Controlled Flow of Spin-Entangled Electrons via Adiabatic Quantum Pumping

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We propose a method to dynamically generate and control the flow of spin-entangled electrons, each belonging to a spin singlet, by means of adiabatic quantum pumping. The pumping cycle functions by periodic time variation of localized two-body interactions. We develop a generalized approach to adiabatic quantum pumping as traditional methods based on a scattering matrix in one dimension cannot be applied here. We specifically compute the flow of spin-entangled electrons within a Hubbard-like model of quantum dots, discuss possible implementations, and identify parameters that can be used to control the singlet flow.

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Entanglement is one of the most intriguing features that distinguish the quantum world from the classical. In recent years [1], potential applications in quantum information and computation have stimulated renewed vigor in the study of entangled states. Since Bohm's [2] reformulation of the Einstein-Podolsky-Rosen paradox [3] in terms of pairing of spins, spin singlets have become the canonical example of an elementary entangled state. From an experimental standpoint, manipulating singlet pairs of electron spins, in particular, is promising because of the vast expertise already available in solid-state electronics [4].

Several proposals have emerged [5–8] that aim to generate a controlled flow of electron spin singlets using Coulomb blockade and tunneling through quantum dots. This is a challenging endeavor because it is necessary to suppress the natural tunneling of unwanted single electrons without hampering the singlets, which must be delivered well before the device decoherence time. In this Letter, we present a scheme that generates a selective flow of only electrons belonging to singlets. As a result, it is not necessary to introduce mechanisms that remove single electrons but can end up impeding the singlet flow. Our scheme relies on a generalized form of adiabatic quantum pumping that is induced by localized interactions. Along with other attractive characteristics that we discuss below, this process is appealing since the absence of bias and the adiabatic nature of the pumping could ease its integration into a quantum information device.

The notion of quantum pumping has its roots in a speculative paper by Thouless [9] in 1983, but advances in nanoscale transport have led to a renewed and growing interest in the phenomenon in recent years both theoretically and experimentally [10–17]. Quantum pumping is a coherent process that creates a direct current in the absence of any bias through a nanoscale device, by changing its scattering properties periodically through independent adiabatic variation of two or more physical parameters. Adiabatic quantum pumping of charge [10], spin [13–15], and thermal [16] currents has been considered. However, previous studies have generally relied on a theoretical description based on transmission and reflection coefficients which cannot be used to describe the pumping of singlets due to localized interactions. In this Letter, we develop a significant generalization of the theory of quantum pumping that can describe such a flow of electron singlet pairs.

Singlet current.—We consider a single available channel in a quasi-one-dimensional mesoscopic conductor connected to macroscopic contacts [Fig. 1(a)]. In the presence of two-body interactions, the charge current can be defined in terms of the two-particle reduced density matrix ρ_2. If the interaction does not affect spins, the reduced density matrix separates into four independent spin subspaces, one singlet with a symmetric spatial part and three triplets with antisymmetric spatial parts; the singlet current is therefore equivalent to the charge current associated with the symmetric spatial part (ρ_2^S) of the two-particle reduced density matrix

$$J_S(x_1, t) = \frac{e\hbar}{m} \int dx_2 \, \mathrm{Im}\{\partial_{x_1} \rho_2^S(x_1, x_2; x_1', x_2; t)\}_{x_1 = x_1'}, \quad (1)$$

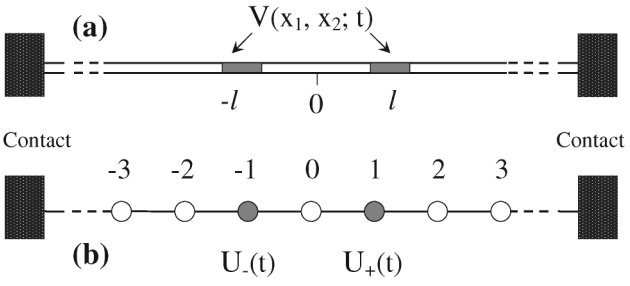

FIG. 1. (a) Schematic figure of a one-dimensional system in which the two-body interaction $V(x_1, x_2; t)$ acts when x_1 and x_2 are both within a finite interval near $-l$ or when both are within a finite interval near l. (b) A tight-binding model where the two-body interaction is present only at lattice sites $m = \pm 1$; the interaction strengths $U_{\pm}(t)$ at the two sites are the time-dependent pumping parameters.

where Im signifies the imaginary part. We consider a situation in which two-body interactions are localized, meaning that they are nonvanishing only when the particles are in certain finite intervals, as shown in Fig. 1(a). Such interactions lend themselves to a scattering description, and the current can be approximately evaluated by expanding the density matrix in terms of two-particle scattering states

$$J_S(x_1,t) \simeq \frac{e\hbar}{2m}\int dE F(E)\int\frac{dk_1}{2\pi}\int\frac{dk_2}{2\pi}\delta\left(\frac{\hbar^2k_1^2}{2m}+\frac{\hbar^2k_2^2}{2m}-E\right) \times\int dx_2\,\mathrm{Im}\{\partial_{x_1}\Psi_{k_1,k_2}(x_1,x_2,t) \times\Psi^*_{k_1,k_2}(x_1',x_2,t)\}_{x_1=x_1'}. \quad (2)$$

Here E denotes the energy required to remove a pair of particles from the many-body ground state, and $F(E)$ is the distribution of this pair energy. The effect of interaction on the current is determined completely by the two-particle singlet scattering states Ψ_{k_1,k_2} arising from free singlet states $\Phi_{k_1,k_2}(x_1,x_2)=\frac{1}{\sqrt{2}}[\phi_{k_1}(x_1)\phi_{k_2}(x_2)+\phi_{k_1}(x_2)\phi_{k_2}(x_1)]$, where $\phi_k(x)$ denotes a single particle plane wave state with momentum $\hbar k$.

Pumped current through adiabatic perturbation.—The two-particle scattering states, and therefore the current, are determined by the interaction $V(\bar{x},t)$ between a pair of particles; we take it to be time-dependent and to occur only in a finite region $|x_i|<l$. Most importantly, the interaction $V(\bar{x},t)$ is chosen to be localized so that it affects only singlets, thereby naturally eliminating the flow of triplets in the absence of a bias. When the characteristic period ω of the time variation of the potential is slow compared to the time δt the particles dwell in the scattering region [18] $\omega\times\delta t\ll 1$, we can apply adiabatic perturbation theory to express the scattering states of the time-dependent Hamiltonian in terms of the instantaneous states up to linear order

$$\Psi_{\bar{k}}(\bar{x},t)\simeq\Psi^t_{\bar{k}}(\bar{x})-i\hbar\int d\bar{x}'G^t(\bar{x},\bar{x}';E)\frac{\partial}{\partial t}\Psi^t_{\bar{k}}(\bar{x}'). \quad (3)$$

We use the notation $\bar{x}\equiv\{x_1,x_2\}$ and $\bar{k}\equiv\{k_1,k_2\}$, so that $G^t(\bar{x},\bar{x}';E)$ is the two-particle instantaneous retarded Green's function for the full Hamiltonian. The instantaneous state $\Psi^t_{\bar{k}}(\bar{x})$ is a solution of the time-independent Lippmann-Schwinger equation for the potential $V(\bar{x}',t)$ at the specific time t,

$$\Psi^t_{\bar{k}}(\bar{x})=\Phi_{\bar{k}}(\bar{x})+\int d\bar{x}'G_0(\bar{x},\bar{x}';E)V(\bar{x}',t)\Psi_{\bar{k}}(\bar{x}'), \quad (4)$$

where G_0 is the free two-particle retarded Green's function. Taking the time derivatives of the defining equations for $\Psi^t(\vec{x}')$ and $G^t(\bar{x},\bar{x}';E)$ enables us to express the second term in Eq. (3), which is linear in ∂_t, as

$$\Delta\Psi_{\bar{k}}(\bar{x},t)=-i\hbar\int d\bar{x}'\int d\bar{x}''G^t(\bar{x},\bar{x}';E)G^t(\bar{x}',\bar{x}'';E) \times\dot{V}(\vec{x}'',t)\Psi^t_{\bar{k}}(\bar{x}''). \quad (5)$$

If there is no bias or time dependence, the laws of thermodynamics demand that there should be no current; we explicitly confirm that our expression for the current satisfies this essential physical requirement. The net current in the absence of time dependence is evaluated by using the zeroth order term from Eq. (3) for the scattering state in Eq. (2): $\Psi_{k_1,k_2}(x_1,x_2,t)\equiv\Psi^t_{k_1,k_2}(x_1,x_2)$. The resulting expression can be simplified by relating the imaginary part of the retarded Green's function to the free singlet states: $\mathrm{Im}\{G_0(\bar{x},\bar{x}';E)\}=-\pi\int dk_1\int dk_2\delta[(\hbar^2k_1^2/2m)+(\hbar^2k_2^2/2m)-E]\Phi_{\bar{k}}(\bar{x})\Phi^*_{\bar{k}}(\bar{x}')$. Then repeated use of the Lippmann-Schwinger equation and properties of the Green's functions shows that the net current corresponding to the zeroth order $\Psi^t(\bar{x})$ vanishes.

After confirming that our expression cannot produce spontaneous current, we evaluate the singlet current induced by the adiabatic time evolution. To linear order in the time dependence, this involves the evaluation of

$$\int dx_2\,\mathrm{Im}\{\partial_{x_1}\Psi^t_{\bar{k}}(x_1,x_2)\Delta\Psi^*_{\bar{k}}(x_1',x_2,t)\}_{x_1=x_1'} \quad (6)$$

within the expression for the current in Eq. (2). A calculation employing standard Green's function identities, similar to that for the zeroth order, leads to an expression for the net amount of singlet entangled electron pairs pumped in a complete cycle of period τ, yielding the main result of this Letter:

$$Q_S(\tau)=\frac{e\hbar^2}{2\pi m}\int_0^\tau dt\int dE F(E)\frac{\partial}{\partial E}\Big[\int d\bar{x}'\dot{V}(\bar{x}',t) \times\int dx_2\,\mathrm{Im}\{G^{t*}(\bar{x},\bar{x}';E)\partial_{x_1}G^t(\bar{x},\bar{x}';E)\}\Big]. \quad (7)$$

Singlet pumping in a turnstile model.—We illustrate our results with a tight-binding model [Fig. 1(b)] with two Hubbard impurities located at sites $-m$, m

$$V(\bar{n},t)=U_-(t)\delta_{n_1,-m}\delta_{n_2,-m}+U_+(t)\delta_{n_1,m}\delta_{n_2,m}. \quad (8)$$

Two electrons can interact only if they are both together at one of those two sites; therefore, due to the Pauli principle, only singlets are affected. The strengths of the interactions $U_\pm(t)$ are the two time-dependent pumping parameters. This concept is similar to a "turnstile model" [11] but differs significantly in that, instead of time-varying external potentials, the two-body interaction among electrons is varied in time. We separately derived a discrete version of Eq. (7); the end result amounts to replacing the coordinate arguments with site indices $x\to n$, integrals by sums, and derivatives with a finite difference form. A lengthy calculation leads to an expression for the singlets pumped in a complete cycle in terms of the free two-particle lattice

Green's function, specifically, two of its matrix elements $G_0(0) = G_0(\bar{m}, \bar{m}; E)$ and $G_0(2\bar{m}) = G_0(\bar{m}, -\bar{m}; E)$

$$Q_S(\tau) = \frac{-e}{2\pi}\int_0^\tau dt \int dE F(E) \frac{\partial}{\partial E} \sum_{\pm} \dot{U}_{\pm}(t) \frac{|T_{\pm}(t)|^2[\mathrm{Im}\{G_0(0)\}(1+|T_{\mp}(t)G_0(2\bar{m})|^2) \pm 2\mathrm{Im}\{T_{\mp}(t)G_0(2\bar{m})G_0^{\pm}(2\bar{m})\}]}{U_{\pm}(t)^2|1 - T_{\mp}(t)T_{\pm}(t)G_0(2\bar{m})G_0(2\bar{m})|^2}. \quad (9)$$

Here $\bar{m} \equiv \{m, m\}$, and $T_{\pm}(t) = 1/(U_{\pm}^{-1}(t) + G_0(0))$ is the T matrix for a single Hubbard impurity, and $G_0^{\pm} \equiv G_0(2\bar{m})$ for $\lambda = \pm i|\lambda|$, when expressed in the form $G_0(2\bar{m}) = \int_0^{\pi} \frac{dk}{2\pi}[e^{2m(ik-\lambda)}/\sinh(\lambda)]$ with $\cosh(\lambda) = E/2 - \cos(k)$.

Exact analytical forms exist for the lattice Green's functions G_0 in terms of elliptic integrals [19]. For the purpose of numerical estimates, we assume a square-profile time dependence [12] in the plane of the parameters $U_{\pm}(t)$, shown in Fig. 2(a), where the two parameters change alternately between a minimum value $U_{\min}$ and a maximum value $U_{\max}$. The pair distribution function is taken to be a Fermi function $F(E) \simeq 1/[e^{(E-\mathcal{E})/kT} + 1]$. At low temperatures, an integration by parts with respect to energy yields an expression for the pumped singlets in terms of the maximum energy $\mathcal{E}$ available for a pair.

We express all energies in units of the nearest neighbor coupling strength J. The on site energy of each tight-binding site is taken to be zero. In Fig. 2(b), we plot the net singlets pumped in a single cycle as a function of the size and location of the square footprint of the time cycle in the space of the parameters $U_{\pm}$; the flow depends on the enclosed region. Figure 3 shows the dependence of the singlet current on the parameter $\mathcal{E}$ that measures the available energy for pairs of electrons determined by the chemical potential in the contacts. The two curves in the figure correspond to different locations of the Hubbard impurities, at lattice sites $m = \pm 1$ and $m = \pm 2$, illustrating the significant effect the spatial separation of two impurities has on the pumping rate. The direction of flow can also reverse for certain values of the various parameters; reversing the time cycle is not the only way to reverse the direction of the current [10]. Quantum pumping has the intrinsic property that the magnitude of the pumped quantity, in this case singlets, is continuous in nature, so that the delivery rate per cycle can be continuously adjusted. Thus, there are several ways to precisely control the magnitude and direction of the flow of singlets dynamically.

Discussion and outlook.—The turnstile model could be implemented by taking the interaction sites to be quantum dots, with the Coulomb interaction among electron pairs varied periodically by changing the shape of the confining potential. With two independently controllable parameters for each dot, the interaction strength can be varied, while keeping the single electron energy in the dot fixed and far off resonance with the chemical potential in the leads. Such a setup will suppress single electron pumping which relies on resonance [11]. Furthermore, arranging the pumping cycle to follow a path where only singlets can form in the quantum dots while triplets and other higher energy states are not energetically accessible, one can assure that only singlets will be pumped.

One way to realize this would be to use two concentric top gates with independently controllable voltages. Adjusting the difference between the inner and outer gate voltages would change the transverse confining potential and hence the electron-electron interaction. Simultaneously changing the sum of the potentials would allow a compensating shift of the single electron energy to keep it fixed. Estimates show that a 20% change in the gate voltages would yield a 10% variation of the interaction strength and, hence, of the pumping parameters $U_{\pm}$ [20].

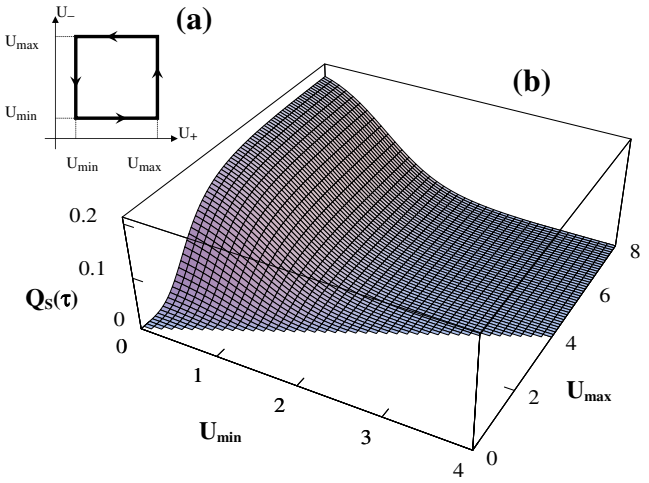

FIG. 2 (color online). (a) Pumping cycle in the space of parameters $U_{\pm}$. (b) Net singlets pumped in a single cycle as a function of the location and size of the square pumping cycle as $U_{\min}$ and $U_{\max}$ are varied. Here $U_{\min}$ and $U_{\max}$ are in units of intersite coupling strength J.

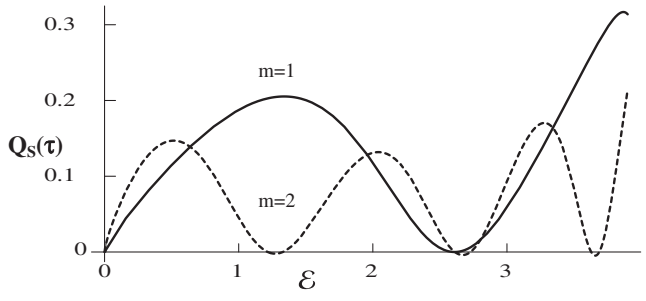

FIG. 3. Net singlets pumped per cycle as a function of the maximum available energy $\mathcal{E}$ for pairs. The two curves correspond to different separations of the Hubbard impurities: the solid line for impurities lattice sites ± 1 and the dashed line for impurities at sites ± 2.

The adiabatic condition requires that the period satisfy $\tau \gg \delta t$, where the dwell time is given by $\delta t = d/v$, with d the size of the scattering region and v the carrier velocity [18]. (We get similar results if we adopt the alternate condition $\tau \gg t_{\rm esc}$, where $t_{\rm esc} \sim h/\Delta E$ is the escape time associated with the dot energy level spacing ΔE.) Using typical estimates of interdot coupling $\sim$1 meV and interdot separations of about 50 nm, we estimate group velocities of order 10^5 m/s, so that for a scattering region $d \sim 100$ nm the adiabatic condition would allow $\sim 10^3$ cycles per nanosecond, leaving plenty of room to achieve pumping rates of several singlets per nanosecond. In typical Coulomb-blockade-based schemes, the most optimistic estimates yield a delivery rate of the order of 1 ns per singlet [6,7]. Thus, our approach also has the potential to be faster.

Our result Eq. (7) has the merit that it can also be applied to quantum pumping in systems that allow an independent particle description, as considered in previous studies. All the elements in Eq. (7) then reduce to single particle functions: $F(E) \to f(E)$ is the Fermi distribution function and the Green's function is a single particle one with an asymptotic form $\to -i[m/(\hbar^2 k)]e^{ikx}\psi_k^{t*}(x')$ in terms of the scattering state $\psi_k^t(x')$ and wave vector $k = [2mE/\hbar^2]^{-1/2}$. This yields the pumped charge

$$Q(\tau) = \frac{em}{2\pi\hbar^2}\int_0^\tau dt \int dE f(E)\frac{\partial}{\partial E}\left\{\frac{1}{k}\langle\psi^t|\dot{V}|\psi^t\rangle\right\}, \quad (10)$$

which agrees with expressions derived in earlier works [10,12]. But, unlike most previous treatments, we never use 1D scattering matrix elements, as they do not have useful generalizations when particles interact with each other in a region rather than scatter off an external potential.

To summarize, we have proposed a method based on two-body adiabatic quantum pumping for generating a dynamically controlled flow of spin-entangled electrons. The process is inherently coherent, potentially faster than most current proposals, has reduced noise because the lack of bias suppresses the natural current of single electron flow, and allows for continuous adjustment of the flow through numerous physical parameters. All of these features can be developed and incorporated into a comprehensive scheme to generate a controlled flow of entangled electrons. Our goal here has been to present the basic idea, develop a theoretical framework for its description, and discuss a possible physical model for implementation. We have in the process generalized the treatment of quantum pumping to incorporate interactions that cannot be treated in a scattering matrix approach.

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Note added.—While developing our original proposal [21] into this Letter, we became aware of a creative idea [22] to generate entangled electron-hole pairs using a one-body potential in a scattering matrix approach.

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