Enhanced Effect of Temporal Variation of the Fine Structure Constant and the Strong Interaction in ^{229}Th

V. V. Flambaum

School of Physics, The University of New South Wales, Sydney NSW 2052, Australia

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The relative effects of the variation of the fine structure constant $\alpha = e^2/\hbar c$ and the dimensionless strong interaction parameter $m_q/\Lambda_{\rm QCD}$ are enhanced by 5–6 orders of magnitude in a very narrow ultraviolet transition between the ground and the first excited states in the ^{229}Th nucleus. It may be possible to investigate this transition with laser spectroscopy. Such an experiment would have the potential of improving the sensitivity to temporal variation of the fundamental constants by many orders of magnitude.

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Unification theories applied to cosmology suggest a possibility of variation of the fundamental constants in the expanding Universe (see, e.g., the review in Ref. [1]). There are hints of variation of α and $m_{q,e}/\Lambda_{\rm QCD}$ in quasar absorption spectra, big bang nucleosynthesis, and Oklo natural nuclear reactor data (see [2], and references therein). Here $\Lambda_{\rm QCD}$ is the quantum chromodynamics (QCD) scale, and m_q and m_e are the quark and electron masses, respectively. However, the majority of publications report only limits on possible variations (see, e.g., reviews in Refs. [1,3]). A very sensitive method to study the variation in a laboratory consists of the comparison of different optical and microwave atomic clocks (see recent measurements in Refs. [4–10]). An enhancement of the relative effect of α variation can be obtained in a transition between almost degenerate levels in a Dy atom [11]. These levels move in opposite directions if α varies. An experiment is currently underway to place limits on α variation using this transition [12], but, unfortunately, one of the levels has quite a large linewidth and this limits the accuracy. An enhancement of 1–3 orders exists in narrow microwave molecular transitions [13]. Some atomic transitions with enhanced sensitivity are listed in Ref. [14].

A very narrow level (3.5 ± 1) eV above the ground state exists in the ^{229}Th nucleus [15] [in Ref. [16], the energy is (5.5 ± 1) eV]. The position of this level was determined from the energy differences of many high-energy γ transitions (between 25 and 320 KeV) to the ground and excited states. The subtraction produces the large uncertainty in the position of the 3.5 eV excited state. The width of this level is estimated to be about 10^{-4} Hz [17]. This would explain why it is so hard to find the direct radiation in this very weak transition. The direct measurements have only given experimental limits on the width and energy of this transition (see, e.g., [18]). A detailed discussion of the measurements (including several unconfirmed claims of the detection of the direct radiation) is presented in Ref. [17]. However, the search for the direct radiation continues [19]. The aim of the present work is to provide a new strong motivation.

The ^{229}Th transition is very narrow and can be investigated with laser spectroscopy. This makes ^{229}Th a possible reference for an optical clock of very high accuracy and opens a new possibility for a laboratory search for the variation of the fundamental constants [20]. Below, I will show that there is an additional very important advantage. The relative effects of variation of α and $m_q/\Lambda_{\rm QCD}$ are enhanced by 5–6 orders of magnitude.

Note that there are other narrow low-energy levels in nuclei, e.g., the 76 eV level in ^{235}U with the 26.6 min lifetime (see, e.g., [20]). One may expect a similar enhancement there. Unfortunately, this level cannot be reached with usual lasers. In principle, it may be investigated using a free-electron laser or synchrotron radiation. However, the accuracy of the frequency measurements is much lower in this case.

The ground state of the ^{229}Th nucleus is $J^P[Nn_z\Lambda] = 5/2^+[633]$; i.e., the deformed oscillator quantum numbers are $N = 6$, $n_z = 3$, the projection of the valence neutron orbital angular momentum on the nuclear symmetry axis (internal z axis) is $\Lambda = 3$, the spin projection $\Sigma = -1/2$, and the total angular momentum and the total angular momentum projection are $J = \Omega = \Lambda + \Sigma = 5/2$. The 3.5 eV excited state is $J^P[Nn_z\Lambda] = 3/2^+[631]$; i.e., it has the same $N = 6$ and $n_z = 3$. The values $\Lambda = 1$, $\Sigma = 1/2$, and $J = \Omega = 3/2$ are different. The energy of both states may be described by an equation [21] $E = E_0 + C\Lambda\Sigma + D\Lambda^2$; i.e., the energy difference between the excited and ground state is $\omega = E_e - E_g = 2C - 8D$. The values of the constants C and D are presented, for example, in Ref. [21]. Note that ω is 5 orders of magnitude smaller than C and D. Therefore, for consistency of this simple valence model, we must take $2C \approx 8D$. Based on the data from [21], we will use the following numbers: $2C \approx 8D \approx -1.4$ MeV. The relative variation of the transition frequency may be presented as

$$\frac{\delta\omega}{\omega} = \frac{\delta(2C) - \delta(8D)}{\omega} \approx 0.4 \times 10^6 \left(\frac{\delta D}{D} - \frac{\delta C}{C}\right). \quad (1)$$

The large factor here appeared from the ratio $2C/\omega \approx 8D/\omega \approx -0.4 \times 10^6$ for $\omega = 3.5$ eV. The orbit-axis interaction constant D vanishes for zero deformation parameter β_2. Therefore, we should assume that $D \approx \text{const} \times V_0\beta_2$, where V_0 is the depth of the strong potential. The nuclear deformation reduces the energy of the Coulomb repulsion between the protons. Without this repulsion, the deformation parameter β_2 would probably be zero [22]. Therefore, it is natural to assume that $\beta_2 \approx \text{const} \times \alpha$. Thus, we have $D \approx \text{const} \times V_0 \times \alpha$ and

$$\frac{\delta D}{D} \approx \frac{\delta V_0}{V_0} + \frac{\delta\alpha}{\alpha}. \tag{2}$$

To estimate variation of V_0, we will use the Walecka model [23], where the strong nuclear potential is produced by the sigma and the omega meson exchanges

$$V = -\frac{g_s^2}{4\pi}\frac{e^{-rm_\sigma}}{r} + \frac{g_v^2}{4\pi}\frac{e^{-rm_\omega}}{r}. \tag{3}$$

Using Eq. (3), we can find the depth of the potential well [24,25]

$$V_0 = \frac{3}{4\pi r_0^3}\left(\frac{g_s^2}{m_\sigma^2} - \frac{g_v^2}{m_\omega^2}\right). \tag{4}$$

Here $r_0 = 1.2$ fm is an internucleon distance. Note that the nuclear potential in this model is a highly tuned small difference of two large terms. Therefore, the contribution of the variation of r_0 is not as important as the contribution of the meson mass variation, which is enhanced due to the cancellation of two terms in V_0. The result is [24]

$$\frac{\delta V_0}{V_0} \approx -8.6\frac{\delta m_\sigma}{m_\sigma} + 6.6\frac{\delta m_\omega}{m_\omega}. \tag{5}$$

The final result will depend on the variation of the dimensionless parameter $m_q/\Lambda_{\mathrm{QCD}}$. During the following calculations, we shall assume that Λ_{QCD} does not vary and so we shall speak about the variation of masses (this means that we measure masses in units of Λ_{QCD}). We shall restore the explicit appearance of Λ_{QCD} in the final answers. The dependence of the meson masses on the current light quark mass $m_q = (m_u + m_d)/2$ has been calculated in Ref. [26]: $\delta m_\sigma/m_\sigma = 0.013(\delta m_q/m_q)$, $\delta m_\omega/m_\omega = 0.034(\delta m_q/m_q)$. This gives us

$$\frac{\delta V_0}{V_0} \approx 0.11\frac{\delta m_q}{m_q}. \tag{6}$$

The relatively small contribution of the light quark mass is explained by the fact that $m_q \approx 5$ MeV is very small. The contribution of the strange quark mass $m_s \approx 120$ MeV may be much larger. According to the calculation in Ref. [24], $\delta m_\sigma/m_\sigma \approx 0.54(\delta m_s/m_s)$, $\delta m_\omega/m_\omega \approx 0.15(\delta m_s/m_s)$, and $\delta V_0/V_0 \approx -3.5(\delta m_s/m_s)$. By adding all contributions, we obtain

$$\frac{\delta D}{D} \approx \frac{\delta\alpha}{\alpha} + 0.11\frac{\delta m_q}{m_q} - 3.5\frac{\delta m_s}{m_s}. \tag{7}$$

The reason for the enhancement here ($\sim$5 times) is the cancellation of the σ and ω meson contributions to V_0 [see Eq. (4)], which appears in the denominator of the relative variation of D: $\frac{\delta D}{D} = (\delta V_0/V_0) + \ldots$. However, the σ and ω mesons contribute with equal sign to the spin-orbit interaction constant C [27]. Therefore, there is no "cancellation" enhancement here. However, there is another efficient mechanism. The spin-orbit interaction is inversely proportional to the nucleon mass M_N squared, $C \propto 1/M_N^2$, $\frac{\delta C}{C} = -2(\delta M_N/M_N)$. The nucleon mass depends on the quark masses: $\delta M_N/M_N = K_q(\delta m_q/m_q) + K_s(\delta m_s/m_s)$, where $K_q = 0.045$ and $K_s = 0.19$ in Refs. [24,28], $K_q = 0.037$ and $K_s = 0.011$ in Ref. [29], and $K_q = 0.064$ in Ref. [26]. All three values of K_q are close to the average value $K_q = 0.05$. However, different methods of calculations give very different values of K_s. Fortunately, this is not important, since the strange mass dependence of the σ meson is much stronger than that of the proton [due to the SU(3) symmetry, σ contains a valence $\bar{s}s$ pair, another factor is strong repulsion of σ from the close K^+K^-, $\bar{K}^0K^0$, $\eta\eta$ states [24]]; also, there is the cancellation enhancement of the σ contribution to D. As a result, we have

$$\frac{\delta D}{D} - \frac{\delta C}{C} \approx \frac{\delta\alpha}{\alpha} + (0.11 + 0.10)\frac{\delta m_q}{m_q} - (3.5 - 0.2)\frac{\delta m_s}{m_s}. \tag{8}$$

The final estimate for the relative variation of the ^{229}Th transition frequency in Eq. (1) is

$$\frac{\delta\omega}{\omega} \approx 10^5\left(4\frac{\delta\alpha}{\alpha} + \frac{\delta X_q}{X_q} - 10\frac{\delta X_s}{X_s}\right)\frac{3.5\ \mathrm{eV}}{\omega}, \tag{9}$$

where $X_q = m_q/\Lambda_{\mathrm{QCD}}$ and $X_s = m_s/\Lambda_{\mathrm{QCD}}$. When obtaining this estimate, we made several assumptions [the Walecka model for the nuclear forces, linear dependence of the deformation parameter on α, and the SU(3) model for the σ meson]. We plan to do a more sophisticated numerical calculation which is not based on these assumptions. Unfortunately, it is quite complicated, and one should not expect the result soon. However, it cannot change the most important conclusion: There is a 5–6-order enhancement in the relative variation of the transition frequency.

To measure the variation of the fundamental constants, one should do repeated measurements of the ratio of the ^{229}Th frequency to a frequency of any other narrow optical or microwave transition (e.g., to the frequency standard, the Cs microwave clock). The variation of the reference frequency can be neglected in comparison with the enhanced variation of the ^{229}Th frequency. Current atomic clock limits on the variation of the fundamental constants are approaching 10^{-15} per year. Therefore, even without any improvement of the measurement accuracy, one can bring the sensitivity to the variation of the fundamental constants to better than 10^{-20} per year. Another advantage is that the width of this nuclear transition is several orders of magnitude smaller than a typical atomic clock width (~1–100 Hz). In principle, this may give another few orders of magnitude improvement. We conclude that the ^{229}Th transition has an enormous potential for a laboratory search for the variation of the fundamental constants.

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[1] J.-P. Uzan, Rev. Mod. Phys. **75**, 403 (2003).
[2] M. T. Murphy, J. K. Webb, and V. V. Flambaum, Mon. Not. R. Astron. Soc. **345**, 609 (2003); V. F. Dmitriev, V. V. Flambaum, and J. K. Webb, Phys. Rev. D **69**, 063506 (2004); S. K. Lamoreaux and J. R. Torgerson, Phys. Rev. D **69**, 121701(R) (2004); E. Reinhold, R. Buning, U. Hollenstein, A. Ivanchik, P. Petitjean, and W. Ubachs, Phys. Rev. Lett. **96**, 151101 (2006).
[3] S. G. Karshenboim, V. V. Flambaum, and E. Peik, in *Handbook of Atomic, Molecular and Optical Physics*, edited by G. W. F. Drake (Springer, New York, 2005), Chap. 30, pp. 455–463.
[4] J. D. Prestage, R. L. Tjoelker, and L. Maleki, Phys. Rev. Lett. **74**, 3511 (1995).
[5] H. Marion *et al.*, Phys. Rev. Lett. **90**, 150801 (2003).
[6] S. Bize *et al.*, physics/0502117.
[7] E. Peik, B. Lipphardt, H. Schnatz, T. Schneider, Chr. Tamm, and S. G. Karshenboim, Phys. Rev. Lett. **93**, 170801 (2004).
[8] S. Bize *et al.*, Phys. Rev. Lett. **90**, 150802 (2003).
[9] M. Fischer *et al.*, physics/0311128.
[10] E. Peik, B. Lipphardt, H. Schnatz, T. Schneider, Chr. Tamm, and S. G. Karshenboim, Laser Phys. **15**, 1028 (2005).
[11] V. A. Dzuba, V. V. Flambaum, and J. K. Webb, Phys. Rev. A **59**, 230 (1999).
[12] A. T. Nguyen, D. Budker, S. K. Lamoreaux, and J. R. Torgerson, Phys. Rev. A **69**, 022105 (2004).
[13] V. V. Flambaum, Phys. Rev. A **73**, 034101 (2006).
[14] E. J. Angstmann, V. A. Dzuba, V. V. Flambaum, S. G. Karshenboim, and A. Yu. Nevsky, J. Phys. B **39**, 1937 (2006).
[15] R. G. Helmer and C. W. Reich, Phys. Rev. C **49**, 1845 (1994).
[16] Z. O. Guimaraes-Filho and O. Helene, Phys. Rev. C **71**, 044303 (2005).
[17] E. V. Tkalya, A. N. Zherikhin, and V. I. Zhudov, Phys. Rev. C **61**, 064308 (2000); A. M. Dykhne and E. V. Tkalya, Pis'ma Zh. Eksp. Teor. Fiz. **67**, 233 (1998) [JETP Lett. **67**, 251 (1998)].
[18] I. D. Moore, I. Ahmad, K. Bailey, D. L. Bowers, Z.-T. Lu, T. P. O'Connor, and Z. Yin, Argonne National Laboratory Report No. PHY-10990-ME-2004; Y. Kasamatsu, H. Kikunaga, K. Takamiya, T. Mitsugashira, T. Nakanishi, Y. Ohkubo, T. Ohtsuki, W. Sato, and A. Shinohara, Radiochimica Acta **93**, 511 (2005); S. B. Utter, P. Beiersdorfer, A. Barnes, R. W. Lougheed, J. R. Crespo Lopez-Urrutia, J. A. Becker, and M. S. Weiss, Phys. Rev. Lett. **82**, 505 (1999).
[19] Z.-T. Lu (private communication); E. Peik (private communication).
[20] E. Peik and Chr. Tamm, Europhys. Lett. **61**, 181 (2003).
[21] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1974), Vol. 2.
[22] In principle, the shell effect may also produce some small nonzero deformation. Therefore, we should write $\beta_2 = c_0 + c_1 \times \alpha + c_2 \times \alpha^2 \ldots$. However, ^{229}Th is not so far from the area of spherical nuclei, and the linear term must be important. This justifies our estimate.
[23] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).
[24] V. V. Flambaum and E. V. Shuryak, Phys. Rev. D **67**, 083507 (2003).
[25] Note that the suppression of the nucleon wave function at the small separation due to the repulsive core reduces the depth of the effective potential V. However, this effect is not so important in the ratio $\frac{\delta V}{V}$.
[26] V. V. Flambaum, A. Hoell, P. Jaikumar, C. D. Roberts, and S. V. Wright, Few-Body Syst. **38**, 31 (2006); A. Holl, P. Maris, C. D. Roberts, and S. V. Wright, nucl-th/0512048.
[27] R. Brockmann and W. Weise, Phys. Rev. C **16**, 1282 (1977).
[28] V. V. Flambaum and E. V. Shuryak, Phys. Rev. D **65**, 103503 (2002).
[29] V. V. Flambaum, D. B. Leinweber, A. W. Thomas, and R. D. Young, Phys. Rev. D **69**, 115006 (2004).