P H Y S I C A L R E V I E W L E T T E R S

Higher Harmonics In Vacuum From Nonlinear QED Effects without Low-Mass Intermediate Particles

J. Tito Mendonça,[1] J. Dias de Deus,[2] and P. Castelo Ferreira[2]

[1]*CFP and CFIF, Instituto Superior Técnico, 1096 Lisboa, Portugal*
[2]*CENTRA, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal*
(Received 8 June 2006; published 6 September 2006)

We show that in the presence of a slowly rotating strong transverse magnetic field there is an infinite spectrum of harmonic wave functions A_n due to the first order QED correction (in α^2) given by the Euler-Heisenberg Lagrangian. The frequency shifts are integer multiples $\pm \omega_0 n$ of the magnetic field angular frequency rotation $\omega_0 = 2\pi \nu_m$, and the several modes n are coupled to the nearest harmonics $n \pm 1$. This is a new effect due to QED vacuum fluctuations, not exploited before, that can explain, both qualitatively and quantitatively, the recent experimental results of the PVLAS collaboration without the need of a low-mass intermediate particle, hence may dismiss the recent claim of the discovery of the axion.

In quantum field theory, the vacuum is not empty, in the sense that virtual particle pair creation and annihilation are taking place. Considering the case of QED -Quantum Electrodynamics- in the presence of external electromagnetic fields, the vacuum shows properties close to the properties of any other optical medium, as dichroism and birefringence. These effects can be considered as first order corrections to the Maxwell theory [1–4], and the basic diagrams correcting the photon propagator, are shown in Fig. 1. In the presence of low-mass neutral particles coupling to two photons, like the π^0 and, more interesting, the axion -Primakov effect- or the graviton, there are additional nonlinear effects (see Fig. 1), which may be useful in directly detecting the axion [5,6].

In this Letter, we present and exploit an effect not considered before: in vacuum, a strong rotating magnetic field generates an infinite set of higher harmonics. We show that recent experimental results [7] can be explained on the basis of pure QED which dismisses the recent claim of the discovery of the axion [8].

In the QED, lowest order correction (radiative corrections in α^2) to the Maxwell Lagrangian $\mathcal{L}_{\text{Maxwell}} = -F_{\mu\nu}F^{\mu\nu}$ is given by the regularized Euler-Heisenberg Lagrangian $\mathcal{L}^{(2)}$ [1,2]

$$\mathcal{L}^{(2)} = \xi[4(F_{\mu\nu}F^{\mu\nu})^2 + 7(\epsilon^{\mu\nu\delta\rho}F_{\mu\nu}F_{\delta\rho})^2] \quad (1)$$

where $\xi = 2\alpha/(45B_c^2) \sim \alpha^2$ and the critical field is $B_c = m_e^2 c^2/(e\hbar) = 4.414 \times 10^9$ T [4]. The usual procedure is to take a decomposition of the gauge connection F into an external component F_0 plus an internal component $f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, such that we obtain $F_{\mu\nu} = F_{0,\mu\nu} + f_{\mu\nu}$, and consider the lowest order diagrams of photon interactions due to the background electromagnetic fields. There are two main processes that may be considered, *photon splitting* and *photon dispersion*. Depending on the practical application and experimental setups, there are other processes, such as, *photon-photon interactions* [9] and other

photon alternative processes [10] that are relevant. In this Letter, we address only the process of *photon dispersion* in the low frequency and weak field limit, i.e. $\omega \ll 2m_e c^2/\hbar$ and $|F| \ll m_e^2 c^2/(\hbar e)$. The relevant diagrams are the ones containing one internal fermion loop with two external photons and an even number of exchanged photons between the fermion loop and the external electromagnetic field (see Fig. 1).

We obtain that the relevant radiative corrections to the usual classical wave equation in order α^2 is linear in the photon field A [4]

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\mathbf{A} = \Lambda \cdot \mathbf{A} + O(\alpha^4), \quad (2)$$

where the matrix Λ has the eigenvalues [3,4]
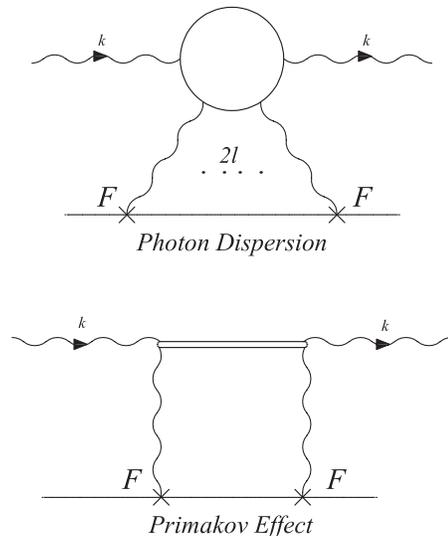

FIG. 1. Feynman diagrams corresponding to the electron loop that contributes to the photon dispersion effect and the axion exchange.

$$\lambda_\parallel = 14\xi(\omega/c)^2|\mathbf{Q}|^2 \quad \text{and} \quad \lambda_\perp = 8\xi(\omega/c)^2|\mathbf{Q}|^2, \quad (3)$$

where $\mathbf{Q} = \mathbf{k} \times \mathbf{E_0}/|\mathbf{k}| + \mathbf{k} \times (\mathbf{k} \times \mathbf{B_0})/|\mathbf{k}|^2$ and $\mathbf{k}$ stands for the light wave vector. $\mathbf{E_0}$ and $\mathbf{B_0}$ are the external fields, and the directions $\parallel$ and $\perp$ correspond, respectively, to the parallel and transverse directions to the external magnetic field.

We consider linear polarized light travelling in vacuum in the z direction with wave number $\mathbf{k}$ and frequency ω under a slowly rotating transverse magnetic field rotating with angular frequency $\omega_0 = 2\pi\nu_m \ll \omega$ such that we have $\mathbf{A} = (A_x, A_y, 0)$, $\mathbf{k} = (0, 0, k)$, $w = ck$, $\mathbf{B_0}(t) = B_0[\sin(\omega_0 t), \cos(\omega_0 t), 0]$, $\mathbf{E_0} = \mathbf{0}$ and $|\mathbf{Q}| = cB_0$. In these conditions, the gauge field depends only on z, and we can write the Eq. (2) for $\mathbf{A}$ in terms of the parallel direction ($\parallel$) and transverse direction ($\perp$) as derived by Adler [4]. However, these directions are rotating with an angular frequency ω_0; therefore, we take the directions of the decomposition of the fields to be the directions of the magnetic field at some initial time $t = 0$. Then the eigenvalues of the matrix Λ become time dependent since they are valid with respect to the magnetic field direction such that we have

$$\Lambda_\parallel(t) = \lambda_\parallel \cos(\omega_0 t) - \lambda_\perp \sin(\omega_0 t)$$
$$\Lambda_\perp(t) = \lambda_\parallel \sin(\omega_0 t) + \lambda_\perp \cos(\omega_0 t) \quad (4)$$

with $\lambda_\parallel$ and $\lambda_\perp$ given in (3). Considering the above eigenvalues, we obtain the field equation

$$\left(\partial_z^2 - \frac{1}{c^2}\partial_t^2\right)(A_\parallel^0, A_\perp^0) = [\lambda_\parallel \cos(\omega_0 t) - \lambda_\perp \sin(\omega_0 t)](A_\parallel^0, 0)$$
$$+ [\lambda_\parallel \sin(\omega_0 t) + \lambda_\perp \cos(\omega_0 t)]$$
$$\times (0, A_\perp^0), \quad (5)$$

where the $A_\parallel^0$ and $A_\perp^0$ correspond to the decomposition of the field along the directions parallel ($\parallel$) and transverse ($\perp$) to the magnetic field at the initial time $t = 0$. For constant magnetic field ($\omega_0 = 0$), these equations imply that the vacuum acquires a birefringence due to the existence of two different refractive indices $\mathcal{N}_i = 1 - \lambda_i/2$ along both the directions $i = \parallel, \perp$. For slowly rotating magnetic fields, the usual procedure is to take the zeroth order expansion of the cosines and sines that hold the same effect [6]. However we show next that even for small ω_0, Eq. (5) implies not just a vacuum birefringence, but instead a coupling between the neighbor harmonics separated by frequency shifts of $\pm\omega_0$. It is important to note that Eq. (5) is valid for slow rotating external fields (see [4] for further details), i.e. $\omega_0 \ll \omega$.

We can cast Eq. (5) in complex form by using the definitions $\lambda_\pm = \lambda_\parallel \pm i\lambda_\perp$, $A_\pm = A_\parallel \pm iA_\perp$ and the usual relations for the trigonometric functions and consider a mode decomposition of the form

$$A_\pm(z, t) = \sum_{n=-\infty}^{+\infty} A_n^\pm(t) e^{i[kz - \omega_n t + \theta_n^\pm]},$$

$$ck_n = \omega_n = \omega + \omega_0 n, \qquad \theta_n^\pm = \left(-\theta_\lambda \pm \frac{\pi}{4}\right)n, \quad (6)$$

$$\lambda_0 = \sqrt{\lambda_\parallel^2 + \lambda_\perp^2}, \qquad \theta_\lambda = \arctan\left(\frac{\lambda_\perp}{\lambda_\parallel}\right).$$

Given the above mode decomposition, the differential equation becomes a recursion relation in the several frequency modes A_n

$$i\frac{k}{\sqrt{2}c\lambda_0}\dot{A}_n = A_{n+1} + A_{n-1} \quad (7)$$

where $A_n = A_n^+ = e^{-i\pi/2}A_n^-$. We neglected a second derivative term $\ddot{A}_n$ and considered a dispersion relation that renders a tower of refractive indices in each direction. This is the same approach of [4] in consistent field approach. From (6) we have for each mode n and each direction $i = \parallel, \perp$ a different refractive index

$$kc = \mathcal{N}_i^n(\omega)\omega, \qquad \mathcal{N}_i^n(\omega) = 1 + \frac{\omega_0}{\omega}n. \quad (8)$$

The solutions up to an overall phase for the recursive relation (7) are modified Bessel function of the first kind

$$A_n(t) = (-1)^n I_n(i\tau), \qquad \tau = \frac{\sqrt{2}c\lambda_0 t}{k}. \quad (9)$$

We note that indeed the second derivative of A_n is of order α^4 and can be safely neglected. These functions are normalized for any τ, and for even n the functions are pure real while for odd n the functions are pure imaginary. The only mode that is non null at $\tau = 0$ is $I_0(i\tau)$ such that only for larger values of t will the amplitudes of the other modes increase. These properties are important when imposing the initial conditions, i.e. matching our solution with the incident wave at the initial time $t = 0$.

The results obtained so far are very interesting and describe both vacuum birefringence and a generalized dichroism in the presence of a strong rotating magnetic field. The tower of refractive indices (8) correspond to a birefringence effect quantized in terms of the quantity ω_0/ω. The Bessel functions solutions account for a generalized dichroism effect that decreases the amplitude of the incident wave in one mode and increases it in the nearest modes. We note that the QED effects considered here do preserve energy-momenta. As studied in detail in [4], the process of photon dispersion (see Fig. 1) preserves four momenta. In the framework present here, this means that (in order α^2) the total radiation amplitude is conserved, being only transferred from one mode to the nearest modes. These effects can be understand as follows: from Eq. (7), we see that the variation of each mode amplitude A_n over time contributes to the nearest modes $A_{n\pm1}$. This accounts for the generalized dichroism effect in the sense

that no real absorption happens; therefore, we have a *true* rotation of polarization (in the sense considered in [7]). As for birefringence, we note that by combining the nearest modes (e.g. $n_\parallel = n$ and $n_\perp = n \pm 1$) in different directions, we will have a net birefringence effect given by $\mathcal{N}_\perp - \mathcal{N}_\parallel = \pm \omega_0/\omega$ such that each mode combination corresponds to a wave with elliptic polarization. It is also interesting to note that, although the description for the propagating wave is given in terms of the collective field A, we may as well interpret these solutions as cross section for photons. In such interpretation, the allowed configurations for a single photon are independent in both spatial dimensions ($\parallel, \perp$), such that indeed we have elliptic polarized waves.

A solution for the electric field compatible with linear polarized waves at the initial time $\tau = 0$ is

$$\mathbf{E} = E_0 \left\{ \sum_{n=-\infty}^{+\infty} [I_{2n}(i\tau)\cos(kz - \omega_{2n}t + \theta_{2n}) - iI_{2n-1}(i\tau)\right.$$

$$\left. \times \sin(kz - \omega_{2n-1}t - \theta_{2n-1})] \right\}[\sin(\theta_0), \cos(\theta_0)].$$

(10)

Here E_0 is the electric field amplitude, $\omega_n = \omega + \omega_0 n$, and $\theta_n = -(\theta_\lambda + \pi/4)n$. This field is defined in the referential of the parallel and transverse direction to the magnetic field at the initial time $t = 0$ such that θ_0 stands for the angle between the original linear polarization direction and the magnetic field direction at $t = 0$.

Take the $n_\parallel = n_1$ mode in one of the directions together with one of the other modes $n_\perp = n_2 > n_1$ in the other direction such that this combination has now an elliptic polarization and the difference of argument of the trigonometric functions corresponding to each direction corresponds to a phase difference of $\Delta\varphi_{n_2-n_1} = (n_2 - n_1)(\omega_0 t + \theta_\lambda + \pi/4)$.

We proceed to compare our results with the recent experimental results of the PVLAS collaboration [7]. For the apparatus of that experiment, an incident polarized laser beam traverses a region under a strong rotating magnetic field such that it is possible to measure polarization rotations (that correspond to vacuum dichroism) and ellipticities (that correspond to vacuum birefringence).

We can write the amplitude measured in the experience for our solutions as

$$I = \left[2 \sum_{n=1}^{+\infty} \Gamma_n |I_n(i\tau)| \cos[n(\omega_0 t + \theta_\lambda + \theta_0 + \pi/4)] \right.$$

$$\left. + \eta_0 \cos(\omega_S t + \theta_S) + \Gamma_0 \right]^2$$

(11)

Here $\eta_0 \cos(\omega_S t + \theta_S)$ stands for the modulator wave [7], and heterodyne detection is used to identify the several harmonics present due to rotations. Γ_n are corrections that encode uncompensated dispersions for the several modes

n, due either to systematic deviations or unaccounted processes. In (11), we sum over rotations in one direction only. That is why we have a factor of 2 in front of the sum. By expanding the square of this expression, we obtain the angular frequency spectrum and the respective amplitudes and phases for each of them. We list the more significant frequencies that can be measured near the modulator frequency ω_S and list them in Table I.

We take the values of Fig. 2 of the original reference [7] (expressed in I.S. units)

$$\omega = 1.772 \times 10^{+15} \text{ s}^{-1} \qquad \omega_0 = 0.7\pi \text{ s}^{-1}$$

$$\Delta z = c\Delta t = 4.6 \times 10^{+4} \text{ m} \qquad B_0 = 5.5 \text{ T.}$$

(12)

The theoretical results obtained above are valid for these experimental conditions; in particular, the magnetic field is slowly rotating ($\omega_0 \ll \omega$) and obeys the low frequency and weak field approximations, respectively $\omega \sim 10^{15} \ll 2m_e c^2/\hbar \sim 10^{21}$ and $B_0^2 \sim 10 \ll m_e c^2/(e\hbar) \sim 10^{39}$. For the values given in (12) and using the finesse of the Fabry-Perot cavity $\mathcal{F} \approx 7 \times 10^4$ (corresponding to an increase of optical path $N \approx 4.46 \times 10^4$) [7], we have from (9) with $\tau = 0.0088$ [3] that $|I_0(i\tau)| \approx 0.9998$, $|I_1(i\tau)| \approx 4.384 \times 10^{-3}$, $|I_2(i\tau)| \approx 9.609 \times 10^{-6}$, and $|I_3(i\tau)| \approx 1.404 \times 10^{-8}$. We use these values to fit our solutions to the data expressed in Fig. 2 of reference [7]. From graphical inspection, we considered that the background noise is given by the approximately constant amplitude independent of the frequency, i.e. ≈ -135 dB V$_{\text{rms}}$. Also, we have assumed values for η_0 and the Γ_i's that allow us to fit the experimental results. Based on these assumptions, we list a possible set of values that fits the experimental results of [7] in Table II. The values presented here are given as an example and are not based in a detailed analysis. In order to properly test our theoretical results, a full analysis of the existent data is necessary. We note that the value obtained for the mode $n = \pm 3$ (≈ -145.65 dB V$_{\text{rms}}$) is below the noise level (≈ -135 dB V$_{\text{rms}}$); hence it could not be detected for the available data.

To conclude, we compute the expected rotation values corresponding to the second harmonic $n = 2$. By direct geometrical analysis, we conclude that the angular rotations of polarization are given by the generic expression $\varphi = \theta_0 - \arctan(E_\perp^{\text{out}}/E_\parallel^{\text{out}})$, where $\mathbf{E}^{\text{out}} = (E_\parallel^{\text{out}}, E_\perp^{\text{out}})$ stands for the radiation electric field reaching the detector.

TABLE I. The relevant angular frequency spectrum measured near the modulator frequency ω_S with respective amplitudes and phases. There is an infinite set of other frequencies that have lower amplitudes.

Frequency	I	Phase
0	$\Gamma_0^2 + 2\sum_{n=1}^{+\infty}(\Gamma_n I_n)^2 + \frac{\eta_0^2}{2}$	$\cdots$
ω_S	$2\Gamma_0 \eta_0$	θ_S
$\omega_S \pm \omega_0 n$	$2\Gamma_n I_n \eta_0$	$\theta_S \pm n(\theta_\lambda + \theta_0 + \pi/4)$

TABLE II.   We list the experimental values taken from Fig. 2 of Reference [7] with the theoretical values given by the $I_n(i\tau)$ up to $n = \pm 3$. We considered $\eta_0 \approx -68.63$ dB V_{rms}. This value, as well as the values of the Γ_n's, are only given as an example that it is possible to fit the data to our solutions. An analysis of the existent data is required to fully test our results. The theoretical value found for the amplitude of a rotation corresponding to $n = \pm 3$ is ≈ -145.65 dB V_{rms}, being below the background noise level considered (≈ -135 dB V_{rms}).

n	frequency (Hz)	I_{exp} (dB V_{rms})	$\eta_0 I_n$ (dB V_{rms})	Γ_n
$\cdots$	506	≈ -51	$\cdots$	$\approx +16.93$
± 1	506 ± 0.35	≈ -91	≈ -90.70	$\approx +0.704$
± 2	506 ± 0.70	≈ -118	≈ -117.30	≈ -0.704
± 3	506 ± 1.05	< -135	≈ -145.65	$\cdots$

For a given harmonic n, we have the possible combinations $\mathbf{E}_n^{out+} = (I_0 \sin(\theta_0) \cos[\delta(t, z)], I_n \cos(\theta_0) \cos[\delta(t, z) + \omega_0 \Delta t + \arctan(7/4) + \pi/4])$ and $\mathbf{E}_n^{out-} = (I_n \cos(\theta_0) \times \cos[\delta(t, z) + \omega_0 \Delta t + \arctan(7/4) + \pi/4], I_0 \sin(\theta_0) \times \cos[\delta(t, z)])$. Here the phase $\delta(t, z) = kz - \omega t$ is the phase of the wave when reaching the detector, and the sine and cosine of θ_0 are due to the angle between the original polarization and the magnetic field when the radiation enters the magnetic field as expressed in the solution (10). Based in the values (12) with $\theta_0 = \pi/4$ (that corresponds to the maximum effect) and assuming $\delta(t, z) = 0$, we obtain for the rotations corresponding to $n = 2$, $\varphi_{n=2}^+ \approx 3.78 \times 10^{-6}$ rad. This value is 1 order of magnitude higher than the experimental values measured ($\sim 10^{-7}$). A possible explanation for this result may be due to a different initial value of the angle between the polarization and the magnetic field, θ_0. If we consider for instance $\theta_0 = \pi/48$ we obtain $\varphi_{n=2}^+ \approx 2.48 \times 10^{-7}$ rad, which is closer to the experimental observed rotation. Other possibilities may be due to an over-estimate the effective path length due either to decoherence, unaccounted losses, or other sources of uncertainty.

We have presented a new effect of vacuum polarization rotation due to external strong magnetic fields based in pure QED. This effect generates an infinite number of harmonics and explains, both qualitatively and quantitatively, the PVLAS experimental results [7]. Our results show that the PVLAS experiment gives (as far as we know for the first time) a beautiful demonstration of the QED properties of vacuum and have beforehand registered an effect that has not been justified at theoretical level. Our results may dismiss the recent claim of the finding of the axion [8]. Both for future use in experiments currently being planned and for a correct interpretation of already detected (and to be detected) physical phenomena [11], a full analysis of existent data is required. In particular, we note that, if the noise level can be decreased, the third peak ($n = \pm 3$) should be detected.

[1] W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936).
[2] J. Schwinger, Phys. Rev. **82**, 664 (1951).
[3] Z. Bialynicka-Birula and I. Bialynicka-Birula, Phys. Rev. D **2**, 2341 (1970).
[4] S. L. Adler, Ann. Phys. (N.Y.) **67**, 599 (1971).
[5] L. Maiani, R. Petronzio, and E. Zavattini, Phys. Lett. B **175**, 359 (1986).
[6] G. Raffelt and L. Stodolski, Phys. Rev. D **37**, 1237 (1988).
[7] E. Zavattini *et al.*, Phys. Rev. Lett. **96**, 110406 (2006).
[8] S. Lamoreaux, Nature (London) **441**, 31 (2006).
[9] E. Lundström *et al.*, Phys. Rev. Lett. **96**, 083602 (2006).
[10] M. Marklund and P. K. Shukla, Rev. Mod. Phys. **78**, 591 (2006).
[11] E. Massó, hep-ph/0607215; Y. N. Gnedin, M. Y. Piotrovich, and T. M. Natsvlishvili, astro-ph/0607294; U. Kotz, A. Ringwald, and T. Tschentscher, hep-ex/0606058; E. Massó and J. Redondo, hep-ph/0606163; A. V. Afanasev, O. K. Baker, and K. W. McFarlane, hep-ph/0605250.