Quantum Two-Level Systems in Josephson Junctions as Naturally Formed Qubits

A. M. Zagoskin,[1,2] S. Ashhab,[1] J. R. Johansson,[1] and Franco Nori[1,3]

[1]*Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Wako-shi, Saitama, Japan*
[2]*Department of Physics and Astronomy, The University of British Columbia, Vancouver, B.C., Canada*
[3]*MCTP, CSCS, Department of Physics, The University of Michigan, Ann Arbor, Michigan, USA*
(Received 31 March 2006; published 14 August 2006)

The two-level systems (TLSs) naturally occurring in Josephson junctions constitute a major obstacle for the operation of superconducting phase qubits. Since these TLSs can possess remarkably long decoherence times, we show that such TLSs can themselves be used as qubits, allowing for a well controlled initialization, universal sets of quantum gates, and readout. Thus, a single current-biased Josephson junction can be considered as a multiqubit register. It can be coupled to other junctions to allow the application of quantum gates to an arbitrary pair of qubits in the system. Our results indicate an alternative way to realize superconducting quantum information processing.

DOI: 10.1103/PhysRevLett.97.077001 PACS numbers: 85.25.Dq, 03.67.Lx, 85.25.Cp

Several advances in the field of quantum information processing using superconducting circuits have been made in recent years [1]. A major obstacle to further advances, however, is the problem of decoherence. In particular, recent experiments on current-biased Josephson junctions (CBJJs) revealed resonances that suggest the presence of quantum two-level systems (TLSs) that are strongly coupled to the CBJJ when it is biased near one of those resonances [2,3]. So far, the TLSs have been treated as a nuisance that prevent the operation of the CBJJ as a qubit near any resonance. In this Letter, we show that the TLSs themselves can be used as qubits. The CBJJ then acts as a bus, enabling state initialization, one- and two-qubit operations, and readout. Moreover, the results of Refs. [2,3] show that the decoherence times of the TLSs are longer than those of the CBJJ. That property can be used in a scalable design such that the decoherence time of the entire system scales as the CBJJ decoherence time $T_d^{\rm CBJJ}$, *as opposed to the usual* $T_d^{\rm CBJJ}/N$, where N is the number of CBJJs in the circuit. Our results therefore demonstrate an alternative way to achieve a scalable qubit network in a superconducting system, in addition to illustrating a method to perform multiqubit experiments with available experimental capabilities.

Model.—The phase qubit, which is comprised of a single CBJJ, is one of the simplest experimental implementations of a qubit in superconducting systems [1,4]. The working states $|0\rangle$ and $|1\rangle$ are the (metastable) ground and first excited states in a local minimum of the washboard potential produced by the CBJJ. The (undriven) system is described by the Hamiltonian

$$H = \frac{\hat{Q}^2}{2C} - \frac{I_c\Phi_0}{2\pi}\cos\hat{\varphi} - \frac{I_b\Phi_0}{2\pi}\hat{\varphi} \approx \frac{\hbar\omega_{10}}{2}\sigma_z. \quad (1)$$

Here $\hat{Q}$ is the operator of the charge on the junction, C is the junction's capacitance, $\hat{\varphi}$ is the operator of the Josephson phase difference, and I_c and $I_b(\le I_c)$ are the critical and bias currents, respectively. The nonlinearity of the potential allows one to consider only the two lowest energy states. The Pauli matrix σ_z operates in the subspace $\{|0\rangle, |1\rangle\}$. The transition frequency $\omega_{10} \approx \omega_p = (2\pi I_c/\Phi_0 C)^{1/2}(1-j^2)^{1/4}$, the plasma frequency in the biased junction (the corrections due to nonlinearity are $\sim$10%), and $j = I_b/I_c$ (see, e.g., [4]). The terms in the Hamiltonian proportional to σ_x and σ_y, which enable a complete set of one-qubit gates, appear in the rotated frame of reference when applying microwave pulses of bias current at the resonance frequency, $I_b \to I_{\rm DC} + I_{\mu wc}(t) \times \cos(\omega_{10}t) + I_{\mu ws}(t)\sin(\omega_{10}t)$, as explained in Ref. [4].

The simplicity of the qubit design and manipulation contributed to its successful experimental realization [1,2] and a spectroscopic demonstration of the formation of entangled two-qubit states [5]. Nevertheless, the experiment in Ref. [2] also demonstrated that TLSs present in the tunneling barrier tend to destroy the coherent operation of the qubit (such as Rabi oscillations).

Such TLSs are ubiquitous in solid state systems wherever disorder is present and can be thought of as groups of atoms capable of tunneling through a potential barrier between two degenerate configurations. They are currently believed to be the main source of $1/f$ noise in solids.

The observed coherent oscillations between a TLS and a phase qubit [6] proved that a TLS can be considered as a coherent quantum object described by the pseudospin Hamiltonian

$$H_{\rm TLS} = -\frac{\Delta}{2}\tilde{\sigma}_x - \frac{\epsilon}{2}\tilde{\sigma}_z, \quad (2)$$

with a decoherence time *longer* than that of the qubit. The Pauli matrices $\tilde{\sigma}_x$ and $\tilde{\sigma}_z$ operate on the TLS states. Note that, since the nature of the TLSs is currently unknown, one cannot derive the values of the TLS parameters from first principles. As will become clear below, however, neither a derivation of those parameters nor an understanding of their physical origin is necessary in order to make use of the TLSs.

Although the available experimental data give rather limited information about the TLS-TLS interaction, it is highly unlikely that the TLSs interact directly with each other or with the external fields, because of their supposedly microscopic dipole moments and relatively large spatial separation.

The TLS-CBJJ coupling is believed to be due to one of the following mechanisms: (A) through the critical current dependence on the TLS position [2] or (B) the direct dipole coupling to the junction charge $\hat{Q}$ (which is currently considered more likely) [3,7]:

$$\hat{H}^{(A)}_{\rm int} = -\frac{I_c\Phi_0}{2\pi}\frac{\delta I_c}{2I_c}\cos\phi\tilde{\sigma}_z; \qquad \hat{H}^{(B)}_{\rm int} = \alpha\hat{Q}\tilde{\sigma}_z. \quad (3)$$

Both produce the coupling term $h_x\sigma_x\tilde{\sigma}_z$. In addition, in the former case there appears an $h_z\sigma_z\tilde{\sigma}_z$ term, which reflects the change in the interlevel spacing ω_{10} due to the plasma frequency dependence on the bias. The ratio $\gamma \equiv h_z/h_x = (E_C/2E_J)^{1/4}(1-j^2)^{-5/8}$, where $E_C = 2e^2/C$ and $E_J = I_c\Phi_0/2\pi$. If $(E_C/2E_J) = 10^{-7}$ (achievable using the external capacitor technique [3]) and $j = 0.90$, then $\gamma = 0.05$ [8].

Hereafter, we consider the case where $\gamma \ll 1$. Otherwise, the effective TLS-CBJJ decoupling is impossible even when out of resonance (on the positive side, this effect would provide the means to definitively establish the mechanism of TLS-CBJJ coupling). Then, performing a basis transformation on the TLS states, relabeling $[\Delta^2 + \epsilon^2]^{1/2} \to \Delta$, and using the rotating-wave approximation, we finally obtain the effective Hamiltonian for the TLS-CBJJ system:

$$\hat{H} = -\frac{\hbar\omega_{10}}{2}\sigma_z - \sum_j\left(\frac{\Delta_j}{2}\tilde{\sigma}^j_z + \lambda_j\sigma_x\tilde{\sigma}^j_x\right), \quad (4)$$

with the effective coupling coefficients λ_j. The coupling term acts only in resonance, when $|\hbar\omega_{10} - \Delta_j| < \lambda_j$.

The Hamiltonian (4) was derived under the experimentally relevant assumption that the coupling $\lambda \ll \hbar\omega_{10} \sim \Delta$. We neglected a term of the form $\mu\sigma_x\tilde{\sigma}^j_z$ in Eq. (4), because its influence on the system's dynamics is negligible when $\mu \ll \hbar\omega_{10}$ and $\mu^2/\hbar\omega_{10} \ll \lambda$.

Qubit operations.—The interlevel spacing $\hbar\omega_{10}$ is tuned by the bias current, which, together with the resonant behavior of the TLS-CBJJ coupling, allows the independent manipulation of each TLS (due to the natural dispersion of their characteristic energies Δ_j). The condition $\lambda \ll \Delta$, $\hbar\omega_{10}$ allows us to consider changes in the bias current as adiabatic from the point of view of internal CBJJ and TLS evolution but instantaneous with respect to the CBJJ-TLS dynamics.

First, consider the single-qubit operations, assuming for the time being that the decoherence times of the TLSs $T^{(j)}_d$ exceed the decoherence time of the CBJJ $T^{\rm CBJJ}_d$, which in turn is much larger than the characteristic interaction time $\hbar/\lambda$. On resonance with the jth TLS, the Hamiltonian (4) contains a block that acts as $-\lambda_j\sigma_x$ in the subspace of degenerate states $\{|1\rangle\otimes|g\rangle, |0\rangle\otimes|e\rangle\}$. Its operation leads to quantum beats between the states of the TLS and the CBJJ [see Fig. 1 (inset)], with the period $\tau_j \equiv \hbar/\lambda_j$, as was observed in Ref. [2]. Therefore, single-qubit operations on a TLS and its initialization to an arbitrary state can be achieved as follows: After initializing the CBJJ in the state $|0\rangle$, we bring it in resonance with the TLS for the duration $\tau_j/2$; as a result, the states of TLS and qubit are swapped:

$$|0\rangle\otimes(\alpha|g\rangle+\beta|e\rangle)\to e^{i(\pi/2)}(\alpha e^{i(\pi/2)(1+\Delta/\lambda)}|0\rangle+\beta|1\rangle)\otimes|g\rangle. \quad (5)$$

Then the CBJJ is taken out of resonance with the TLS, a rotation of its state is performed, and the resulting state is again transferred to the TLS [always compensating for the parasite phase shifts $\frac{\pi}{2}(1+\Delta/\lambda)$]. The readout of the qubit state can be performed after the swap (5), e.g., by using the technique of Ref. [6], where the transition to the resistive state, with its potentially problematic coupling to the TLSs, is avoided. The decoherence time $T^{(j)}_d$ can be determined using a similar sequence of operations, by initializing the TLS in a superposition state, decoupling it from the CBJJ, and measuring the probability to find the TLS in a given state after a given time. Note also that if the Rabi frequency of CBJJ greatly exceeds the TLS-CBJJ coupling, the manipulations with the state of the CBJJ can be performed while in resonance with a TLS, reducing the overall operation time [9].

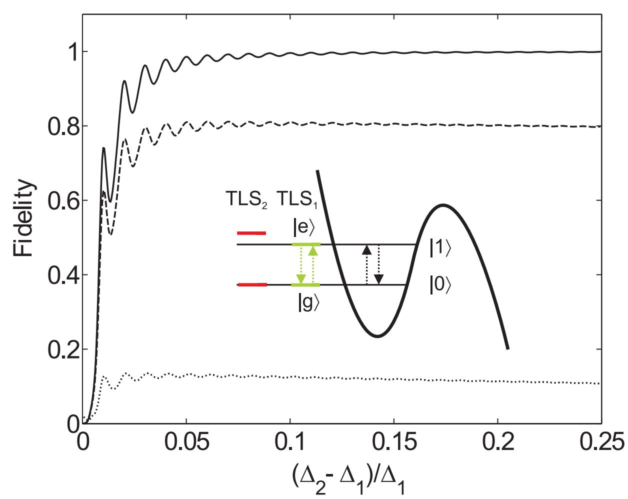

FIG. 1 (color online). Fidelity of the ISWAP gate on two TLS qubits in a CBJJ as a function of the relative difference between the TLS energy splittings (Δ_1 and Δ_2, respectively). The solid line corresponds to no decoherence; the dashed and dotted lines correspond to CBJJ decoherence rates $\Gamma_1 = \lambda/20\pi$ and $\Gamma_1 = \lambda/2\pi$, respectively, where in both cases $\Gamma_2 = 2\Gamma_1$. The much slower decoherence of the TLSs have been neglected. $\lambda = 0.005\Delta_1$. Inset: Schematic depiction of the quantum beats between the CBJJ and the TLS_1 in resonance; TLS_2 is effectively decoupled from the CBJJ.

Note that as soon as the CBJJ is biased away from the resonance with the TLS, they effectively decouple, and any perturbation of the quantum state of the CBJJ does not affect the survival of quantum coherence in the TLS. Therefore, the quality factor (the number of gates, of average duration $\tau_{\rm gate}$, that can be performed in the system before it loses quantum coherence) is no less than $T_d^{\rm CBJJ}/\tau_{\rm gate}$ and can exceed this value depending on the specific decoherence decay law.

The application of two-qubit gates between two TLSs inside the same CBJJ can be achieved similarly. For example, the universal ISWAP gate can be performed by the following sequence of operations: Tune the CBJJ in resonance with TLS_1 for $\tau_1/2$, then tune it in resonance with TLS_2 for $\tau_2/2$, and finally again in resonance with TLS_1 for $\tau_1/2$. (In the process, each TLS qubit was also rotated by $\pi/2$ around the z axis, which can be trivially repaired by performing the single-qubit rotation as described in the previous paragraph.) At the end of the above sequence, the CBJJ is decoupled *and* disentangled from both TLSs.

Numerical estimates.—The idealized picture of two-qubit operations neglected (i) the finite detuning between TLS qubits, (ii) the influence of other TLSs in the CBJJ, (iii) decoherence, and (iv) the finite time of the CBJJ bias current adjustment. We numerically investigated the impact of these on the fidelity F:

$$F = \mathrm{minimum}_{|\Psi_i\rangle}(\langle\Psi_i|U^\dagger_{\rm ideal}\rho_f U_{\rm ideal}|\Psi_i\rangle), \tag{6}$$

where $|\Psi_i\rangle$ is an initial state, $U_{\rm ideal}$ is the (ideal) desired operation, and ρ_f is the numerically obtained final density matrix. [Note that the quantity inside the parentheses in Eq. (6) depends on the initial state.]

In the numerical simulations, we took the ISWAP operation as a representative quantum gate and used 900 different initial states. The results are as follows (see Fig. 1): For two TLS qubits, with $\lambda_1 = \lambda_2 = 0.005\Delta_1$ (values close to the experimental data [2,6]), the fidelity first reaches 90%, when $\delta \equiv |\Delta_1 - \Delta_2| = 3.5\lambda$, and 99%, when $\delta \approx 10\lambda$. When $\delta = 8\lambda$, the fidelity is 98.8%. Adding an idle TLS with $\lambda = 0.002\Delta_1$ in resonance with one of the qubit TLSs reduces the fidelity to 80% (94% for $\lambda = 0.001\Delta_1$). Finally, with two qubit TLSs with reduced energies 1 and 1.04, four idle TLSs with reduced energies 0.99, 1.01, 1.03, and 1.05, and the reduced coupling strengths $\lambda_{\rm qubit} = 0.005$ and $\lambda_{\rm idle} = 0.002$, respectively, we find that the fidelity of the ISWAP gate $\approx 95\%$. Therefore, we conclude that the finite detuning and presence of idle TLSs *per se* is not dangerous.

We now take into account the decoherence in the CBJJ, neglecting the much weaker one in the TLSs. The ISWAP fidelity with $\delta = 8\lambda_j$ and without idle TLSs is 81% when $\Gamma_1 = (1/2)\Gamma_2 = \lambda/20\pi$, but it drops to 13% when $\Gamma_1 = (1/2)\Gamma_2 = \lambda/2\pi$.

Finally, we now consider the effects of a finite CBJJ bias switching time between the resonant frequencies of the TLSs. Taking a simple linear ramp with $t = 2$ ns and no pulse optimization, and neglecting decoherence, we find that the fidelity drops to 80%. Finite decoherence $\Gamma_1 = (1/2)\Gamma_2 = \lambda/20\pi$ further suppresses it to 63%. The above estimates show that the operation of the proposed two-qubit gate can be realized with the current experimental techniques used, e.g., in Ref. [3].

Scalability.—To ensure the scalability of the system, we must be able to perform two-qubit gates on TLSs located in different CBJJs. It can be done by swapping the states of the capacitively coupled adjacent CBJJs like in Ref. [10], but a better solution is based on the method suggested in Ref. [11]. Here the qubit-carrying CBJJs are coupled capacitively to a common linear LC circuit with resonance frequency ω_0 much higher than the characteristic frequencies of separate CBJJs (Fig. 2).

The Lagrangian of the system is

$$\begin{aligned}\mathcal{L} &= \frac{C_0\dot\phi_0^2}{2} + \sum_{j=1}^N\left[\frac{C_j\dot\phi_j^2}{2} + \frac{C'_j(\dot\phi_j - \dot\phi_0)^2}{2}\right] - \frac{\phi_0^2}{2L_0} \\ &\quad - \sum_{j=1}^N\left[-I_j\phi_j - E_j\cos\frac{2e\phi_j}{\hbar}\right] \\ &\equiv \frac{1}{2}\sum_{j,k=0}^N \mathcal{C}_{jk}\dot\phi_j\dot\phi_k - U(\{\phi\}),\end{aligned} \tag{7}$$

where $\dot\phi_j(t) \equiv V_j(t)$, the voltage between node j and the ground. The corresponding Hamiltonian becomes

$$H = \frac{1}{2}\sum_{j,k=0}^N \mathcal{C}^{-1}_{jk}\hat p_j\hat p_k + \frac{1}{2}\sum_{j=0}^N \frac{\omega_j^2}{\mathcal{C}^{-1}_{jj}}\hat\phi_j^2 + \cdots,$$

where $\hat\phi = (\hbar/2e)\hat\varphi$ [see Eq. (1)], $[\hat\phi_j, \hat p_j] = i\hbar$, $\mathcal{C}^{-1}$ is the inverse capacitance matrix, ω_0 is the frequency of the LC bus, ω_j is the frequency of the jth CBJJ in the harmonic approximation, and the ellipsis stands for the nonlinear corrections. After introducing the Bose operators a and $a^\dagger$ via $\hat\phi_j = \Lambda_j(a_j + a_j^\dagger)/2$, $\hat p_j = \hbar(a_j - a_j^\dagger)/(i\Lambda_j)$, $\Lambda_j = ((2\hbar\mathcal{C}^{-1}_{jj})/\omega_j)^{1/2}$, the Hamiltonian becomes

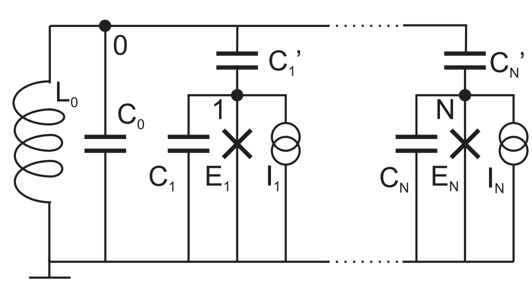

FIG. 2. Scalability of the structure: Two-qubit operations between the TLSs on different CBJJs ($j = 1, \ldots, N$) are enabled by the common LC circuit, which is capacitively coupled to the CBJJs. The Josephson energy, capacitance, and bias current of the jth CBJJ are E_j, C_j, and I_j, respectively.

$$H = \sum_{j=0}^{N} \hbar\omega_j \left(a_j^\dagger a_j + \frac{1}{2}\right) - \sum_{k>j=0}^{N} g_{jk}(a_j - a_j^\dagger)(a_k - a_k^\dagger) + \cdots$$

with the effective coupling

$$g_{jk} = \hbar(\omega_j\omega_k)^{1/2}\mathcal{C}_{jk}^{-1}/[2(\mathcal{C}_{jj}^{-1}\mathcal{C}_{kk}^{-1})^{1/2}].$$

By assumption, $\omega_0 \gg \omega_j$, $j = 1, \ldots, N$. Therefore, the Hamiltonian can be projected on the ground state of the LC bus [11]. The nonlinearity of the CBJJ allows us to further restrict the Hamiltonian to the subspace spanned by the states $|0\rangle$ and $|1\rangle$ of each CBJJ, producing

$$H_{\rm eff} = \frac{1}{2}\sum_{j=1}^{N} \hbar\omega_j \sigma_z^j + \sum_{k>j=1}^{N} g_{jk}\sigma_y^j\sigma_y^k. \qquad (8)$$

In the interaction representation with respect to $H_0 = \frac{1}{2}\sum_{j=1}^{N} \hbar\omega_j\sigma_z^j$, it is easy to see that $\sigma_y^j(t) = \sigma_y^j(0)\times \cos\omega_j t + \sigma_x^j(0)\sin\omega_j t$ [12]. Therefore, the pairwise couplings in (8) will be effective only for the CBJJs tuned in resonance with each other, in which case ($\omega_j = \omega_k$) the effective interaction term is $\tilde{H}_{\rm eff}^{jk} = g_{jk}(\sigma_x^j\sigma_x^k + \sigma_y^j\sigma_y^k)/2$. On the subspace spanned by the states $|0\rangle_j \otimes |1\rangle_k$ and $|1\rangle_j \otimes |0\rangle_k$, the operator $(\sigma_x^j\sigma_x^k + \sigma_y^j\sigma_y^k)/2$ acts as σ_x (while it is exactly zero outside). Therefore, this coupling allows the same universal two-CBJJ manipulations as in Ref. [10]. Universal two-qubit gates on TLSs situated in different CBJJs can then be performed by transferring the states of the $\mathrm{TLS}_{1,2}$ to the corresponding $\mathrm{CBJJ}_{1,2}$, performing the two-qubit operations on the states of $\mathrm{CBJJ}_{1,2}$, and retransferring the resulting states back to the TLSs.

The number of TLS per CBJJ is of the order of 10 and depends on the fabrication. The decoherence times of TLSs are determined by their local environment and are insensitive to the number of CBJJs linked to the same LC circuit. Similarly, the influence of the LC circuit on the decoherence of the CBJJ is negligible as long as $\omega_0 \gg \omega_{10} \sim \omega_p$ [4], the same requirement we need to obtain the coupling $\tilde{H}_{\rm eff}^{jk}$. Therefore, the scalability of the system is limited by the condition $[L(C_0 + NC'_{1,\rm eff})]^{-1} \gg I_c/[\Phi_0 C_1]$, or, to the same accuracy, $N \ll [\Phi_0/LI_c][C_1/C'_1]$. Therefore, N can be of the order of a few dozen without violating the applicability conditions for the above scalable design.

The usefulness of TLSs for our purpose could be questioned because their parameters undergo spontaneous changes. Nevertheless, since such changes typically happen on the scale of days [13], a prerun calibration is sufficient for any realistic task. Another concern that could be raised is the fact that any operation on the TLSs is done through the CBJJ, so that the latter's decoherence must be a limiting factor on the number of operations that can be performed on the TLSs, no matter how long their decoherence times are. However, an important point to note here is that, in the above design with N CBJJs, only one or two of them are used during any gate operation. Therefore, the decoherence time of the entire system is of the order of $T_d^{\rm CBJJ}$ *rather than* $T_d^{\rm CBJJ}/N$, which one would normally obtain when using N CBJJs as phase qubits.

Note that there is some control, albeit very limited, over the properties of the TLSs and their number [3]. They will not be the only naturally formed objects to allow quantum manipulation (see, e.g., [14]).

Summary.—In conclusion, we have demonstrated that quantum TLSs naturally occurring in CBJJs can be used as qubits. The one- and two-qubit gates, initialization, and readout can be readily performed, and the system can be scaled beyond a single CBJJ. Being microscopis objects, TLSs have a higher probability of possessing long decoherence times, which are only weakly affected by the TLS-CBJJ interactions, due to that interaction being switched off for most of the time. The tunability of the CBJJs compensates for our currently limited control over the parameters of the TLSs. In any case, both our numerical simulations and especially the observation of quantum beats between TLS and CBJJ [2] show that the experimental realization of our scheme is within the reach of current experiments [3].

We are grateful to M. Grajcar and Y. X. Liu for valuable comments. This work was supported in part by the ARO, LPS, NSA, and ARDA under AFOSR Contract No. F49620-02-1-0334 and also supported by the NSF Grant No. EIA-0130383. A. Z. acknowledges partial support by the NSERC Discovery Grants Program. S. A. was supported by the JSPS.

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