Self-Localization of Bose-Einstein Condensates in Optical Lattices via Boundary Dissipation

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We introduce a technique to obtain localization of Bose-Einstein condensates in optical lattices via boundary dissipations. Stationary and traveling localized states are generated by removing atoms at the optical lattice ends. Clear regimes of stretched-exponential decay for the number of atoms trapped in the lattice are identified. The phenomenon is universal and can also be observed in arrays of optical waveguides with mirrors at the system boundaries.

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Localized states of Bose-Einstein condensates (BECs) in optical lattices have been predicted by exploiting the similarity of the dynamical equation from the Gross-Pitaevskii description of the condensate and the nonlinear Schrödinger equation used in nonlinear optics [1–6]. In continuous models, these localized solutions include lattice, gap, and matter-wave solitons [5] and, more recently, gap waves [6]. Self-trapped states (STS) and traveling breathers are instead distinctive of the discrete nonlinear Schrödinger equations [1–3,7]. Localization of BECs increases spatial control of the atomic cloud in a variety of experimental realizations to study, e.g., the fundamentals and use of interacting atom gases.

Experiments on BECs in optical lattices [8–10] and matter-wave bright solitons [11] have been recently completed with the observation of nonlinear self-trapping of matter waves in optical lattices by preparing wave packets and measuring their progressive expansion [12]. The techniques developed in Ref. [12] are the matter-wave counterpart of those employed to obtain discrete optical solitons in arrays of optical waveguides [13]. It is the aim of this Letter to present an alternative technique where the preparation of initial wave packets is not necessary and where localization emerges spontaneously due to dissipative cooling applied at the lattice ends. Our method should allow for the realization of very narrow spatially localized states and of dynamical breathers in BECs trapped in optical lattices. Figure 1 shows the BEC configuration simulated and analyzed here. A given BEC atomic density is initially distributed uniformly with random fluctuations (in phase as well as amplitude) over the sites of a one-dimensional optical lattice. At the two ends of the lattice, atoms are extracted via a spatially confined dissipation. This can be achieved by applying two separate continuous microwave or optical-Raman fields to locally spin flip atoms inside the BEC [14]. Spatially localized microwave fields focused below the wavelength can be obtained at the tips of tapered waveguides. The spin-flipped atoms do not experience the magnetic trapping potential and are released through gravity in two atomic beams at the ends of the optical lattice. The number of atoms extracted can be modified by changing the extracting field intensity or the distance of the waveguide tips from the BEC.

Recent theoretical work described relaxation phenomena induced by absorbing boundaries in nonlinear lattices. Energy localization via dissipative cooling was observed in a variety of models of chains of nonlinear oscillators. These include Morse [15] and ϕ^4 [15,16] oscillators, as well as rotators [17]. In these systems, statistical description of the relaxation process induced by the absorbing boundaries revealed that energy can be released in sudden bursts. These bursts correspond to the progressive disappearance of localized structures and to stretched-exponential decays [18]. We apply these statistical methods to models of BECs in one-dimensional optical lattices and show that relaxation to single self-trapped and/or traveling breather states are attainable for appropriate values of the boundary dissipation but generic initial configurations of the BEC spatial density. Monitoring of the atomic output allows for the identification of the localized states inside the optical lattice and possible interactions [19]. A clear stretched-exponential regime of relaxation is identified in the presence of self-trapped walls that act as insulating boundaries of hotter (denser) central cores. We also show that the localized atomic distributions are stable

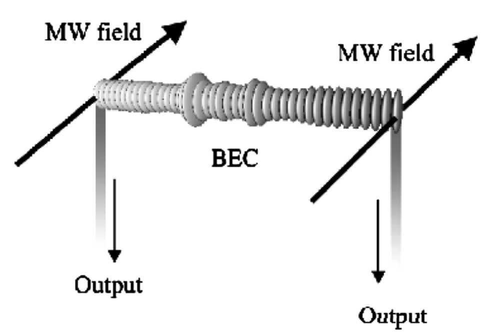

FIG. 1. BEC in a one-dimensional optical lattice with atomic dissipation at the lattice ends. The atoms are released from the trap by the action of two microwave (MW) or laser fields and gravity.

solutions of the matter-wave system once the boundary dissipations are removed.

We numerically simulate a dilute gas of bosonic atoms in the superfluid regime and in a spatially modulated external trap (the optical lattice) with losses of atoms at the trap ends. The dynamics of such a system is described by the Gross-Pitaevskii equation. Nevertheless, the periodicity of the potential casts this equation in a discrete form within the tight-binding approximation. By using a variational principle when the condensate's order parameter is written in terms of Wannier functions as $\psi(\mathbf{x}, t) = \sum_j z_i(t) w_j(\mathbf{x})$ [2,3,20], the system dynamics is described by the semiclassical Hamiltonian of a linear chain of M coupled BECs:

$$\mathcal{H} = \sum_{i=1}^{M} [U|z_i|^4 + \xi_i |z_i|^2] - \frac{T}{2} \sum_{i=1}^{M-1} (z_i^* z_{i+1} + \text{c.c.}), \quad (1)$$

where in the jth lattice site $z_j(t) = \sqrt{n_j(t)} e^{i\phi_j(t)}$ is the time-dependent amplitude of the wave function satisfying the Poisson brackets $\{z_j^*, z_\ell\} = i\delta_{j\ell}/\hbar$, $n_j(t)$ is the number of atoms, and $\phi_j(t)$ is the phase. The other parameters are the on-site atomic interaction $U = 4\pi\hbar^2 a_s \int d^3x |w_j(\mathbf{x})|^4/m$, the on-site chemical potential ξ_j, and the tunneling amplitude $T = \int d^3x w_j^*(\mathbf{x})[-(\hbar^2\nabla^2/2m) + V(\mathbf{x})] w_{j+1}(\mathbf{x})$, where the trapping potential V is periodic in one direction and harmonic in the other two. These parameters have been chosen in the simulations to fulfill $U/T < 3.84$ (value corresponding to the 1D Mott transition with filling one) and $19 < UN/T < 64$ according to experimental values (see [21]) where the tight-binding approximation has been verified to work successfully [9]. We combine the equations of motion associated to the Hamiltonian $\mathcal{H}$ with local dissipations at the end of the lattice to obtain

$$i\dot{z}_j = (\Lambda|z_j|^2 + \epsilon_j) z_j - \tfrac{1}{2}[z_{j-1}(1 - \delta_{j,1}) + z_{j+1}(1 - \delta_{j,M})] - i\gamma_1 z_j \delta_{j,1} - i\gamma_M z_j \delta_{j,M}, \quad (2)$$

where $j = 1, \ldots, M$, $\Lambda = 2U/T$, $\epsilon_j = \xi_j/T$, the time has been normalized via $\tau = Tt/\hbar$, and γ_1, γ_M describe the atomic losses due to the extracting fields shown in Fig. 1. The conservative part of the system dynamics is generated by Eq. (2) and $\gamma_1 = \gamma_M = 0$.

We have numerically integrated Eq. (2) on a grid of 128, 256, and 512 sites with increased accuracy to check intermediate and asymptotic states. Typical durations of the numerical simulations range from $\tau = 10^3$ to $\tau = 10^5$, corresponding to fractions and tens of seconds of the original time units, respectively.

Self-trapped states.—Figure 2 shows two typical evolutions of the atomic density on the optical lattice with losses at the ends of the trapping potential. In the first (second) case, a double (single) STS has been excited via progressive losses of atoms at the boundaries. Once the final states of Fig. 2 have been reached, the extracting fields at the lattice ends can be switched off, leading to an asymptotic trapping of the localized solutions. The initial conditions of the time evolutions displayed in Fig. 2 have $N(0)$ atoms, randomly distributed phases, and a constant amplitude with small fluctuations across the lattice. Such initial conditions are first thermalized during a conservative transient of, typically, $\tau = 500$ with $\gamma_1 = \gamma_M = 0$. After such a transient is completed, dissipations at the lattice boundaries are switched on, leading to a progressive loss of atoms. Depending on the values of $\Lambda N(0)$, one observes the formation of one or more walls. The values of γ_1 and γ_M control the speed at which localized structures are attained. Within two contiguous walls, the dynamics is very similar to that of the conservative system since there is almost no interaction with the lattice boundaries. There is a wide range of parameter values where a single wall is generated. In spite of the boundary dissipations, single walls are extremely long-lived [that one of Figs. 2(b) and 2(c) is still present after 8.6×10^5 time units] and owe their stability to a self-trapping mechanism. The latter is a well known phenomenon of a wide class of discrete equations with application in many research areas [7]. In the presence of self-trapping, large energy (and particle) amounts stored over few lattice sites have difficulty in diffusing to nearby sites through the hopping term of the Hamiltonian. It is remarkable that boundary dissipations take generic initial conditions towards such self-trapped states. To assure that self-localization has taken place, it is important to check that the STS of Fig. 2(c) is a localized solution of the original Hamiltonian (1).

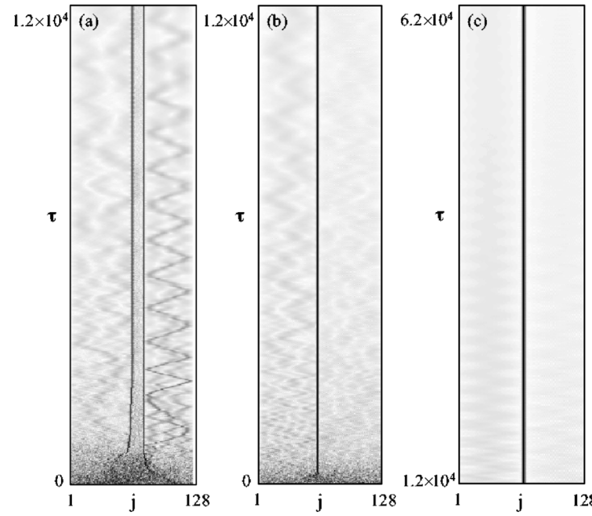

FIG. 2. Time evolution of the atomic density for $\gamma_1 = \gamma_M = 0.3$, $\epsilon_j = 0$ for all j, $M = 128$, (a) $\Lambda N(0) = 89.6$ and (b)–(c) $\Lambda N(0) = 44.8$, where $N(0)$ is the initial number of atoms. (c) is the continuation of (b).

The localized solution of Fig. 2 contains more than 99.5% of the remaining atomic density in just three lattice sites and presents an exponential decay of the particle number away from the central site. Moreover, it bears a strong resemblance to the *intrinsic localized mode* (ILM) for positive scattering lengths discussed in Ref. [3]. The ILM is symmetric with respect to a central site and can be represented by a perturbative expansion in the smallness parameter $\kappa = (2\Lambda N)^{-1}$. Up to $O(\kappa^4)$ in our normalizations, it is given by the expression:

$$z_0 = \pm\sqrt{N}\alpha e^{i\omega\tau}; \qquad z_j = z_{-j} = -\frac{\kappa}{\alpha^2} z_{j-1}, \qquad (3)$$

with $j = 1, 2, 3$ ($z_j = 0$ for $j \geq 4$) and where

$$\omega = \Lambda N(\alpha^2 + 2(\kappa/\alpha)^2), \qquad \alpha = \sqrt{\frac{1 + \sqrt{1 - 4\kappa^2}}{2}}. \qquad (4)$$

We have compared the atomic density of this approximate ILM solution with the long-lived STS obtained from the simulations of Figs. 2(b) and 2(c) with $\Lambda N \approx 2.8$ and extending up to $\tau = O(10^6)$. The agreement remains below 0.3%, thus indicating that the localized states generated by the boundary dissipation correspond (within the numerical approximation) to the solitonic (breatherlike) structures of the conservative system. We want to outline that the dissipative evolution naturally selects highly localized long-lived solutions of the conservative dynamics of the Hamiltonian (1), which have almost empty sites where the dissipation operates, i.e., at the lattice ends.

An interesting dynamical regime due to boundary dissipations is obtained for larger values of $\Lambda N(0)$ and lower dissipations with respect to those of Fig. 2. In this case, several walls are formed after a transient. The effect of the boundary dissipations is to progressively destroy the closest walls until a single or no STS remains in the lattice. In Fig. 3, we show that the progressive destruction of walls is a sequence of rare events where the number of atoms $N(\tau)$ presents a stretched-exponential relaxation of the kind

$$N(\tau) = N(0)\exp(-\tau^{\sigma_N}). \qquad (5)$$

From the data of Fig. 3, we measured σ_N to be close to 0.60. In Fig. 3, the time τ has been normalized by $M/2\gamma$, with $\gamma = \gamma_1 = \gamma_M = 0.2$ and $M = 128$ to guarantee a unitary slope of the curve for the early stages of the decay. The stretched-exponential behavior is typical of relaxations in chains of nonlinear oscillators [16] and rotators [17]. Note that the behavior of the total number of atoms trapped in the lattice can be experimentally monitored by the output at the boundaries so that a direct measurement of the stretched-exponential decay should be feasible with an arrangement like that of Fig. 1.

Traveling breathers.—The relaxation due to boundary dissipations allows for the localization of energy in traveling breathers as well as in STS. Figure 4(a) shows the relaxation dynamics for a value of $\Lambda N(0)$ below the formation of walls. By removing the dissipations after an appropriate time [see Fig. 4(b)], a single traveling breather is trapped in the optical lattice. Appropriate conditions can be found to trap more than one traveling breather or traveling breathers interacting with either STS or walls [see, for example, Fig. 2(a)]. Interaction of localized solutions is under investigation [19].

Localization of atomic density takes place initially close to the lossy boundaries where gradients of the amplitudes $A_j = |z_j|$ are formed and progressively steepen. It is, in fact, the lattice hopping mechanism of the Hamiltonian (1) that favors large differences of local amplitudes. The dynamics of the amplitudes A_j away from the boundaries is, in fact, ruled by

$$\dot{A}_j = -\tfrac{1}{2}[A_{j+1}\sin(\phi_{j+1} - \phi_j) + A_{j-1}\sin(\phi_{j-1} - \phi_j)], \qquad (6)$$

where ϕ_j is the phase of $z_j = A_j \exp(i\phi_j)$. The evolution of a largely populated lattice site slows down for local

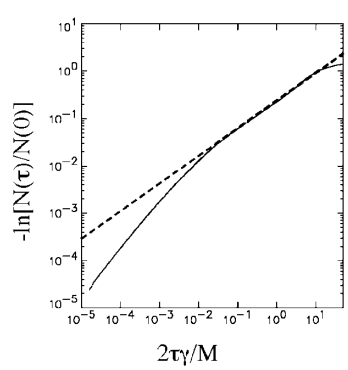

FIG. 3. Logarithmic plot of $-\ln[N(\tau)/N(0)]$ versus time averaged over 50 different initial conditions for $\Lambda N(0) = 128$, $\gamma = \gamma_1 = \gamma_M = 0.2$, and $M = 128$ (solid line). The dashed line fits the exponent σ_N and has a slope of 0.60.

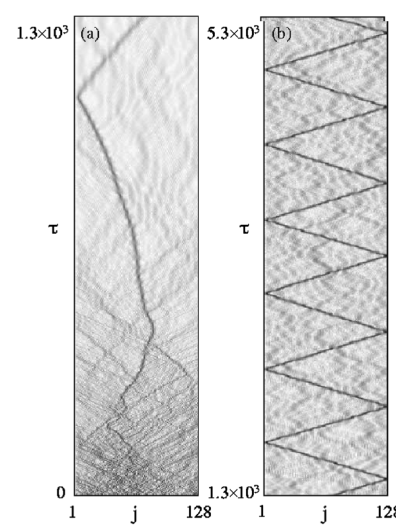

FIG. 4. Time evolution of the atomic density for $\Lambda N(0) = 38.4$, $\epsilon_j = 0$ for all j, $M = 128$, (a) $\gamma_1 = \gamma_M = 0.3$ and (b) $\gamma_1 = \gamma_M = 0$. (b) is the continuation of (a) after the atomic losses at the trap ends have been switched off.

phase differences close to 0 or π and/or for small values of the neighboring densities. Moreover, close to large amplitude gradients, local phase differences relax to values where their cosine is negative, thus favoring phase jumps close to π. Walls separating regions of high and low atomic density are then formed and survive for long times since both energy and atomic density are not redistributed on the side where the dissipation operates. We note that local values of the phase (i.e., momentum) may create traveling breathers as shown in Fig. 2(a). Traveling breathers, however, are discouraged in the central region between two walls where the average value of ΛN is large (see, e.g., the parameter diagram of Ref. [2]). They may form on the side of the walls where the local atomic density is low and where they progressively decay by hitting the lossy boundaries. As a consequence, slow-moving amplitude walls and self-localization of the atomic density are the long term features generated by the dissipative dynamics for suitably large values of $\Lambda N(0)$.

Self-localization due to boundary dissipations is a universal phenomenon and should be observable under general conditions of operation of BECs in optical lattices as well as other experimental realizations. In the first case, dissipative dynamics as shown in Fig. 2 is observable in BEC models with extended losses at the lattice ends, continuous variables, and in two dimensions. For the second case, we note that Eq. (2) describes, for example, the dynamics of light in arrays of coupled waveguides [13,22] where selective feeding of discrete optical solitons is feasible in large aspect ratio experiments. With progressive miniaturization and the use of photonic crystals, however, boundary dissipations with suitable mirrors can offer an important alternative for the generation and control of spatially localized light beams.

Beyond its practical interest for BECs and optics, the boundary cooling technique has an intrinsic conceptual interest. In fact, it allows one to perform a low-energy limit which selects naturally, by exploiting the statistics of energy fluctuations (see [17,18]), genuine nonlinear quantum states in the form of long-lived periodic (static or moving) localized solutions.

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