$\pi N \to \eta N$ Data Require the Existence of the $N(1710)$ P_{11} Resonance, Reducing the 1700-MeV Continuum Ambiguity

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In spite of long-lasting discussions, the agreement on the existence of the $N(1710)$ P_{11} resonance has not yet been reached, so the Particle Data Group declares it as a 3-star resonance only. We show that the proper inclusion of inelastic channels in the coupled-channel formalism indisputably demands the existence of the $N(1710)$ P_{11} state, and that it presumably stays hidden within the continuum ambiguity of a typical single-channel partial-wave analysis. Consequently, the Particle Data Group confidence rating of this state should be raised to a 4-star resonance.

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A central task of baryon spectroscopy is to establish a connection between resonant states predicted by various low-energy QCD models and hadron scattering observables. A reasonable way to proceed is to identify poles of analytic scattering amplitudes which simultaneously describe all experimental data in all attainable channels with theoretically predicted resonant states. The next, but nontrivial step is to uniquely extract resonant parameters out of obtained poles. The "missing resonance problem", a failure to experimentally confirm a number of predicted quark model states (standard, glue enriched, or created from color neutral diquark or multiquark molecules) [1], poses a dilemma whether to suspect the reliability of quark models or whether to mistrust the resonance identification in partial-wave analyses.

Having in mind a possible number of new states to be identified, a dispute about the existence of the already reported resonant states, seen by one group and not confirmed by another, presents the matter which should be cleared up with forceful efficiency. In this Letter we concentrate on the $N(1710)$ P_{11} problem.

One of the latest and widely accepted partial-wave analysis (PWA), Virginia Polytechnic Institute/George Washington University (VPI/GWU) [2], does not see the $N(1710)$ P_{11} state. When the energy-dependent coupled-channel Chew-Mandelstam K matrix formalism is applied to fit experimental data, all available for πN elastic and only up to $T_\pi = 800$ MeV for $\pi N \to \eta N$, the obtained FA02 (SP06) solution has no poles in the P_{11} partial wave in the vicinity of 1710 MeV. On the other hand, the error bars of single-energy solutions (SES) accompanying it on the web page [3] do show a disproportional increase in the energy domain of 1700 MeV. As this analysis relies heavily on the elastic channel, the immanent continuum ambiguity problem [4,5] is handled by iteratively stabilizing the solution during the minimization procedure through applying the fixed t dispersion relations.

Other partial-wave analyses do report the presence of the $N(1710)$ P_{11} pole. Owing to the constraints coming from the crossed channels, a single-channel and fully analytic PWA (Karlsruhe-Helsinki KH80 [6]) undoubtedly sees the $N(1710)$ P_{11} state, and qualifies it as being strongly inelastic. Including inelastic channels leaves very little doubt about the $N(1710)$ P_{11} existence. The coupled-channel T-matrix Carnegie-Mellon Berkeley (CMB) type models [7–9], the coupled-channel K-matrix analyses/University of Giessen [10], coupled-channel K-matrix analyses of kaon-hyperon interactions [11], and the Kent State analysis [12] are all very affirmative about the existence of the $N(1710)$ P_{11} resonance. The consequence of the current failure to achieve unanimous agreement is that only a 3-star confidence rating is attributed to the $N(1710)$ P_{11} resonance by the Particle Data Group (PDG) [13]. This rating should be changed.

In this Letter we present the essence of the mechanism how inelastic channels enforce the existence of the $N(1710)$ P_{11} state. We use the technically improved version of the T-matrix coupled-channel formalism of the CMB type [7,8] and apply it to the P_{11} partial-wave T matrices. We use the most recent available input for the πN elastic scattering [2,3], and the experimentally constrained P_{11} partial wave for the $\pi N \to \eta N$ process [8]. We show that fitting only the elastic channel requires just the $N(1440)$ P_{11} state and that the P_{11} T matrix is smooth in the 1700 MeV range. When inelastic channels are also fitted, no doubt is left about the existence of the $N(1710)$ P_{11}, but the smoothness of the P_{11} T matrix is spoiled. This finding contradicts the smooth energy behavior of the offered VPI/GWU FA02 (SP06) solution, but *is not* in controversy with the collection of single-energy solutions of the same group. We believe that the enlargement of error bars reported by the VPI/GWU SES in the critical energy range is not only of experimental, but primarily of theoretical nature, and we offer the explanation that a $N(1710)$ P_{11} resonance is "hidden" within the continuum ambiguity [4,5] in the 1700 MeV range. The way how the $\pi N \to \eta N$ data have been used in the VPI/GWU analysis is also not being of much help for eliminating the 1700 MeV continuum ambiguity: all data above $T_{\rm lab} = 800$ MeV, important to see the $N(1710)$ P_{11} resonance, unfortunately have not been included in this analysis. So, no wonder that the $N(1710)$ P_{11} is not seen in FA02 (SP06).

Finally, let us just mention that the P_{11} T matrix energy behavior obtained in this publication is almost identical with the (C-p-$\pi+$) solution reported by the Giessen coupled-channel K-matrix analysis [10].

Setting up the model.—We use the CMB model [7–9]. It is the separable coupled-channel partial-wave analysis with the channel propagator ϕ analyticity ensured by the once subtracted dispersion relation. The Bethe-Salpeter equation for the dressed resonance propagator G is explicitly solved by using bare propagators G^0 and the self-energy Σ. The parameters of the model are bare propagator poles s_0, their number N_P, and the channel-to-resonance mixing real matrix γ. Once the number of channels N_C, and the number of bare propagator poles are chosen, the model contains a total of $N_P + (N_P \times N_C)$ free parameters per partial wave.

Our model contains resonant contributions and a nonresonant background. The resonant contribution is generated by "dressing" the bare poles of the resonant propagator with the self-energy term. The background is generated by two nonphysical poles lying beyond the measurable energy range. The initial number of genuine resonances, two less than N_P, is denoted by N_R.

The self-energy is parameterized as $\Sigma = \gamma^T \Phi \gamma$, so the model will satisfy the unitarity demands. Φ is a diagonal matrix of channel propagators ϕ, and Imϕ are defined as in Ref. [8] for each channel. The unitary normalized T matrix in the physical limit is given by $T = \sqrt{\mathrm{Im}\Phi}\gamma G \gamma^T \sqrt{\mathrm{Im}\Phi}$.

We use the model with three channels: two physical two-body channels πN and ηN, and the third (effective) channel $\pi^2 N$, which represents *all remaining two- and three-body* processes in the form of a two-body process with π^2 being a quasiparticle with a different mass chosen for each partial wave.

The data base.—In principle, the model parameters should be obtained by fitting the experimental data directly. However, since the number of free parameters gets reduced because the formalism separates individual partial waves, we fit the most recent πN elastic and the available $\pi N \to \eta N$ partial-wave amplitudes, understanding them as an effective representation of all existing measurements [14].

For the πN elastic partial waves we used the P_{11} VPI/GWU single-energy solutions [2,3] having 122 data points with the corresponding error bars.

Similarly as in Ref. [15], where the Pittsburgh results of the coupled-channel PWA [9] are used as the experimentally constrained $T^{S_{11}}_{\pi N \eta N}$, we used the coupled-channel model of the same type from Batinić *et al.* [16] to obtain the P_{11} T matrix in the ηN channel. However, instead of using smooth theoretical curves, we constructed 78 "experimental" data points by normally distributing the model input in order to simulate the statistical nature of really measured data. The standard deviation σ was set to 0.02, similar to the average error value of the GWU data. By using this procedure we were able to produce a set of $\pi N \to \eta N$ partial-wave data that, when fitted, gave realistic χ^2 values comparable with those obtained by SES fits.

The fitting procedure.—We started with a minimal set of resonances and increased their number until the satisfactory fit was achieved. We first fitted each channel independently and obtained separate collections of poles. Then we compared them. If the obtained sets disagreed (various channels required dissimilar poles), we concluded that different channels exhibited more sensitivity to different resonances. As all once identified poles have to exist, they all have to be included to create the T-matrix singularity structure. Therefore, when we combined all channels, we fitted them simultaneously using the unique, analytic, multichannel T matrix whose analytic structure was identical for all channels and was defined by all poles established in individual fits regardless of the fact that extra poles might spoil the level of agreement in individual channels. We increased the number of poles until the quality of the fit, measured by the lowest reduced χ^2 value, could not be improved. In addition, a visual resemblance of the fitting curve to the data set as a whole was used as a rule of thumb; i.e., we rejected those solutions which had a tendency to accommodate for the rapidly varying data points regardless of the χ^2 value.

Results and conclusions.—Pole positions and the reduced χ^2_R (defined as χ^2 divided by the difference of number of data points and number of fitting parameters) are given for the total of 10 solutions in Table I.

Elastic channel only … NO N(1710) P_{11}: We fitted the elastic channel only and obtained solutions (sol) 1–3. To achieve the overall agreement of the model with the experimental input of Ref. [2,3], it suffices to use only one physical pole in the vicinity of 1400 MeV (Roper). Adding new poles is just visually improving the quality of the high-energy end of the fit making the χ^2_R only slightly improved. Changes are of a cosmetic nature only, so the existence of the second pole near 1950 MeV is not essential but is only consistent with the data.

The agreement of the input $T_{\pi N \pi N}$ values with the results of the inelastic channel fit for the best three-pole solution (sol 6) is shown with the dotted line in Fig. 2, the predictions of sols 4 and 5 are not shown because they are very similar to those of sol 6.

The T-matrix values, predicted for the $\pi N \to \eta N$ channel when only the πN elastic channel is fitted, deviate strongly from the ηN input, and are given just as an indication with unlabeled thin gray lines in Fig. 2. These lines demonstrate that by fitting only one channel it is in principle not possible to get some reliable constraints on other channels without any additional input. Such a fairly general statement can be easily understood within the framework of the coupled-channel formalism. The analytic structure of the utilized unique, multichannel T matrix must be identical for all channels, but in the case of the single-channel fit, it is determined solely by the chosen

TABLE I. The extracted P_{11} partial-wave T-matrix poles.

sol	$N_P(N_R)$	χ_R^2	πN only/MeV $\binom{\mathrm{Re\,W}}{-2\,\mathrm{Im\,W}}$	$\binom{\cdots}{\cdots}$	$\binom{\cdots}{\cdots}$	χ_R^2	ηN only/MeV $\binom{\mathrm{Re\,W}}{-2\,\mathrm{Im\,W}}$	$\binom{\cdots}{\cdots}$	$\binom{\cdots}{\cdots}$	χ_R^2	πN and ηN/MeV $\binom{\mathrm{Re\,W}}{-2\,\mathrm{Im\,W}}$	$\binom{\cdots}{\cdots}$	$\binom{\cdots}{\cdots}$	$\binom{\cdots}{\cdots}$
1/4/7	3(1)	3.37	$\binom{1325}{175}$	$\cdots$	$\cdots$	1.24	$\binom{1170}{75}$	$\binom{1735}{180}$	$\binom{2175}{215}$	8.2	$\binom{1345}{150}$	$\binom{1880}{375}$	$\cdots$	$\cdots$
2/5/8	4(2)	3.08	$\binom{1335}{155}$	$\binom{1820}{290}$	$\binom{1970}{105}$	1.28	$\binom{1405}{200}$	$\binom{1730}{170}$	$\binom{2175}{290}$	3.48	$\binom{1350}{170}$	$\binom{1710}{80}$	$\binom{1970}{330}$	$\cdots$
3/6/9	5(3)	2.94	$\binom{1320}{150}$	$\binom{1945}{140}$	$\binom{1975}{190}$	1.36	$\binom{1350}{195}$	$\binom{1730}{170}$	$\binom{2150}{310}$	2.71	$\binom{1350}{170}$	$\binom{1640}{330}$	$\binom{1730}{150}$	$\binom{2120}{400}$
10	6(4)	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	2.86	$\binom{1350}{190}$	$\binom{1730}{215}$	$\binom{1760}{260}$	$\binom{2175}{170}$

channel. That means two things: first, the set of poles obtained in the one-channel procedure describes only the chosen channel, and poles which dominantly couple to other channels might be missed; and second, only the submatrix of the γ matrix coefficients corresponding to the chosen channel is constrained; other parameters are more or less free. The consequence is that there exist a number of solutions for the channel-resonance coupling constants which perfectly describe one channel, and give an almost unpredictable result for other channels. In spite of applying analyticity and unitarity conditions very strictly, an arbitrary large number of data in one channel is insufficient to constrain the data in other channels.

Inelastic channel only ... $N(1710)$ P_{11} required: We fitted the inelastic channel only and obtained solutions 4–6. To achieve the overall agreement of the model with the experimental input of Ref. [16] we need at least three physical poles; one in the vicinity of 1300 MeV (below Roper), the second one around 1700 MeV, and the third one near 2100 MeV. One of these resonances corresponds directly to the investigated $N(1710)$ P_{11}.

It is indicative that the model is by itself producing three physical poles for sols 4 and 5. In this case, in addition to the two background poles only one or possibly two bare propagator poles are allowed in the physical region. We say that the model is spontaneously requiring a three-physical-pole solution in spite of the fact that it has not been planned for.

In Table I we observe that the χ_R^2 increases with increasing number of resonances. It only reflects the fact that the quality of the fit, achieved already for sol 4, is not significantly improved by adding new resonances. We added them only because we expect that a physical resonance should be produced by the bare propagator pole in the physical region, and not by the interference effect of the nonphysical background poles.

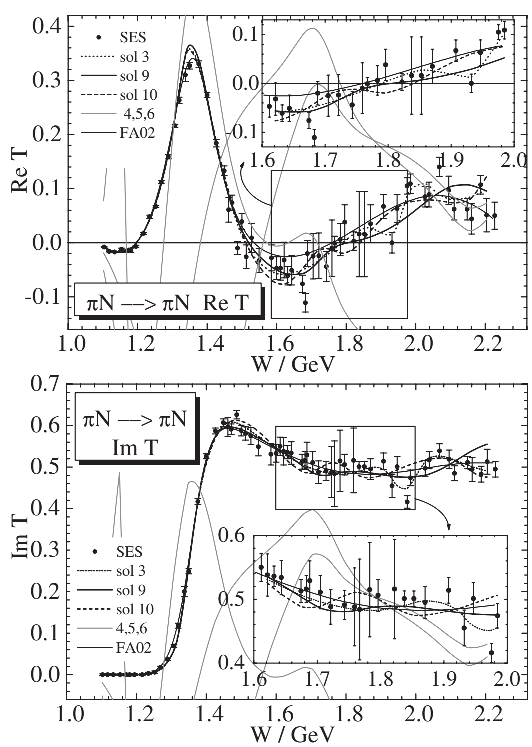

FIG. 1. The agreement of the input $T_{\pi N \pi N}$ values with the results of the fit.

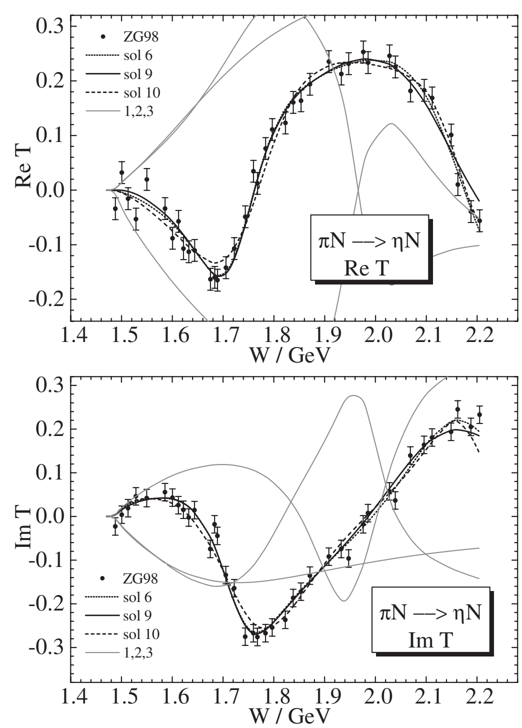

FIG. 2. The agreement of the input $T_{\pi N \eta N}$ values with the results of the fit.

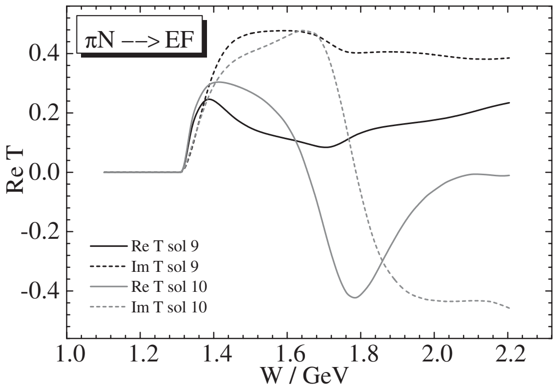

FIG. 3. The prediction for the $T_{\pi N \pi^2 N}$ amplitude for the best three- and the only four-bare-propagator pole solution.

The agreement of the input $T_{\pi N \eta N}$ values with the results of the inelastic channel fit for the best three-pole solution (sol 6) is shown with the dotted line in Fig. 2, the predictions of sols 4 and 5 are not shown because they are very similar to those of sol 3.

The T-matrix values, predicted for the πN elastic channel when only the $\pi N \to \eta N$ channel is fitted, strongly deviate from the input as is to be expected, and are just as an indication given with the unlabeled thin gray lines in Fig. 1.

Elastic+inelastic channels ... N(1710) P_{11} *survives*: We simultaneously fitted elastic and inelastic channels and obtained solutions 7–9. The overall agreement of the model with the experimental input is achieved using at least three physical poles; one in the vicinity of 1400 MeV (Roper), at least one around 1700 MeV, and the next one near 2100 MeV. The investigated $N(1710)$ P_{11} is needed again, and hence confirmed. However, let us just mention that the 1700 MeV state seems to be degenerated into two nearby poles, but the existing experimental data in the πN and ηN channels are insufficient to make a firm statement. The χ_R^2 is falling from sol 7 to sol 9 indicating that the quality of the fit is being increased by adding new bare propagator poles.

The sol 9, the best result of the combined elastic + inelasic channel fit with three bare propagator poles, is shown as a full line in Figs. 1 and 2 and compared with the input $T_{\pi N \pi N}$ and $T_{\pi N \eta N}$.

Other inelastic channels: The situation that the interference of background poles produces a physical resonant state, which occurred when we fitted the inelastic channel, is repeated for a combined fit in sol 9 where the 3-bare-propagator pole solution is generating 4 physical resonances. Therefore, we allow for a 4-bare-propagator pole solution and obtain sol 10, which is shown with dashed lines in Figs. 1 and 2. The χ_R^2 slightly increases with respect to sol 9 indicating again that the quality of the fit itself is not improved.

The obtained 4-pole solution does not differ significantly from sol 9 for the πN and ηN channels, but we show in Fig. 3 that the huge difference between the two solutions appears in the third, effective channel.

Consequently, we do need to measure other inelastic channels ($K\Lambda$ channel, for example) in order to distinguish the two solutions and to get a unique one, and that should automatically give us a better insight into the exact structure of all possible P_{11} poles in the 1700 MeV range.

Summary.—We have shown the mechanism how the inelastic data enforce the existence of the $N(1710)$ P_{11} state. Our findings coincide with all coupled-channel analyses, our model curve is almost identical with the (C-p-$\pi +$) University of Giessen solution [10], so we claim that the inelastic channels indubitably require at least one P_{11} singularity in the 1700 MeV range.

The PDG confidence rating of the $N(1710)$ P_{11} should be raised to 4 stars.

Our analysis suggests the presence of yet another resonant state in the 1700 MeV energy range, but its existence and its properties have to be confirmed by including the data from other inelastic channels.

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