Dissipation-Driven Quantum Phase Transitions in a Tomonaga-Luttinger Liquid Electrostatically Coupled to a Metallic Gate

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The dissipation induced by a metallic gate on the low-energy properties of interacting 1D electron liquids is studied. As a function of the distance to the gate, or the electron density in the wire, the system can undergo a quantum phase transition from a Tomonaga-Luttinger liquid to two kinds of dissipative phases, one of them with a finite spatial correlation length. We also define a dual model, which describes an attractive one-dimensional metal with a Josephson coupling to a dirty metallic lead.

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Introduction.—In the study of properties of quantum wires (and other mesoscopic systems), proximity to metallic gates is frequently regarded as a source of static screening of the interactions in the wire. Within a classical electrostatic picture, this arises because electrons in the wire interact, not only amongst themselves but also with their image charges in the gate. Thus, the Coulomb potential becomes a more rapidly decaying dipole-dipole potential. Moreover, metallic environments (e.g., gates) are also a source of dephasing and dissipation [1–4] (for experimental research on related topics, see [5]), which arise because the electrons in the wire can exchange energy and momentum with the low-energy electromagnetic modes of the gate. This effect is described by the dissipative part of the screened potential, and, as we show below, it can lead to backscattering (i.e., a scattering process where one or several electrons reverse their direction of motion) in a one-dimensional (1D) quantum wire. We find that, for an arbitrarily small coupling to the gate, the backscattering can drive a quantum phase transition provided that the interactions between the electrons in the wire are sufficiently repulsive.

The effects of dissipation on quantum phase transitions have attracted much attention [6–9]. We discuss here transitions induced by dissipation, as in Refs. [10–12]. Some aspects of this work are also related to previous research on 1D systems coupled to environments [13–18]. In particular, the system studied here can be considered a microscopic realization of the model studied in Ref. [10]. A discussion of the relations between our findings and this work will be given below.

The model.—The electrostatic coupling of the wire to a metallic gate is described by

$$H_{gw} = \int d\mathbf{r}d\mathbf{r}' v_c(\mathbf{r}-\mathbf{r}')\rho_w(\mathbf{r})\rho_g(\mathbf{r}'), \tag{1}$$

where $v_c(\mathbf{r}) = e^2/4\pi\epsilon|\mathbf{r}|$ is the (statically screened) Coulomb potential, $e < 0$ being the electron charge and ϵ the dielectric constant of the insulating medium located between the gate and the wire. The operators $\rho_g(\mathbf{r})$ and $\rho_w(\mathbf{r})$ describe the density fluctuations in the gate and the wire, respectively. For a 1D quantum wire, $\rho_w(\mathbf{r}) \simeq \rho_w(x)\delta(y)\delta(z-z_0)$, where $z_0 \gg w > 0$ is the distance measured from the surface of the gate and w is the width of the wire. Following Refs. [19,20], we integrate out the density modes of the metallic gate and obtain the following effective action for the 1D wire (L is the length of the wire and $T = \beta^{-1}$ is the temperature):

$$S_{\rm diss} = \frac{1}{2\hbar}\int_0^L dxdx' \int_0^{\hbar\beta} d\tau d\tau' \rho_w(x,\tau) \times V_{\rm scr}(x-x', z_0, \tau-\tau')\rho_w(x',\tau'), \tag{2}$$

where we have assumed a translationally invariant flat gate so that the screened interaction $V_{\rm scr}(\mathbf{r},\mathbf{r}',\tau) = -\langle V(\mathbf{r},\tau)V(\mathbf{r}',0)\rangle_g/\hbar$, depends only on $\mathbf{R}-\mathbf{R}'$ [$\mathbf{r} = (\mathbf{R}, z) = (x, y, z)$]; $V(\mathbf{r},\tau) = \int d\mathbf{r}' v_c(\mathbf{r}-\mathbf{r}')\rho_g(\mathbf{r}')$ and $\langle\cdots\rangle_g$ denotes average over the gate degrees of freedom. To obtain an effective description of the effect of (2) on the low-temperature and low-frequency properties of the 1D wire, we first employ bosonization [21–23]. In the absence of the gate, the wire [24] is a Tomonaga-Luttinger liquid, described by the action [21–23]:

$$S_0[\phi] = \frac{1}{2\pi g}\int dxd\tau\left[\frac{1}{v}(\partial_\tau\phi)^2 + v(\partial_x\phi)^2\right], \tag{3}$$

where $\phi(x,\tau)$ is a plasmon field [25] that varies slowly on the scale of lattice constant a, and $v = v_F/g$ is the phase velocity of the plasmons, v_F being the *bare* Fermi velocity. Treating the ions in the wire as a positive uniform background of the same density ρ_0 as the electrons, we focus on the density fluctuations about ρ_0. In bosonized form, $\rho_w(x,\tau) = \frac{1}{\pi}\partial_x\phi(x,\tau) + \frac{1}{2\pi a}\sum_{m\neq 0} e^{2im(k_F x + \phi(x,\tau))}$. Each of the terms describes the low-energy density fluctuations of momentum $q \approx 2mk_F$, where m is an integer and k_F the Fermi momentum. Replacing $\rho_w(x,\tau) \to \frac{1}{\pi}\partial_x\phi(x,\tau)$ in (2) yields a term that describes forward ($q \approx 0$) scattering between electrons. Such a term leads to the (static) screening of the Coulomb interaction described in the introduction, and its dissipative part yields a contribution to the action of the form $\int dq d\omega |\omega| q^2 f(q) |\phi(q, \omega)|^2$. We find at small q that $f(q) \sim \ln(1/q)$ for a semi-infinite 3D diffusive gate or a granular gate. Thus, the forward scattering term is irrelevant in the renormalization-group (RG) sense. For a 2D diffusive gate gate, $f(q) \sim q^{-1}$ and the term is marginal in the RG sense, but it can be shown [13,26] that it does not modify the power-law correlations of the system at zero temperature. Moreover, in the limit of strongly repulsive interactions which interests us here, its effect is small. We therefore focus on the (backscattering) terms with $q \approx 2k_F$, which describe Friedel oscillations. In the limit of very repulsive interactions, the electrons in 1D wire are a Tomonaga-Luttinger liquid (TLL) that is close to a Wigner-crystal state [22,23] and density correlations are dominated by this oscillating term. Thus,

$$S_{\rm diss}[\phi] = -\frac{\eta}{\pi} \int_0^L dx \int_0^{\hbar\beta} d\tau d\tau' \mathcal{K}(\tau - \tau') \times \cos 2p[\phi(x, \tau) - \phi(x, \tau')], \quad (4)$$

where $\eta \propto a^{-2} S(Q_p, z_0)$, $S(q, z_0)$ being the Fourier transform of the spatial dependence of the dissipative part of $V_{\rm scr}(x, z_0, \tau) \simeq W_{\rm scr}(x, z_0)\delta(\tau) + \mathcal{K}(\tau) S(x, z_0)$ at low energies [19,20]. The static part $W_{\rm scr}(x, z_0)$ has the effect of screening the interactions in the wire and, therefore, leads to an effective dependence of g on z_0: As $z_0 \to 0$, the Coulomb interactions become more screened and therefore $g \to 1$. The dissipative kernel $\mathcal{K}(\tau) = (\pi/\hbar\beta)^{1+s} |\sin(\pi\tau/\hbar\beta)|^{-1-s}$, for $\tau \gg \tau_c$, and $\tau_c = \omega_c^{-1}$, where $\omega_c = \min\{E_F/\hbar, \omega_c^G\}$, E_F being the bandwidth of the wire and ω_c^G the characteristic response frequency of the gate electrons [27]. We have generalized the model to consider general dissipative environments (characterized by s, a metallic gate corresponding to Ohmic dissipation, $s = 1$) as well as generic backscattering processes for $q \approx Q_p = 2p^2 k_F$. Spinless electrons correspond to $p = 1$, spin-$\frac{1}{2}$ electrons to $p = \sqrt{2}$ (provided that $g < \frac{1}{3}$ [23]), and nanotubes to $p = 2$ (provided that $g < \frac{1}{5}$ [28]).

The dependence of the coefficient η on z_0 can be obtained for various models of the gate: $\eta \propto a^{-2}(\pi Q_p \sigma_{2D})^{-1} L_0(2Q_p z_0)$ for a 2D gate, and $\eta \propto a^{-2}(\pi\sigma_{3D})^{-1} K_0(2Q_p z_0)$ for a 3D gate, where σ_{2D} and σ_{3D} are gate conductivities measured in units of e^2/h, and $L_0'(x) = K_0(x)$, $K_0(x)$ being the modified Bessel function of the 2nd kind. Thus, the values of η and g can be tuned either by bringing the wire closer to the gate or, if the wire is connected to leads, by charging the gate to vary the chemical potential (and, therefore, the density $\rho_0 \propto k_F \propto Q_p$) of the wire. In deriving Eq. (4), we have assumed that the wire is away from half-filling. The analysis of the half-filled case is more involved and will reported elsewhere [26].

Weak coupling RG analysis.—To assess the stability of the TLL when perturbed by $S_{\rm diss}[\phi]$, we have studied the RG flow of the above model. Assuming that the dimensionless coupling $\alpha = (v\tau_c)(\tau_c)^{1-s}\eta$ is small, we perturbatively integrate high-frequency density fluctuations to lowest order in α and obtain the following RG equations:

$$\frac{dv}{d\ell} = -4p^2 g v \alpha, \qquad \frac{dg}{d\ell} = -4p^2 g^2 \alpha, \quad (5)$$

$$\frac{d\alpha}{d\ell} = (2 - s - 2p^2 g)\alpha, \quad (6)$$

where $\ell = \ln(\omega_c/T)$. These equations describe a Kosterlitz-Thouless-like transition around a quantum critical point where $\alpha = \alpha^* = 0$ and $g = g^* = (2-s)/2p^2$ ($g^* = 1/2p^2$ for the Ohmic case). At the critical point, correlations decay as power laws with universal exponents determined by g^*. For example, density correlations at $2k_F$ decay as $(x^2 + v^2\tau^2)^{-g^*}$, which implies that the dynamical exponent $z = 1$. It also interesting to point out that dissipation does not drive any phase transition for $s > 2$. Furthermore, for an infinite-range dissipative kernel ($s = -1$) and $p = 1$, (5) and (6), reduce to the equations derived by Voit and Schulz [29] and Giamarchi and Schulz [30] for spinless electrons in the presence of phonons and disorder, respectively.

In the phase where α flows towards strong coupling, the system is characterized by a length scale which diverges as $\xi_1 \approx k_F^{-1} e^{-\pi/[(2-s)p\sqrt{\alpha - \alpha_c}]}$, with the distance $\alpha - \alpha_c$ to the transition, and behaves as $\xi_1 \approx k_F^{-1}[\alpha(0)]^{(1/2p^2(g-g^*))}$ far from it. On the side where α scales down to zero, the system is a TLL with infinite conductivity at zero temperature (frequency). However, at finite temperature (frequency) the (optical) conductivity is finite a behaves as a power law: $\sigma(T) \sim \frac{1}{\alpha} T^{2-\mu}$ [$\mathrm{Re}\sigma(\omega > 0) \sim \alpha\omega^{\mu-4}$, and $\mathrm{Re}\sigma(\omega > 0) \sim \alpha/(\omega \ln^2(\omega))$ at the transition], where the exponent $\mu = 2p^2 g + s + 1$. In the phase where $S_{\rm diss}[\phi]$ is relevant, a rather crude estimate of the conductivity, hopefully valid in the large η limit, can be obtained by expanding the cosine in Eq. (4) and keeping the quadratic terms in $\phi(x, \tau)$ only. Using the Kubo formula [23], $\sigma(\omega) = i\mathcal{D}/(\omega + i/\tau_d)$, where $\mathcal{D} = ge^2 v/\hbar\pi$ is the Drude weight and $\tau_d^{-1} = 4\pi p^2 g v \eta$. However, for $\omega \gg \tau_d^{-1}$ (but $\omega \ll \omega_c$), we expect a crossover to a power law such that $\mathrm{Re}\sigma(\omega) \sim \alpha\omega^{\mu-4}$.

Self-consistent harmonic approximation (SCHA).—A better approximation than expanding the cosine in (4) can be obtained by using the SCHA, which approximates $S_{\rm diss}$ by a quadratic term [18,23,31–33] $-\frac{1}{2} \times \int dx d\tau d\tau' \Sigma(\tau - \tau')[\phi(x, \tau) - \phi(x, \tau')]^2$. Assuming that $\Sigma(\tau) \simeq \tilde{\eta}/(\pi\tau^2)$ at long times ($s = 1$) and optimizing the free energy, we find that $\tilde{\eta} \sim (v\tau_c)^{-1}[\eta(v\tau_c)]^{(1/1-2p^2 g)}$. Note that $\tilde{\eta} \to 0$ as $g \to g^* = 1/2p^2$, thus signaling a transition, in agreement with the RG analysis. For $g < g^*$, the SCHA yields a diffusive plasmon propagator, $G^{-1}(q, \omega) \simeq \tilde{\eta}|\omega| + vq^2/(\pi g)$ at small ω, signaling the breakdown of the TLL. Moreover, $\Phi(x, \tau) = \langle e^{2ip\phi(x,\tau)} e^{-2ip\phi(0,0)} \rangle_{\rm SCHA} \simeq N_0^2[1 + C_0(\tilde{\eta}x, \tilde{\eta}v\tau/g)]$, where N_0 is nonuniversal and $C_0(\tilde{\eta}x, 0) \sim (\tilde{\eta}|x|)^{-1}$, while $C_0(0, \tilde{\eta}v\tau/g) \sim (\tilde{\eta}|v\tau/g|)^{-1/2}$.

Large N approach.—Further insight into the properties of the model can be obtained by means of the large N approach, which captures many of the properties of dissipative quantum rotor models [32,34–36]. Let $\mathbf{n}(x,\tau) = (\cos 2p\phi(x,\tau), \sin 2p\phi(x,\tau))$, so that the action $S = S_0 + S_{\rm diss}$ becomes

$$S[\mathbf{n},\lambda] = \frac{1}{2}\int \frac{dqd\omega}{(2\pi)^2} G_0^{-1}(q,\omega)|\mathbf{n}(q,\omega)|^2 + \frac{i}{2}\int dx d\tau \lambda(x,\tau)[\mathbf{n}^2(x,\tau) - 1], \quad (7)$$

where $G_0^{-1}(q,\omega) = \eta|\omega| + \kappa_p[(\omega/v)^2 + q^2]$, $\kappa_p = 4p^2 v/(\pi g)$, and $\lambda(x,\tau)$ is a Lagrange multiplier ensuring that $\mathbf{n}^2(x,\tau) = 1$. After generalizing the symmetry of the model from $O(2)$ to $O(N)$, the field $\mathbf{n}$ is integrated out. In the large N limit, the path integral is dominated by a saddle point at $\lambda(x,\tau) = -i\kappa_p \xi^{-2}$, which obeys:

$$N\int \frac{dqd\omega}{(2\pi)^2} G(q,\omega|\lambda = -i\kappa_p\xi^{-2}) = 1, \quad (8)$$

where $G^{-1}(q,\omega;\lambda_0) = G_0^{-1}(q,\omega) + i\lambda_0$. For a single Ohmic quantum rotor, only solutions of (8) with $\xi > 0$ exist [32,35]. However, the spatial coupling of the rotors described by the q^2 term of the action allows for solutions with $\xi = 0$. Thus, we find $\xi^{-1} \sim (\eta_c - \eta)$, which implies that the critical exponent $\nu = 1$ and $z = 2$, in the large N limit [36] [more accurate estimates $\nu = 0.689(6)$ and $z = 1.97(3)$ have been reported in Ref. [10]]. Critical correlations $\Phi(x,\tau) = \langle \mathbf{n}(x,\tau)\cdot\mathbf{n}(0,0)\rangle_{N\to\infty} = NC_0(\eta x, v\eta v\tau/g)$, with $C_0(x,\tau)$ defined as above (a more correct form for $N = 2$ can be found in Ref. [10]). Away from criticality, $\Phi(x,0) \sim e^{-|x|/\xi}/|\xi^{-1}x|$ and $\Phi(0,\tau) \sim \xi^2 g^2/(v\tau)^2$ in the phase with $\xi \neq 0$. For $\eta > \eta_c$, we set $n_1(x,\tau) = N_0$ and integrate out the remaining $N-1$ components of $\mathbf{n}$ and proceed as above. Assuming that the phase is ordered, i.e., $N_0 \neq 0$, implies that $\xi^{-1} = 0$. The correlation function $\Phi(x,\tau)$ takes the same asymptotic form as that found using the SCHA.

Phase diagram.—The simplest flow and phase diagram compatible with all above results is shown in Fig. 1. We find three phases: (i) The TLL, where the coupling to the gate flows towards zero, (ii) an ordered dissipative phase, which has diffusive plasmons, and (iii) a disordered phase with a finite spatial correlation length and density correlations at $q \approx Q_p$ decaying as τ^{-2}. Just like for the single dissipative quantum rotor, we expect this result, obtained in the large N limit, to remain valid for $N = 2$. The form of the spatial correlations indicates that the system can be regarded as consisting of independent "puddles" of size $\sim\xi$, each puddle behaving as a single Ohmic quantum rotor. In the TLL phase, an analysis of the leading irrelevant operators shows that $\Phi(0,\tau) \sim \tau^{-2}$, at least. These results are in agreement with Griffiths' theorem [37].

We believe the model considered here to be equivalent, within the bosonization approach and for $\alpha \sim 1$, to the dissipative 2D XY model studied in Ref. [10]. The two

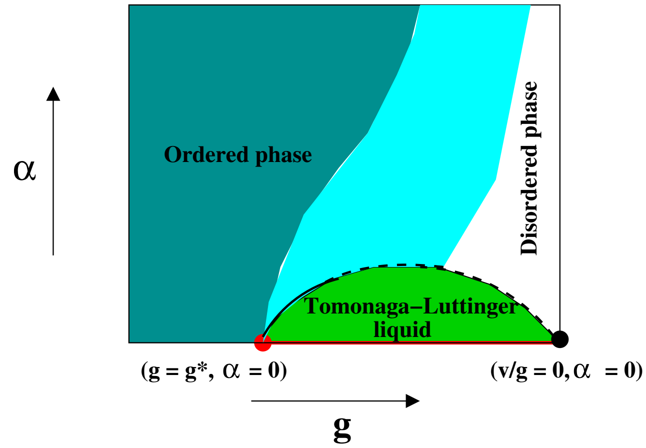

FIG. 1 (color online). Schematic phase diagram. The quantum critical point at ($g = g^*$, $\alpha = 0$) is predicted by the weak coupling RG approach discussed in the text. The dashed lines as well as the phase boundary between the ordered and the disordered phase (lightly shaded region) cannot be inferred by the methods used in this work.

dissipative phases found here correspond to those reported in Ref. [10]. However, we note that on the line $\alpha = 0$ (and for α small, too) the two models differ: Whereas the model of Ref. [10] exhibits a Kosterlitz-Thouless transition for large g and $\alpha = 0$, and, therefore, it is in a disordered (plasma) phase, our model does not exhibit such a transition but instead has a line of fixed points for $\alpha = 0$ (the TLL phase). A detailed comparison of the two models will be published elsewhere.

Dual model.—A model dual to the one discussed above can be realized if one considers an 1D metal of spin-$\frac{1}{2}$ fermions with attractive interactions (i.e., a Luther-Emery liquid) in front of a dirty metallic lead. Such a 1D metal exhibits a spin gap [23], Δ_s. For $T < \Delta_s$, single-electron hopping is suppressed and only hopping of pairs can take place, which leads to a Josephson coupling to the lead:

$$H_J = -t_J \int dx[\Delta^\dagger(x)\Delta_L(x) + \text{H.c.}], \quad (9)$$

where $t_J \sim t^2/\Delta_s$, t being the single-fermion hopping amplitude, and $\Delta_L(x) = \Psi_{\uparrow L}(x)\Psi_{\downarrow L}(x)$ and $\Delta(x) = e^{i\theta(x)}$ the pairing operators in the lead and the 1D metal, respectively. The field $\theta(x)$ is dual to the density field $\phi(x)$. Assuming that t_J is small, we integrate out the lead fermions using that $\langle \Delta_L(x,\tau)\Delta_L^\dagger(0,0)\rangle \simeq |\mathcal{G}_L^0(x,\tau)|^2 e^{-|x|/\ell_p}$, where ℓ_p is the mean-free path and $\mathcal{G}_L^0(x,\tau) = [2\pi(v_{LF}\tau - ix)]^{-1}$ the single-particle Green's function of the lead electrons. Taking into account that $\theta(x,\tau)$ varies slowly on the scale of the correlation length $\xi_s = \hbar v/\Delta_s \gg \ell_p \gg k_{LF}^{-1}$, where k_{LF} is the Fermi momentum of the lead, we obtain

$$\tilde{S}_{\rm diss}[\theta] = -\frac{\eta_J}{\pi}\int dx d\tau d\tau' \frac{\cos[\theta(x,\tau) - \theta(x,\tau')]}{|\tau - \tau'|^2}, \quad (10)$$

for $|\tau - \tau'| > \tau_c$, where $\tau_c \sim \tau_p = \ell_p/v_{LF}$ and $\eta_J = \nu_L \tau_p (t_J/\hbar)^2/\hbar$, ν_L being the density of states of the lead.

The RG flow for this term can be obtained from Eqs. (5) and (6) by replacing $g \to g^{-1}$ and $\alpha \to \alpha_J = (v\tau_c)\eta_J$ and setting $p = \frac{1}{2}$ and $s = 1$. However, in a consistent treatment [26], the lead must be also treated as a source of dissipation for the density fluctuations.

*Conclusions.—*We have analyzed a 1D metallic system with a few channels coupled to a metallic gate. In the absence of this coupling, the system is a Tomonaga-Luttinger liquid. We have shown that this phase is stable if the interactions in the wire are attractive or weakly repulsive, and the coupling is small. For sufficient repulsion, the coupling to the gate induces a phase transition to a gapless phase characterized by diffusive charge excitations, in contrast to the acoustical plasmons found in one-dimensional conductors. For large compressibility $v/g \to 0$, the coupling to the gate can induce a phase with a finite spatial correlation length and Ohmic correlations in the temporal direction.

Although for $g < g^*$ an arbitrarily small coupling to a gate destabilizes the TLL, in practice, finite temperature $T > 0$ or finite length L of the wire will cut off the RG flow before the system can exhibit the full properties of the ordered (or disordered) phase described above. Thus, at finite T and L, it is important to find the optimal conditions for the coupling to the gate to lead to measurable effects. The dimensionless quantity that measures the strength of the coupling to the gate is, e.g., for a 2D gate, $\alpha \approx g(k_{GF}/k_F)^2 e^{-4p^2 k_F z_0}/\sigma_{2D}$, where k_{GF} is the Fermi momentum of the gate electrons. Therefore, the gate should be a high resistance metal, and the Fermi wavelength of the wire needs to be small compared to the distance to the gate. Note, however, that close to the gate the interactions within the wire strongly screened and $g \to 1$, and the TLL is stable. One way around this problem would be to use a granular gate, which provides Ohmic dissipation but not metallic screening of the interactions for $q \approx 0$ [3]. It should also be mentioned that we have also neglected the possibility that disorder destabilizes the TLL before the effects of the gate are significant. These constraints suggest that a reasonable system where some of the phases discussed here can be observable is a weakly doped clean nanotube, where $k_F z_0$ can be made small, coupled to a metallic gate with a short elastic mean-free path or to a granular gate.

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[24] We first consider the simpler case of spinless fermions and discuss further below how to extend our results to other systems such as spin-$\frac{1}{2}$ electrons and nanotubes.
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