Polarization and Propagation of Polariton Condensates

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With the use of the generalized Gross-Pitaevskii equation it is shown that exciton polaritons in semiconductor microcavities form a linearly polarized condensate having two branches of the excitation spectrum. The splitting between these branches is strongly anisotropic. This anisotropy noticeably affects the real-space dynamics of polariton condensates.

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Introduction.—Exciton polaritons in microcavities are composite bosons expected to condense [1] at unusually high temperatures (up to room temperature) due to their light effective masses [2]. While a Wannier-Mott exciton is a solid state analogue of a hydrogen atom, there are a number of important differences between polariton and atomic condensates which have been extensively described in recent years. First of all, polaritons in microcavities have a finite lifetime that usually keeps them out of equilibrium [3]. The second, even more fundamental, peculiarity of the polariton system is linked with the spin structure of a polariton state: being formed usually by heavy-hole excitons, the polaritons have two allowed spin projections on the structure growth axis. In the absence of external magnetic field the "spin-up" and "spin-down" states of noninteracting polaritons, or their linearly polarized superpositions, are degenerate. Interactions mix the linearly polarized polariton states. Moreover, additional mixing comes from the longitudinal-transverse (LT) splitting of polaritons [4].

As a result, the polariton condensates behave differently from the atomic condensates and superfluids even in the thermodynamic limit. In particular, the real-space dynamics of polariton droplets is expected to be qualitatively different from the superfluid dynamics and to reveal strong polarization effects. While theoretical simulations of the real-space propagation of polariton condensates have been reported recently [5], neither dispersion nor the real-space dynamics of polarized polaritons have been addressed till now. The goal of this Letter is to fill this gap presenting the theory of polarization effects in the polariton condensates in planar semiconductor microcavities.

Quasiparticle spectrum.—We first consider analytically the excitations of the polariton condensate in thermal equilibrium. The condensate wave function (the order parameter) can be represented as a complex two-dimensional vector $\boldsymbol{\psi}(\mathbf{r}, t)$. The Hamiltonian density of polariton system can be written as

$$H = \boldsymbol{\psi}^* \cdot \mathbf{T}(-i\boldsymbol{\nabla}) \cdot \boldsymbol{\psi} + \tfrac{1}{2}[U_0(\boldsymbol{\psi}^* \cdot \boldsymbol{\psi})^2 - U_1 \boldsymbol{\psi}^{*2} \boldsymbol{\psi}^2]. \quad (1)$$

The general structure of the Hamiltonian density (1) is similar to that of the two-component atomic condensates [6], Josephson-junction arrays [7], helical magnets [8], and unconventional superconductors [9]. The main differences of the polariton condensates from these systems are the strongly nonparabolic dispersion of noninteracting particles and the dependence of the polariton energy on polarization described by the generalized kinetic energy operator $\mathbf{T}(-i\boldsymbol{\nabla})$. In writing down Eq. (1) we assume the cylindrical symmetry of the cavity, which allows only two isotropic quartic invariants. In terms of nonlinear optics, the U_0 term describes polarization independent properties of the condensate, while the U_1 term defines so-called linear-circular dichroism [10].

In what follows we consider the heavy-hole exciton polaritons having only TM and TE modes, and we neglect the mixing with the light-hole polaritons having a split-up mode polarized normally to the quantum-well plane [11]. With respect to the configuration of the in-plane component of the electric field (defined by the 2D vector $\boldsymbol{\psi}$) and the in-plane 2D wave vector $\mathbf{k}$, the TM and TE modes are longitudinal and transverse, respectively. Their dispersions $\omega_{l(t)}(k)$ are routinely found by the transfer matrix technique [4]. The modes are degenerate at $k = 0$ and the energy will be calculated from the bottom of the band, i.e., $\omega_{l(t)}(0) = 0$. Then, the kinetic energy tensor $\mathbf{T}(\mathbf{k})$ is ($\hbar = 1$)

$$T_{ij}(\mathbf{k}) = \omega_t(k)\delta_{ij} + [\omega_l(k) - \omega_t(k)]\frac{k_i k_j}{k^2}, \quad (2)$$

where $i, j = x, y$. Using (2) one has $\mathbf{T}(\mathbf{k}) \cdot \boldsymbol{\psi} = \omega_{l(t)}\boldsymbol{\psi}$ for $\boldsymbol{\psi}$ parallel (perpendicular) to $\mathbf{k}$.

The equilibrium properties of polariton condensates depend crucially on the presence and the sign of the dichroic U_1 term. For $U_1 > 0$ the free energy density $F = H - \mu(\boldsymbol{\psi}^* \cdot \boldsymbol{\psi})$ is minimized at a linear polarization of the condensate (i.e., when $[\boldsymbol{\psi} \times \boldsymbol{\psi}^*] = 0$). In this case the last term in Eq. (1) contributes most and $F_{\min} = -\mu n/2$, where $n = (\boldsymbol{\psi}^* \cdot \boldsymbol{\psi})$ is the 2D concentration of condensed polaritons and the chemical potential is $\mu = (U_0 - U_1)n$.

Note that μ defines the experimentally measurable blueshift of the polariton emission line due to formation of the condensate. In contrast, in the case $U_1 < 0$ the polariton condensate is formed with a circular polarization ($\boldsymbol{\psi}^2 = 0$), that assures disappearance of the U_1 term. In the absence of the dichroic term ($U_1 = 0$) there is no superfluid transition at any finite temperature, as it follows from mapping the Hamiltonian (1) on the nonlinear sigma model [12]. Clearly, the above analysis of the condensate polarization is valid for the very dilute limit and $|U_1| \gg (na_s^2)U_0$ (a_s is the effective scattering length). Consideration of fluctuations expected to affect the polarization of the condensate in the vicinity of $U_1 = 0$ is beyond the scope of this Letter.

The coupling coefficients U_0 and U_1 can be estimated through the matrix elements of the polariton-polariton scattering in the singlet (α_2) and triplet (α_1) configurations as $U_0 = \alpha_1$ and $U_1 = (\alpha_1 - \alpha_2)/2$. According Ciuti *et al.* [13], one usually has $\alpha_1 \gg |\alpha_2|$, so that $0 < U_1 < U_0$. Therefore, the polariton condensate is expected to be formed with a linear polarization. The condensate ground state can be written then as $\boldsymbol{\psi}_{\text{grd}} = \sqrt{n}\mathbf{e}$, where $\mathbf{e}$ is a real unit vector, $\mathbf{e}^2 = 1$ [14].

The spectrum of excitations of the polariton condensate at equilibrium will be studied on the basis of Gross-Pitaevskii equation

$$i\frac{\partial \psi_i}{\partial t} = \frac{\delta F}{\delta \psi_i^*} = [T_{ij}(-i\nabla) - \mu\delta_{ij}]\psi_j + U_0\psi_j^*\psi_j\psi_i - U_1\psi_j\psi_j\psi_i^*. \quad (3)$$

The excitation spectrum obtained from the Gross-Pitaevskii equation is known [15] to coincide with that of the Bogolyubov model and is valid at $T = 0$ [16]. We follow the method of Ref. [15] (see also Ref. [17]) and look for the solutions of Eq. (3) in the form

$$\boldsymbol{\psi}(\mathbf{r}, t) = \sqrt{n}\mathbf{e} + \mathbf{A}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + \mathbf{B}^* e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}. \quad (4)$$

Linearizing Eq. (3) with respect to the small amplitudes A_i and B_i we obtain

$$[T_{ij}(\mathbf{k}) + (u_0 - 2u_1)e_ie_j + (u_1 - \omega)\delta_{ij}]A_j + [u_0e_ie_j - u_1\delta_{ij}]B_j = 0, \quad (5a)$$

$$[T_{ij}(-\mathbf{k}) + (u_0 - 2u_1)e_ie_j + (u_1 + \omega)\delta_{ij}]B_j + [u_0e_ie_j - u_1\delta_{ij}]A_j = 0, \quad (5b)$$

where $u_0 = nU_0$ and $u_1 = nU_1$. The solutions of Eqs. (5) exist provided the quasiparticle frequency ω satisfies the dispersion equation

$$\omega^4 - [\omega_l^2 + \omega_t^2 + 2(u_0 - u_1)\omega_+ + 2u_1\omega_-]\omega^2 + [\omega_l\omega_t + 2(u_0 - u_1)\omega_-][\omega_l\omega_t + 2u_1\omega_+] = 0, \quad (6)$$

where $\omega_\pm = [\omega_l + \omega_t \pm (\omega_l - \omega_t)\cos(2\varphi)]/2$ and φ is the angle between the condensate polarization $\mathbf{e}$ and the wave vector $\mathbf{k}$. As in the case of two-component atomic condensates [18], in the region of small wave vectors, where $\omega_{l,t} \ll \min\{(u_0 - u_1), u_1\}$, the solutions of (6) give two sound branches of excitation spectrum. In the case of polariton condensates, however, these branches are anisotropic:

$$\omega^2 \simeq 2(u_0 - u_1)\omega_+, \qquad \omega^2 \simeq 2u_1\omega_-. \quad (7)$$

The anisotropy of the quasiparticle spectrum is a result of both the cylindrical-symmetry breaking due to the presence of condensate and the existence of LT splitting. The dispersion becomes isotropic and more simple if one neglects the LT splitting of noninteracting polariton bands by putting $\omega_l = \omega_t = \omega_0$. In this case the result becomes Bogolyubov-like: $\omega^2 = \omega_0^2 + 2(u_0 - u_1)\omega_0 = \omega_0^2 + 2\mu\omega_0$ for the quasiparticles copolarized with the condensate ($\mathbf{A} \parallel \mathbf{B} \parallel \mathbf{e}$), and $\omega^2 = \omega_0^2 + 2u_1\omega_0$ for the cross-polarized quasiparticles ($\mathbf{A} \parallel \mathbf{B} \perp \mathbf{e}$).

The renormalization and the strong anisotropy of the splitting between the two branches of the polariton spectrum is shown in Fig. 1(a) and 1(b). The interaction constants are chosen as $U_0 = 2.4 \times 10^{-18}$ eV m^2 and $U_1 = 0.55U_0$ in accordance with the estimation of Ref. [19] and the parameters used in the next subsection; the condensate density is $n = 10^{15}$ m^{-2}. The bare polariton spectrum corresponds to a CdTe microcavity showing

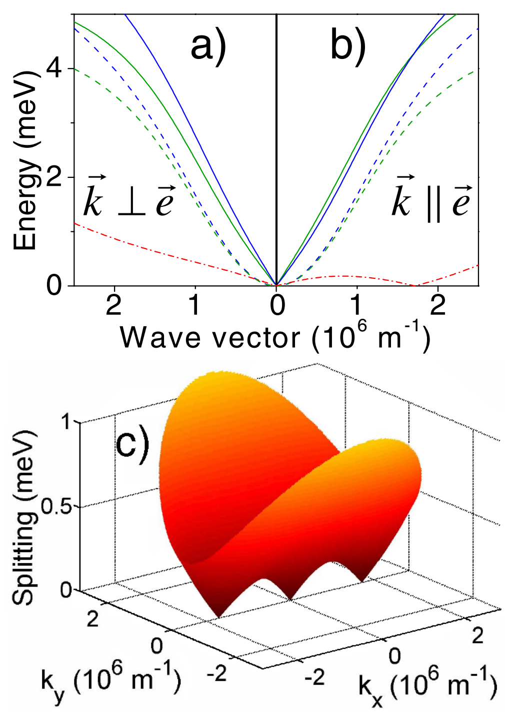

FIG. 1 (color online). Showing the dispersion of bare (dashed lines) and renormalized (solid lines) lower-polariton branches in the region of strong coupling. The splitting is shown in a dash-dotted line. The wave vector is perpendicular to the condensate polarization in panel (a) and collinear with it in panel (b). Panel (c) shows the overall behavior of the splitting. Parameters are given in the text.

a Rabi splitting of 10 meV at zero detuning between exciton and photon modes at $k = 0$. The dispersion of polaritons is clearly strongly modified and becomes linear close to $k = 0$. Note that while the splitting is enhanced in one direction of the wave vector ($\mathbf{k} \perp \mathbf{e}$ in our case), it is suppressed for its perpendicular direction. Moreover, one can observe the crossing of renormalized longitudinal and transverse branches. The strong anisotropy of the splitting is better seen in Fig. 1(c).

The Gross-Pitaevskii equation (3) describes adequately the excitation spectrum only for the case of a weakly depleted condensate. The depletion of the condensate $n_{\rm out}$ can be calculated using the Landau quasiparticle formula, which gives a reliable result for the 2D case [20]. At low temperatures T, when the phononlike parts of the two branches are mostly occupied, the condensate depletion is

$$n_{\rm out} = \frac{3\zeta(3)}{2\pi}\frac{(k_B T)^3}{\hbar^2 m^*}(v_0^{-4} + v_1^{-4}), \qquad (8)$$

where $m^* \sim 10^{-4} m_0$ is the polariton effective mass, $v_0 = [(U_0 - U_1)n/m^*]^{1/2}$ and $v_1 = [U_1 n/m^*]^{1/2}$ are the co- and cross-polarized sound speeds, and we neglect LT splitting for simplicity. It is seen from (8) that $n_{\rm out} \propto n^{-2}$. For $T < 20$ K the depletion becomes negligible at $n = 10^{15}$ m^{-2}; that corresponds to the experimentally observed [21] blueshift $\mu \sim 1$ meV. Note also that the correction to the blueshift $(U_0 - U_1)n_{\rm out}$ is of the order of a few μeV, and it is much smaller than the anisotropic splitting seen in Fig. 1. The above estimation is confirmed by the numerical calculations allowing for the nonparabolicity of the spectrum [22].

Real-space dynamics.—In this subsection we study numerically the impact of the anisotropic splitting on the real-space coherent dynamics of polariton condensates in nonequilibrium conditions. We take into account the pumping and finite lifetime of polaritons following the approach of Ciuti and Carusotto [5]. The model of Ref. [5] is, however, generalized by us to take into account the polarization degree of freedom. In this approach, instead of one polariton wave function $\boldsymbol{\psi}(\mathbf{r}, t)$, we use its two components, photonic $\boldsymbol{\varphi}(\mathbf{r}, t)$ and excitonic $\boldsymbol{\chi}(\mathbf{r}, t)$ parts, which satisfy two coupled vector equations

$$i\dot{\varphi}_i = [T_{ij}^{(\rm ph)}(-i\nabla) - i\tau_{\rm ph}^{-1}\delta_{ij}]\varphi_j + \Omega_R \chi_i + f_i(\mathbf{r}, t), \qquad (9a)$$

$$i\dot{\chi}_i = [T_{ij}^{(\rm ex)}(-i\nabla) - i\tau_{\rm ex}^{-1}\delta_{ij}]\chi_j + \Omega_R \varphi_i + V_0 \chi_j^* \chi_j \chi_i - V_1 \chi_j \chi_j \chi_i^*. \qquad (9b)$$

Here $\mathbf{f}(\mathbf{r}, t)$ describes the exciting pump within a limited spot and Ω_R is the Rabi frequency. We consider the case of zero detuning, where the exciton-exciton interaction parameters, V_0 and V_1, are related to the polariton-polariton ones as $V_0 = 4U_0$ and $V_1 = 0.55V_0$. The kinetic T tensors have a form of Eq. (2) with parabolic free-particle dispersions. Note also that unlike the equilibrium case considered before, the Gross-Pitaevskii equations (9) contain the lifetimes $\tau_{\rm ex(ph)}$ in place of chemical potentials.

We consider a 1.8 ps exciting pulse of light having a lateral size of 15 μm. It resonantly excites the ground state of the lower-polariton branch, as well as some of the excited states, because of its finite broadening. It is polarized horizontally (along the x axis). We assume zero temperature, the cavity photon lifetime of $\tau_{\rm ph} = 7$ ps, and infinite exciton nonradiative lifetime $\tau_{\rm ex}$. Figure 2 shows the 2D plot of the absolute value of the photon part of the wave function at different times for both horizontal and vertical polarizations. The contrast on the plots has been adjusted to get the best visibility. The upper panels in Fig. 2 show the wave function before the arrival of the maximum of the excitation pulse, when the polariton density is very small and nonlinear effects are negligible. The x-polarized component keeps the Gaussian spatial shape of the exciting pulse. At the same time the y-polarized or cross component appears in the diagonal directions, which are the directions where the horizontal and vertical polarizations are no more the quasiparticle eigenstates and the presence of the LT splitting results in the precession of polarization. At a time 2.4 ps after the maximum of the pulse, the x-polarized component forms a ring having the same width as the initial Gaussian pulse (Fig. 2, middle panels). This way of motion is characteristic for a linear dispersion. We believe that the observation of such a ring would be a clear experimental evidence of polariton superfluidity. After 16.4 ps, the pattern again strongly changes. The x-polarized component consists of

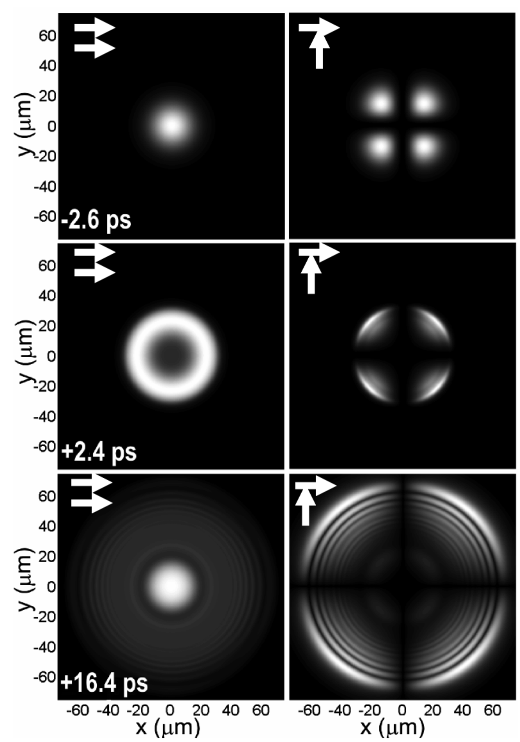

FIG. 2. Real-space image of the photon part of the wave function, showing evolution of a Gaussian shape pulse in nonlinear regime in x and y polarizations. Zero time corresponds to the peak intensity of the pulse.

a central peak, which is not moving or spreading and shows radiative decay only. This static peak can be associated with the ground state polariton condensate. At the same time, in y polarization the rings propagate without deformations, new rings forming from the center while the external ones are expanding. The velocity of expansion is given by the velocity of sound. This velocity depends on the density n of the polariton condensate and it changes in time because of the radiative decay of population. Note also that the black cross seen in the bottom right panel in Fig. 2 is slightly asymmetric: the horizontal band is wider than the vertical one. It happens because of the anisotropy of the splitting between polariton branches.

Recently Langbein [23] reported observation of the crosslike dynamics of polarization propagation in microcavities showing remarkable similarities to our Fig. 2. However, this result cannot be associated with the superfluidity since (i) the interference rings were the result of excitation of polariton excited states and not the ground state, like in our work, and (ii) the observed cross was symmetric. Observation of the superfluid propagation of exciton polaritons remains an important challenge for experimentalists. The problem is mainly related to the presence of imperfections in microcavities able to attract or scatter the condensate wave function, as it was revealed recently in Ref. [21].

Conclusions.—We have derived the Gross-Pitaevskii equation for a two-component Bose condensate of polaritons in microcavities accounting for their nonparabolic dispersion, longitudinal-transverse splitting, and polarization-dependent interactions. We have obtained substantial deviations of the quasiparticle spectrum from the Bogolyubov one. A remarkable peculiarity of the system consists in the anisotropy and strong polarization dependence of the dispersion, which results in characteristic asymmetric crosslike distributions of propagating polaritons in the real space.

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