Mode-Coupling Control in Resonant Devices: Application to Solid-State Ring Lasers

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We report the theoretical and experimental investigation of the effects of mode coupling in a resonant macroscopic quantum device, in the case of a solid-state ring laser. This is achieved by introducing an additional coupling source whose interplay with the already-existing nonlinear effects ensures the coexistence of two counterpropagating cavity modes yielding a rotation-sensitive beat note. The determination of the condition for rotation sensing, both theoretically and experimentally, allows a quantitative study of the role of various mode-coupling mechanisms, in particular, the gain-induced mode coupling. We point out the connection between our work and the theoretical work on mode coupling in superfluid devices. This work opens up the possibility of new types of active rotation sensors.

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Devices using macroscopic quantum effects [1] and their associated phenomenon of interference to detect rotations can be divided into two classes [2]. In the first class, the rotation-induced phase shift is usually detected by looking at the displacement of an interference pattern. Among such devices are the fiber optic gyroscope [3], the gyromagnetic gyroscope [4], and the atomic interferometer [5]. In the second class, the rotation can be detected through a beat signal. The ring laser [6] and the superfluid in a ring container [e.g., liquid helium [7] or Bose-Einstein condensed gas [8]] are two examples of such devices. In these systems, nonlinear coupling between counterpropagating modes, due, for instance, to the gain medium in ring lasers, hinders rotation sensing, and this effect must be fought [9,10].

In the case of helium-neon ring lasers, nonlinear coupling between counterpropagating modes can be almost suppressed by using a two-isotopes mixture and by tuning the laser emission frequency out of resonance with the atoms at rest. Provided the detuning value is bigger than the atomic natural linewidth, the gain medium can be considered as being inhomogeneously broadened and the gain-induced nonlinear coupling practically vanishes [10].

In the case of solid-state ring lasers, for which the gain medium is homogeneously broadened, the gain-induced nonlinear coupling cannot be suppressed and results in unidirectional lasing [11]. In this Letter, we show how a precise understanding of this coupling can lead to its circumvention, allowing us to demonstrate the possibility of using solid-state ring lasers as rotation sensors. Indeed, adding an additional controllable coupling based on polarization-induced effects, we show how to counteract the gain-induced nonlinear coupling and to obtain a stable beat regime. Our experimental study allows us to plot a nonlinear response curve which can be considered as a direct observation of the gain-induced mode-coupling effects. We note that this study is an example of experimental investigation of mode-coupling control in a resonant device, a problem that has been extensively theoretically studied in other quantum devices, such as superfluid systems [12,13].

In order to describe the dynamics of the solid-state ring laser, we use typical Maxwell-Bloch equations with adiabatic elimination of the polarization [14]. We assume a single identical mode in each direction of propagation and no transverse effect, such that the total field inside the cavity can be written as the sum of two counterpropagating waves:

$$E(x,t) = \mathrm{Re}\left\{\sum_{p=1}^{2} \tilde{E}_p(t) e^{i(\omega t + \varepsilon_p k x)}\right\},$$

where $\varepsilon_p = (-1)^p$, $k = 2\pi/\lambda$ is the average spatial frequency of the laser modes associated with the longitudinal coordinate x and ω is the average angular frequency. The slowly varying envelope approximation leads to the following equations [15]:

$$\frac{d\tilde{E}_{1,2}}{dt} = -\frac{\gamma_{1,2}}{2}\tilde{E}_{1,2} + i\frac{\tilde{m}_{1,2}}{2}\tilde{E}_{2,1} + i\varepsilon_{1,2}\frac{\Omega}{2}\tilde{E}_{1,2} + \frac{\sigma(1-i\delta)}{2T} \times \left(\tilde{E}_{1,2}\int_0^l N dx + \tilde{E}_{2,1}\int_0^l N e^{-2i\varepsilon_{1,2}kx} dx\right), \quad (1)$$

$$\frac{\partial N}{\partial t} = W_{\mathrm{th}}(1+\eta) - \frac{N}{T_1} - \frac{aN|E(x,t)|^2}{T_1}, \quad (2)$$

where we use the following notations: $\gamma_{1,2}$ are the cavity mode losses per time unit, σ the stimulated emission cross section, T the cavity round-trip time, l the length of the gain medium, $N(x,t)$ the population inversion density function, η the relative excess of pump power above the threshold value W_{th}, T_1 the population inversion relaxation

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time, and a the saturation parameter. The rotation-induced angular eigenfrequency difference Ω between the counter-propagating modes is linked to the angular velocity $\dot{\theta}$ by the Sagnac formula [16]:

$$\Omega = \frac{8\pi A}{\lambda c T}\dot{\theta}, \tag{3}$$

where A is the area enclosed by the ring cavity and c the speed of light in vacuum. The parameter $S = 4A/(\lambda c T)$ is known as the scale factor of the cavity. The relative detuning of the cavity modes from the center of the gain line is defined as $\delta = (\omega - \omega_{ab})/\gamma_{ab}$, where ω_{ab} and γ_{ab} are, respectively, the position of the center and the width of the gain curve. For typical solid-state configurations, δ is usually smaller than 10^{-2}. Its effects (among which is the gain-induced dispersion of the refractive index) will therefore be neglected in our analysis.

The terms involving $\tilde{E}_{2,1}$ in Eq. (1) are the usual coupling terms for such an optical system, namely: (1) the coupling induced by light backscattering on the cavity elements (mirrors and amplifying crystal), taken into account phenomenologically using the backscattering coefficients $\tilde{m}_{1,2} = m\exp(i\varepsilon_{1,2}\theta_{1,2})$; although this effect is usually small (i.e., $m \ll \gamma_{1,2}\eta$) and decreases when Ω increases, it cannot be neglected for a correct description of the ring laser dynamics; (2) the coupling caused by the establishment of a population inversion grating in the gain medium; this effect is taken into account by the term proportional to the spatial harmonic of N at the order $2k$; as pointed out, for example, by [11], this coupling hinders the coexistence of the two counterpropagating modes because it results in the cross-saturation parameter being bigger than the self-saturation parameter which leads, in the absence of any other coupling source, to single mode (unidirectional) operation.

Under the conditions:

$$|\Omega| \gg \sqrt{\frac{\gamma_{1,2}\eta}{T_1}}, m \quad \text{and} \quad \eta \ll 1, \tag{4}$$

a perturbation method applied to Eqs. (1) and (2) shows that the coupling due to the population inversion grating can be described by the effective coefficient $\tilde{N}$, given by:

$$2\tilde{N} = \frac{\gamma\eta}{1 + \Omega^2 T_1^2}, \tag{5}$$

where $\gamma = (\gamma_1 + \gamma_2)/2$.

In order to counteract the destabilizing effect of the population inversion grating, an additional source of coupling is introduced. This is done by creating losses which depend on the intensity difference between the counter-propagating modes according to the following law:

$$\gamma_{1,2} = \gamma - \varepsilon_{1,2}K(a|\tilde{E}_1|^2 - a|\tilde{E}_2|^2), \tag{6}$$

where K is chosen to be positive so that the mode with the higher intensity gets the higher losses. The associated effective coupling coefficient is on the order of $2K\eta$. A way of generating such losses without introducing any additional element in the laser cavity will be described further in this Letter.

Using typical experimental values, we plotted $\tilde{N}$ and $2K\eta$ as a function of $\dot{\theta}$ (Fig. 1). For simplicity, we consider only the case $\dot{\theta} > 0$. Two ranges of rotation speed appear, delimited by the following critical value:

$$\dot{\theta}_{\text{cr}} = \frac{1}{2\pi S T_1}\sqrt{\frac{\gamma}{4K} - 1}. \tag{7}$$

In the zone where $\dot{\theta} < \dot{\theta}_{\text{cr}}$ the coupling generated by the population inversion grating dominates, while in the zone where $\dot{\theta} > \dot{\theta}_{\text{cr}}$ it is the additional stabilizing coupling that dominates. This latter zone turns out to be the zone of rotation-sensitive operation, as we will see further.

To proceed, Eqs. (1) and (2) have been solved under the conditions (4). Looking for a solution corresponding to the beat regime, i.e., obeying the following conditions:

$$\left|\frac{|\tilde{E}_1|^2 - |\tilde{E}_2|^2}{|\tilde{E}_1|^2 + |\tilde{E}_2|^2}\right| \ll 1 \quad \text{and} \quad |\dot{\Phi} - \Omega| \ll |\Omega|, \tag{8}$$

where Φ is the difference between the arguments of $\tilde{E}_1$ and $\tilde{E}_2$, we obtain the following expression for the relative intensity difference in the beat regime:

$$\frac{|\tilde{E}_1|^2 - |\tilde{E}_2|^2}{|\tilde{E}_1|^2 + |\tilde{E}_2|^2} = \frac{m^2 \sin(\theta_1 - \theta_2)}{2\Omega(\tilde{N} + 2\eta K)}. \tag{9}$$

Considering first the absence of additional coupling ($K = 0$), it can be seen in Eq. (9) using expression (5) that the self-consistency conditions (8) are violated for high values of Ω. This confirms that, indeed, the beat regime does not exist for solid-state ring lasers at high rotation speeds. Instead, the laser turns to unidirectional operation, the direction of emission depending on the sign of Ω [17].

In the case where the additional coupling is present ($K \neq 0$), the relative intensity difference goes to zero for high values of Ω, satisfying the self-consistency conditions (8). It can then be shown that the beat regime is stable if:

$$2\eta K > \tilde{N}. \tag{10}$$

Inequality (10) is equivalent to $\dot{\theta} > \dot{\theta}_{\text{cr}}$, which means that the zone on the right of Fig. 1 is indeed the zone of rotation sensing operation. It is remarkable that even for very small (positive nonzero) values of K the beat regime is stable provided the rotation speed is high enough.

We now report the experimental achievement of mode-coupling control. The additional coupling is introduced in the cw diode-pumped Nd:YAG ring laser cavity using the setup of Fig. 2. This configuration, similar to that used in single mode unidirectional ring lasers [18], is based on the combination of three polarization-related effects. The first effect is a reciprocal rotation of the polarization plane by the use of a slightly nonplanar cavity. The rotation angle α

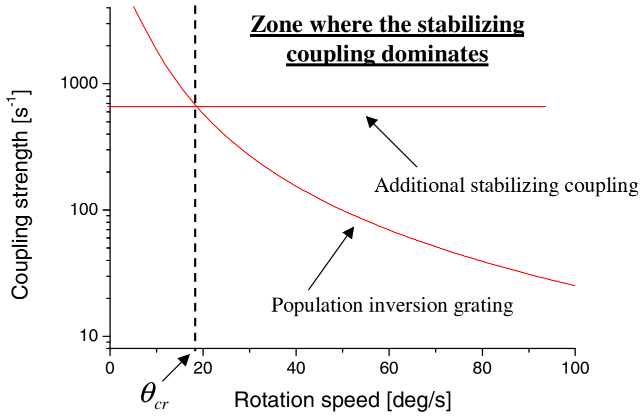

FIG. 1 (color online). Effective coupling coefficients $\tilde{N}$ and $2\eta K$ as a function of the rotation speed $\dot{\theta}$ (logarithmic vertical scale). The two ranges of rotation speed are delimited by the value $\dot{\theta}_{cr}$, given by Eq. (7). Below this value the coupling due to the population inversion grating dominates, while above this value it is the additional stabilizing coupling that dominates.

depends on the geometry of the cavity. The second effect is a nonreciprocal (Faraday) rotation of the polarization plane, produced by a solenoid placed around the Nd:YAG rod. The rotation angle β is proportional to the current flowing through the solenoid. The third effect is a polarizing effect, achieved by replacing one of the cavity mirrors with a polarizing mirror. The differential losses are then to the first order equal to $\gamma_1 - \gamma_2 = 4\alpha\beta/T$. The light intensities of the two counterpropagating modes are monitored by two photodiodes, and the value of β is kept proportional to the difference between those intensities by the mean of an electronic feedback loop acting on the current flowing inside the solenoid. This results in differential losses of the form (6).

The two ranges of Fig. 1 have been identified experimentally, with the measured value $\dot{\theta}_{cr} \simeq 19$ deg/s. Below this critical rotation speed, we observe instabilities of the intensities of the modes, and no beat signal. Above $\dot{\theta}_{cr}$, the intensities of the two modes are stable and similar in magnitude and a beat signal is observed. The measured

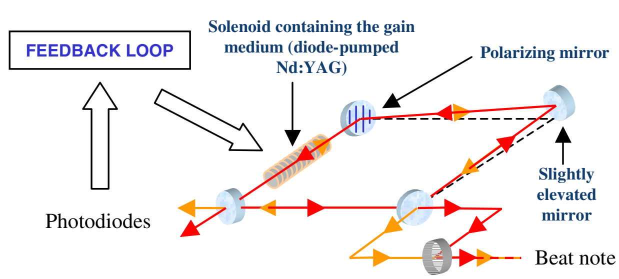

FIG. 2 (color online). Schematic representation of the experimental setup. We use a nonplanar ring laser cavity of about 25 cm of perimeter, enclosing a surface of about 34 cm^2 and with a 2 cm-long Nd:YAG rod. The skew rhombus angle is on the order of 10^{-2} rad. The whole device is placed on a turntable.

frequency of this beat signal as a function of the rotation speed is reported in Fig. 3. This frequency response curve matches the Sagnac line for high values of the rotation speed. For slower rotations (but still above $\dot{\theta}_{cr}$), we observe a deviation from the Sagnac line. This deviation is partly due to the linear coupling induced by backscattering on the cavity elements [also present in He-Ne ring laser gyroscopes [10]] and mostly due to the nonlinear coupling induced by the population inversion grating (typical from solid-state ring lasers). The beat frequency $\langle\dot{\Phi}\rangle$ (where $\langle\rangle$ stands for time averaging) can be estimated analytically under the conditions (8), leading to the following expression:

$$\langle\dot{\Phi}\rangle = \Omega + \frac{m^2\cos(\theta_1 - \theta_2)}{2\Omega} + \frac{\gamma\eta}{2\Omega T_1}. \qquad (11)$$

This expression is in good agreement with our measurements for $\dot{\theta} \gtrsim 50$ deg/s (dashed curve on Fig. 3). The linear dependence of $\langle\dot{\Phi}\rangle$ on the pump power (represented by the parameter η) for a fixed rotation speed has been checked experimentally. The result is given for $\dot{\theta} = 70$ deg/s on Fig. 4. This constitutes a direct observation of the coupling induced by the population inversion grating, whose strength is proportional to η and can thus be controlled easily. This observation is made possible because this coupling is particularly strong in a solid-state ring laser, as compared, for example, to a gas ring laser.

The effects measured in our system can be related to nonlinear phenomena that have been extensively studied for other macroscopic quantum devices [12,13]. To illustrate this, let us consider the two-level toy model of [9] for

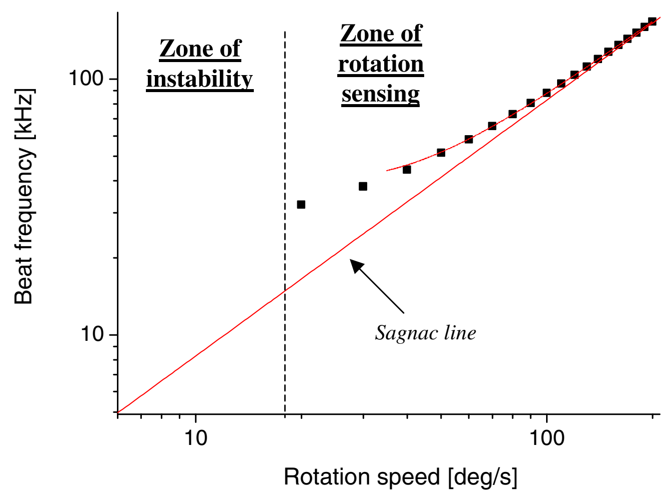

FIG. 3 (color online). Experimental frequency response of the solid-state ring laser gyroscope for $\eta \simeq 0,17$ (logarithmic horizontal and vertical scales). From the asymptotic linear dependence, we get a measurement of the scale factor $S = 0,83 \pm 0,01$ kHz/(deg/s), which is in perfect agreement with the prediction resulting from the Sagnac formula (3). For rotation speeds lower than $\dot{\theta}_{cr} \simeq 19$ deg/s, no beat frequency is measured.

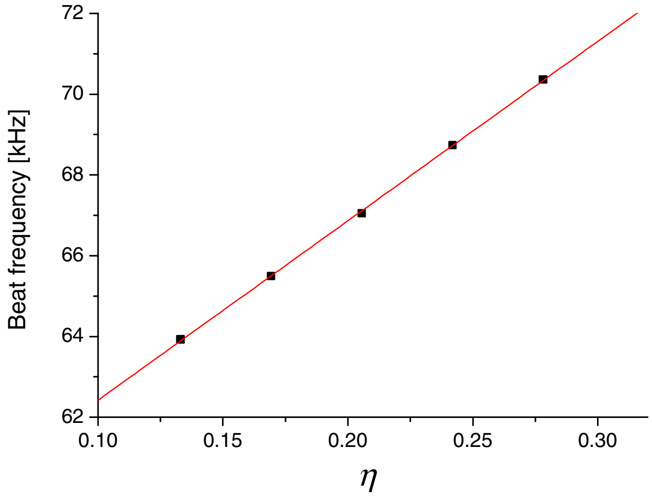

FIG. 4 (color online). Beat frequency as a function of the pump power (represented by the parameter η) for $\dot{\theta} = 70\ \mathrm{deg/s}$ (error bars are smaller than 100 Hz). The linear shift is in agreement with Eq. (11). This is a direct observation of the mode coupling due to the population inversion grating in the solid-state ring laser.

a superfluid in a ring container. In this model, the condition for the system to be rotation sensitive reads:

$$V_0 > g, \tag{12}$$

V_0 being the asymmetry energy and g the mean (repulsive) interaction energy per particle in the s-wave state. Condition (12) reflects the fact that a change in the quantum of circulation around the ring, which is a signature of rotation, is more difficult when repulsive interactions are stronger and is easier when the trap is asymmetric [19]. Condition (12) has to be compared with condition (10). In both cases, these conditions express the fact that the system is rotation sensitive provided that a ''good'' additional coupling is stronger than a ''bad'' intrinsic nonlinear coupling.

In conclusion, the solid-state ring laser turns out to be a good system for studying mode coupling in a resonant macroscopic quantum device. The theoretical and experimental derivation of the rotation sensing condition allowed for quantitative observation and control of the coupling induced by the population inversion grating. Our experimental results are in very good agreement with our theoretical model, in which the couplings usually present in a ring laser cavity are taken into account, and in which mode-coupling control is achieved via an additional loss term. In addition to a better understanding of mode couplings in periodic boundary systems that can help in the study of more complex systems like toroidal superfluids or toroidal Bose-Einstein condensates, this work opens up perspectives for the realization of new active gyrometers.

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[19] This is due to the fact that the quantum of circulation around the ring is a topological invariant. For a vortex to exit or enter the ring, the density has to vanish somewhere on the ring, something which is resisted by repulsive interactions and favored by the presence of an asymmetric potential. For more details, see, for example, D. Rokshar, cond-mat/9709212 or Ref. [9].