \sin[2\beta + \arg(1 + h_d e^{2i\sigma_d})], \\ S_{\psi\phi} &= \sin[2\beta_s - \arg(1 + h_s e^{2i\sigma_s})], \\ A_{\rm SL}^q &= {\rm Im}\{\Gamma_{12}^q/[M_{12}^{q,\rm SM}(1 + h_q e^{2i\sigma_q})]\}, \\ \Delta\Gamma_s^{CP} &= \Delta\Gamma_s^{\rm SM}\cos^2[\arg(1 + h_s e^{2i\sigma_s})]. \end{aligned} \tag{4}$$

Here $\lambda \approx 0.23$ is the Wolfenstein parameter, $\beta_s = \arg[-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)] \approx 1°$ is the angle of a squashed unitarity triangle, and Γ_{12}^q is the absorptive part of the $B_q^0\bar{B}_q^0$ mixing amplitude, which is probably not significantly affected by NP. [We neglect $\mathcal{O}(M_W^2/\Lambda_{\rm NMFV}^2)$ corrections due to NP contributions to SM tree-level $\Delta F = 1$ processes; for a different approach, see [9].]

Looking at Eq. (4) one notices a fundamental difference between the B_d and B_s systems. The SM contributions affecting the B_d system are related to the nondegenerate unitarity triangle. Thus the determination of h_d, σ_d is strongly correlated with that of the Wolfenstein parameters, $\bar{\rho}$, $\bar{\eta}$. On the other hand, the unitarity triangle relevant for the B_s system is nearly degenerate and therefore the determination of h_s, σ_s is almost independent of $\bar{\rho}$, $\bar{\eta}$.

Figure 1 shows the allowed h_s, σ_s parameter space without (left) and with (right) the measurement of Δm_s in Eq. (1) and the bound on $\Delta\Gamma_s^{CP}$, using the CKMFITTER package [10]. (Unless otherwise stated, the input parameters are as in [10].) We used the constraint on the ratio

$$\frac{\Delta m_d}{\Delta m_s} = \left|\frac{1 + h_d e^{2i\sigma_d}}{1 + h_s e^{2i\sigma_s}}\right| \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{m_{B_d}}{m_{B_s}}\xi^2, \tag{5}$$

which is theoretically cleaner than either Δm_d or Δm_s. Since Δm_d depends on h_d, σ_d, $\bar{\rho}$, $\bar{\eta}$, in order to produce the above plots these parameters were scanned over. We can easily see that the new measurement excludes a large part of the previously allowed parameter space. The excluded region around $h_s = 1$ and $\sigma_s = 90°$ would give canceling contributions to Δm_s. The decrease in C.L. around $h_s = 1$ is due to the $\Delta\Gamma_s^{CP}$ constraint, which is useful at present, largely because its central value disfavors any deviation from the SM. After a year of CERN Large Hadron Collider (LHC) data, the bound from this quantity will probably be less important, because of theoretical uncertainties.

The magnitudes of the h_i's provide a measure of how much fine-tuning is required to satisfy the experimental constraints. Generically we do not expect the NP contributions to be MFV-like, i.e., aligned with the SM. Thus we are interested in finding the allowed ranges of h_i, for σ_i not near $0 \bmod \pi/2$. The present constraints are roughly

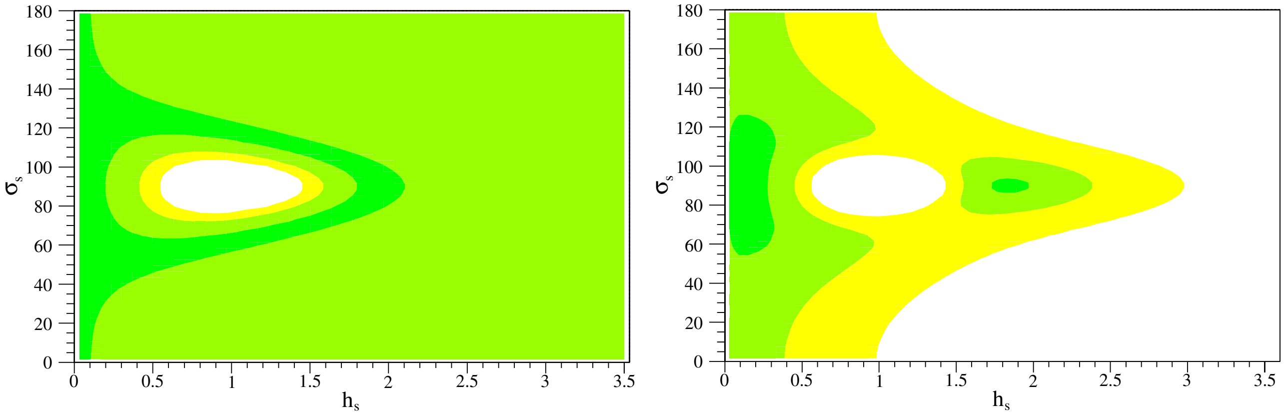

FIG. 1 (color online). The allowed range for h_s, σ_s using the data before (left) and after (right) the recent Δm_s and $\Delta\Gamma_s$ measurements. For Δm_s only the CDF result was used. The dark, medium, and light shaded areas have C.L. > 0.90, 0.32, and 0.05, respectively.

$$h_d \lesssim 0.3, \qquad h_s \lesssim 1.5, \qquad h_K \lesssim 0.6. \tag{6}$$

Let us now discuss some implications of the above results. Equation (6) shows that at present none of the bounds on the NP parameters have reached the 0.1 level, so NMFV survives the current tests. It is then interesting to ask which future measurements will be most important to verify or disfavor the NMFV framework. The constraints on $h_{d,K}$, even though they underwent significant improvements in the last few years due to new SM tree-level measurements [11], are now limited by the statistical errors in the measurements of γ (and effectively α) and the hadronic parameters in the determination of $|V_{ub}|$ from semileptonic decays and $|V_{td}|$ from Δm_d. The improvements in these constraints will be incremental, as they depend on the integrated luminosities at the B factories and on progress in lattice QCD. The constraint from ε_K on the K system is also dominated by hadronic uncertainties. At present, the bound on h_s is weaker than that on h_d, since only one measurement, Δm_s, constrains it, and the hadronic uncertainties are comparable.

However, the B_s system is exceptional because a measurement of $S_{\psi\phi}$ (or a strong bound on it) would provide a very sensitive test of NMFV, which is neither obscured by hadronic uncertainties nor by uncertainties in the CKM parameters. In the SM $S_{\psi\phi}$ is suppressed by λ^2 (the SM CKM fit gives $\sin 2\beta_s = 0.038 \pm 0.003$), whereas Eq. (4) implies

$$S_{\psi\phi} = -\frac{h_s \sin(2\sigma_s)}{|1 + h_s e^{2i\sigma_s}|} + \sin(2\beta_s)\frac{1 + h_s \cos(2\sigma_s)}{|1 + h_s e^{2i\sigma_s}|}, \tag{7}$$

where we set $\cos 2\beta_s$ to unity. Thus when the sensitivity of the measurement of $S_{\psi\phi}$ reaches the SM level, it will provide us with a strong test of NMFV. The precision that will be achieved in forthcoming experiments depends on the value of Δm_s, but since we now know Δm_s, we can use the LHC projections for the SM case. The LHCb experiment expects to reach $\sigma(S_{\psi\phi}) \approx 0.03$ with the first year (2 fb^{-1}) data [12] (in several years the uncertainty may be reduced to 0.01). Figure 2 shows the resulting constraint on h_s, σ_s, assuming an experimental measurement $S_{\psi\phi} = 0.04 \pm 0.03$. This plot demonstrates that already with 1 yr of data the bound on h_s will be better than 0.1, which will severely constrain the NMFV class of models. Even initial data on $S_{\psi\phi}$ at the Tevatron may constrain new physics in B_s mixing comparable to similar bounds on h_d, σ_d in the B_d sector.

Another sensitive probe of this class of models is the CP asymmetry in semileptonic B_s decays, $A^s_{\rm SL}$. In the SM it is unobservably small, because the short distance contributions are much larger than the long distance part, $|\Gamma^s_{12}/M^s_{12}| \propto m_b^2/m_t^2$, and the two contributions are highly aligned, $\arg(\Gamma^s_{12}/M^s_{12}) \propto (m_c^2/m_b^2)\sin 2\beta_s$ [7]. Given the new Δm_s result, we know that even in the presence of NP the first suppression factor can be only moderately affected, while the second one can be significantly enhanced in the presence of new CP violating phases. Figure 3 shows the allowed range of $A^s_{\rm SL}$, taking into account the new constraint from Δm_s. We find

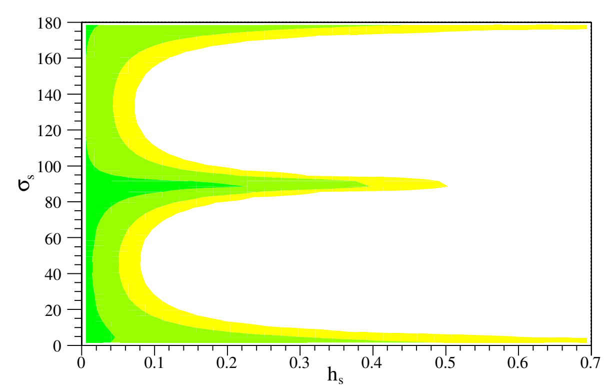

FIG. 2 (color online). The allowed range for h_s, σ_s using the 1 yr LHCb projection, assuming the SM prediction as the central value.

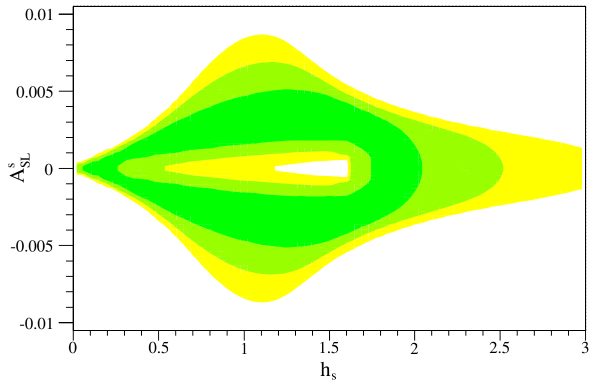

FIG. 3 (color online). The current allowed range of $A^s_{\rm SL}$ as a function of h_s.

$$A^s_{\rm SL} < 0.01, \tag{8}$$

which extends 3 order of magnitude above the SM prediction [13]. In particular, $|A^s_{\rm SL}| > |A^d_{\rm SL}|$ is possible, contrary to the SM, in which $|A^s_{\rm SL}/A^d_{\rm SL}| \approx |V_{td}/V_{ts}|^2$. This demonstrates that while the constraint from the Δm_s measurement is of great importance, it still leaves plenty of room for deviations from the SM within NMFV.

Finally we point out that $A^s_{\rm SL}$ and $S_{\psi\phi}$ are highly correlated in the region in which $h_s, \sigma_s \gg \beta_s$ and h_s is moderate. Defining $2\theta_s \equiv \arg(1 + h_s e^{2i\sigma_s})$, we have $S_{\psi\phi} = \sin(2\beta_s - 2\theta_s)$, so $A^s_{\rm SL}$ can be written as

$$A^s_{\rm SL} = \left|\frac{\Gamma^s_{12}}{M^s_{12}}\right|^{\rm SM} \sin(2\theta_s) + \mathcal{O}\left(h^2_s, \frac{m^2_c}{m^2_b}\right). \tag{9}$$

Thus, we find

$$A^s_{\rm SL} = -\left|\frac{\Gamma^s_{12}}{M^s_{12}}\right|^{\rm SM} S_{\psi\phi} + \mathcal{O}\left(h^2_s, \frac{m^2_c}{m^2_b}\right). \tag{10}$$

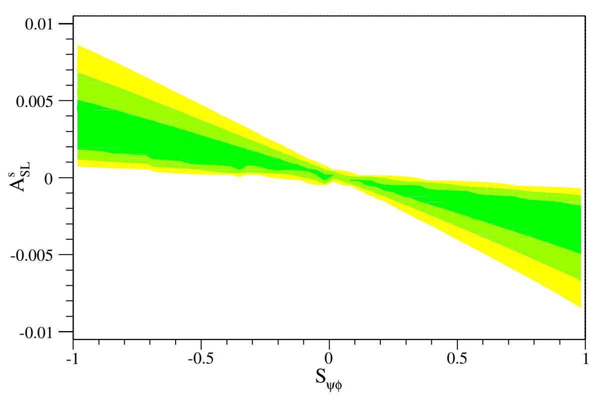

FIG. 4 (color online). The correlation between $A^s_{\rm SL}$ and $S_{\psi\phi}$.

Figure 4 shows $A^s_{\rm SL}$ as a function of $S_{\psi\phi}$, taking into account the constraint from Δm_s [without neglecting the $\mathcal{O}(h^2_s, m^2_c/m^2_b)$ terms]. As explained above, the two observables are strongly correlated. Deviation from this prediction would provide a clear indication of new physics beyond the generic framework defined by (I) and (II).

Sign mistakes in version 1 of this paper were first pointed out in version 2 of [14]. We are grateful to Daria Zieminska for drawing our attention to the relevance of $\Delta\Gamma^{CP}_s$. While we were including it in our analysis, Refs. [15,16] appeared with similar results. We thank A. Cerri, A. Höcker, and C. Paus for helpful discussions, and O. Schneider for pointing out typos in version 1. This work was supported in part by the Director, Office of Science, Office of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231.

[1] V. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **97**, 021802 (2006).
[2] A. Abulencia *et al.* (CDF Collaboration), hep-ex/0606027 [Phys. Rev. Lett. (to be published)].
[3] C. Dib, D. London, and Y. Nir, Int. J. Mod. Phys. A **6**, 1253 (1991).
[4] R. Barbieri, T. Gregoire, and L. J. Hall, hep-ph/0509242.
[5] K. Agashe, M. Papucci, G. Perez, and D. Pirjol, hep-ph/0509117.
[6] A. Ali and D. London, Eur. Phys. J. C **9**, 687 (1999); A. J. Buras, P. Gambino, M. Gorbahn, S. Jager, and L. Silvestrini, Phys. Lett. B **500**, 161 (2001); G. D'Ambrosio, G. F. Giudice, G. Isidori, and A. Strumia, Nucl. Phys. **B645**, 155 (2002).
[7] S. Laplace, Z. Ligeti, Y. Nir, and G. Perez, Phys. Rev. D **65**, 094040 (2002).
[8] Y. Grossman, Phys. Lett. B **380**, 99 (1996); I. Dunietz, R. Fleischer, and U. Nierste, Phys. Rev. D **63**, 114015 (2001).
[9] M. Bona *et al.* (UTfit Collaboration), J. High Energy Phys. 03 (2006) 080.
[10] A. Hocker, H. Lacker, S. Laplace, and F. Le Diberder, Eur. Phys. J. C **21**, 225 (2001); J. Charles *et al.*, Eur. Phys. J. C **41**, 1 (2005), updates at http://ckmfitter.in2p3.fr/.
[11] Z. Ligeti, Int. J. Mod. Phys. A **20**, 5105 (2005).
[12] O. Schneider, Talk given at the Flavour in the Era of the LHC Workshop, CERN, 2005, http://indico.cern.ch/conferenceDisplay.py?confId=a052129.
[13] M. Beneke, G. Buchalla, A. Lenz, and U. Nierste, Phys. Lett. B **576**, 173 (2003); M. Ciuchini, E. Franco, V. Lubicz, F. Mescia, and C. Tarantino, J. High Energy Phys. 08 (2003) 031.
[14] M. Blanke, A. J. Buras, D. Guadagnoli, and C. Tarantino, hep-ph/0604057.
[15] Y. Grossman, Y. Nir, and G. Raz, hep-ph/0605028.
[16] M. Bona *et al.* (UTfit Collaboration), hep-ph/0605213.