Recovering the Acoustic Green's Function from Ambient Noise Cross Correlation in an Inhomogeneous Moving Medium

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We study long-range correlation of diffuse acoustic noise fields in an arbitrary inhomogeneous, moving fluid. The flow reversal theorem is used to show that the cross-correlation function of ambient noise provides an estimate of a combination of the Green's functions corresponding to sound propagation in opposite directions between the two receivers. Measurements of the noise cross correlation allow one to quantify flow-induced acoustic nonreciprocity and evaluate both spatially averaged flow velocity and sound speed between the two points.

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Introduction.—In recent experiments, Weaver and Lobkis [1,2] demonstrated that the cross correlation of the recordings of diffuse noise fields at two spatially separated points gives the time-domain Green's function between the two points, i.e., the wave field that would be observed at one point if a source were placed at the other. Emergence of the Green's function from cross correlations of diffuse noise was demonstrated theoretically by a number of researchers [1–8] under various assumptions, with the most rigorous and general proof being offered by Wapenaar [6]. His reasoning is based on an application of the reciprocity principle and pertains to arbitrary inhomogeneous solids. A similar proof pertaining to fluids of constant density was independently given by Weaver and Lobkis [7]. Proposed applications of noise cross-correlation measurements to passive remote sensing range from ultrasonics and acoustic oceanography to helioseismology and geophysics, at wave frequencies that differ by more than 10 orders of magnitude [see [1,3,5,6,8,9] and references therein].

The work described here is motivated by possible applications to remote sensing of inhomogeneous flows, including precise measurements of flow velocities that are small compared to sound speed, which can be used for environmental monitoring and medical purposes. In this Letter we apply the flow reversal theorem [10] to derive exact and asymptotic relations between the two-point correlation function of noise and the sum of the Green's functions, which correspond to sound propagation in opposite directions between the two points. The cross-correlation function of ambient noise is shown to contain the information necessary to retrieve the flow velocity and the sound speed from acoustic measurements.

The flow reversal theorem.—Linear acoustic fields in an inhomogeneous moving fluid with sound speed $c(\mathbf{x})$, mass density $\rho(\mathbf{x})$, and flow velocity $\mathbf{u}(\mathbf{x})$ are governed by the following equations [[10]; [11], Chap. 8]

$$\rho\frac{d^2\mathbf{w}}{dt^2}+\nabla p+(\mathbf{w}\cdot\nabla)\nabla p_0-\frac{\nabla p_0}{\rho c^2}(p+\mathbf{w}\cdot\nabla p_0)=\mathbf{F}, \quad (1)$$

$$\nabla\cdot\mathbf{w}+(p+\mathbf{w}\cdot\nabla p_0)/\rho c^2=B,\ d/dt=\partial/\partial t+\mathbf{u}\cdot\nabla, \quad (2)$$

where t is time; p, p_0, and $\mathbf{w}$ are acoustic pressure, background pressure (i.e., the pressure in the absence of acoustic waves), and oscillatory displacement of fluid particles due to an acoustic wave. The acoustic wave is generated by sources with volume densities $\mathbf{F}$ of force and dB/dt of volume velocity. For monochromatic waves we will assume and suppress the time dependence $\exp(-i\omega t)$. Equations (1) and (2) have been derived from linearized Euler, continuity, and state equations, neglecting irreversible thermodynamic processes [[10]; [11], Chap. 8]. The linearized boundary condition [[10]; [11], Chap. 8] for the continuous wave (cw) acoustic field at an impedance (that is, locally reacting) surface S within moving fluid is

$$p=(i\omega\zeta-\mathbf{N}\cdot\nabla p_0)\mathbf{w}\cdot\mathbf{N},\quad \mathbf{x}\in S, \quad (3)$$

where $\mathbf{N}$ is a unit normal to S and $\zeta=\zeta(\mathbf{x},\omega)$ is the impedance of the surface. Free and rigid boundaries can be viewed as special cases of the impedance surface corresponding to zero or infinite impedance, respectively.

We define the Green's function $G(\mathbf{x},\mathbf{x}_1,t)$ as the acoustic pressure at $\mathbf{x}$ due to a point source of volume velocity with $B=\delta(\mathbf{x}-\mathbf{x}_1)\delta'(t-t_1)$ and $\mathbf{F}=0$. The oscillatory displacement in such an acoustic field will be denoted $\mathbf{g}(\mathbf{x},\mathbf{x}_1,t)$. In the case of motionless fluid, the Green's function as defined above corresponds to the field of a monopole sound source. The frequency spectrum $G(\mathbf{x}_1,\mathbf{x},\omega)$ of the time-domain Green's function $G(\mathbf{x}_1,\mathbf{x},t)$ has the meaning of the cw Green's function.

Acoustic fields possess an important symmetry with respect to the interchange of source and receiver positions. In the motionless case, the symmetry is expressed by the well-known reciprocity principle [see, e.g., [11], Sec. 4.2]. Although the reciprocity principle does not apply to moving media, its extension, known as the flow reversal theorem (FRT), has been established for arbitrary inhomogeneous moving media with time-independent parameters [10]. FRT relates the acoustic field $p^{(1)}$, $\mathbf{w}^{(1)}$ generated by a set $B^{(1)}$, $\mathbf{F}^{(1)}$ of sources in a medium with flow velocity $\mathbf{u}(\mathbf{x})$, with an acoustic field $p^{(2)}$, $\mathbf{w}^{(2)}$ generated by another set $B^{(2)}$, $\mathbf{F}^{(2)}$ of sources in a medium with the same sound speed and density and *reversed* flow with velocity $-\mathbf{u}(\mathbf{x})$:

$$\int_{\Omega} d^3\mathbf{x}[B^{(2)}p^{(1)} - \mathbf{F}^{(2)}\cdot\mathbf{w}^{(1)} - B^{(1)}p^{(2)} + \mathbf{F}^{(1)}\cdot\mathbf{w}^{(2)}] = \int_{\partial\Omega} ds\mathbf{N}\cdot\mathbf{j}, \quad (4)$$

where Ω is an arbitrary domain with the boundary $\partial\Omega$, $\mathbf{N}$ is an external unit normal to $\partial\Omega$, and

$$\mathbf{j} = p^{(1)}\mathbf{w}^{(2)} - p^{(2)}\mathbf{w}^{(1)} + \rho\mathbf{u}\left(\mathbf{w}^{(1)}\cdot\frac{\tilde{d}\mathbf{w}^{(2)}}{dt} + \mathbf{w}^{(2)}\cdot\frac{d\mathbf{w}^{(1)}}{dt}\right). \quad (5)$$

A tilde represents quantities referring to the medium with reversed flow. In particular, $\tilde{d}/dt = \partial/\partial t - \mathbf{u}\cdot\nabla$. The boundary $\partial\Omega$ is not required to be simply connected. Parts of $\partial\Omega$ may be located at infinity. When Ω is chosen to be the entire volume occupied by the fluid and it is either unbounded or has free, rigid, or impedance boundaries, the right side of Eq. (4) vanishes [[10]; [11], Chap. 8].

For the Green's functions G and $\tilde{G}$ in the original medium and a medium with reversed flow, a simple symmetry relation follows from Eq. (4): $G(\mathbf{x}, \mathbf{x}_1, \omega) = \tilde{G}(\mathbf{x}_1, \mathbf{x}, \omega)$. This result, as well as Eq. (5), remains valid in flow-structure problems where a part of the space is occupied by homogeneous or inhomogeneous, arbitrarily anisotropic elastic solids, provided the background pressure p_0 is uniform along fluid-solid interfaces [12]. The latter assumption can be significantly relaxed [12], but discussion of these details is beyond the scope of this Letter.

In a lossless medium, the parameters c, ρ, and $\mathbf{u}$ are real-valued, and complex conjugated cw fields in the original medium satisfy the same governing equations (1) and (2) as fields due to complex conjugated sources in a medium with reversed flow. The boundary conditions at free and rigid surfaces are also the same for the field and its complex conjugate. At impedance surfaces, the boundary conditions for the field and its complex conjugate are the same only when $\zeta = -\zeta^*$, see Eq. (3). FRT (4) remains valid when $p^{(2)}$, $\mathbf{w}^{(2)}$, $B^{(2)}$, and $\mathbf{F}^{(2)}$ are understood to be complex conjugated fields and sources in the original medium. Such a version of the FRT expresses conservation of acoustic energy averaged over the wave period in a moving fluid [[10]; [11], Chap. 8]. Consider a special case when the fields $p^{(1)}$ and $p^{(2)}$ are due to spatially separated, point sources of volume velocity located at points $\mathbf{x} = \mathbf{x}_1$ and $\mathbf{x} = \mathbf{x}_2$ within Ω. Recalling the definition of the Green's function, we obtain from Eq. (4)

$$G(\mathbf{x}_2, \mathbf{x}_1, \omega) + G^*(\mathbf{x}_1, \mathbf{x}_2, \omega) = i\omega\int_{\partial\Omega} ds(\mathbf{x})\mathbf{N}\cdot(G_1\mathbf{g}_2^* - G_2^*\mathbf{g}_1 + \rho D\mathbf{u}), \quad (6)$$

where $D = \mathbf{g}_2^*\cdot(\mathbf{u}\cdot\nabla)\mathbf{g}_1 - \mathbf{g}_1\cdot(\mathbf{u}\cdot\nabla)\mathbf{g}_2^* - 2i\omega\,\mathbf{g}_1\cdot\mathbf{g}_2^*$, $G_j \equiv G(\mathbf{x}, \mathbf{x}_j, \omega)$, and $\mathbf{g}_j \equiv \mathbf{g}(\mathbf{x}, \mathbf{x}_j, \omega)$. There may be inclusions inside Ω with free or rigid boundaries or impedance boundaries with reactive (i.e., purely imaginary) impedance. Neither such inclusions nor solid inclusions (with uniform background pressure p_0 on their surface) change the appearance of the identity (6) as long as the points $\mathbf{x} = \mathbf{x}_1$ and $\mathbf{x} = \mathbf{x}_2$ are located in the fluid.

Fields of random sources.—Consider an acoustic field generated by random sources of volume velocity in an inhomogeneous medium with deterministic, time-independent parameters. Let the sources be distributed on a surface S defined by the equation $\mathbf{x} = \mathbf{x}(\eta, \xi)$, where η, ξ are orthogonal coordinates on S. The sources' density $B = \partial e(\mathbf{x}, t)/\partial t$ is δ correlated in space, has a zero mean, $\langle e(\mathbf{x}, t)\rangle = 0$, and is stationary in time in the statistical sense:

$$\langle e(\mathbf{x}(\eta_1, \xi_1), t_1)e(\mathbf{x}(\eta_2, \xi_2), t_2)\rangle = E(\mathbf{x}(\eta_1, \xi_1), t_1 - t_2) \times \delta(\eta_1 - \eta_2) \times \delta(\xi_1 - \xi_2). \quad (7)$$

Here, angular brackets denote the average over the statistical ensemble of random sources.

The acoustic pressure

$$P(\mathbf{x}_1, t) = \int_S ds(\mathbf{x})\int_{-\infty}^{+\infty} dt_1 G(\mathbf{x}_1, \mathbf{x}, t - t_1)e(\mathbf{x}, t_1) \quad (8)$$

generated by the random sources has zero mean. For the spectrum $C(\mathbf{x}_1, \mathbf{x}_2, \omega)$ of the two-point correlation function

$$C(\mathbf{x}_1, \mathbf{x}_2, \tau) = \int_{-\infty}^{+\infty} dt\langle P(\mathbf{x}_1, t - \tau)P(\mathbf{x}_2, t)\rangle \quad (9)$$

of noise, we obtain from Eqs. (7) and (8)

$$C(\mathbf{x}_1, \mathbf{x}_2, \omega) = (2\pi)^2 \times \int_S ds(\mathbf{x})G^*(\mathbf{x}_1, \mathbf{x}, \omega)G(\mathbf{x}_2, \mathbf{x}, \omega)E(\mathbf{x}, \omega), \quad (10)$$

where $E(\mathbf{x}, \omega)$ is the frequency spectrum of $E(\mathbf{x}, t)$.

Under certain conditions, the surface integral on the right-hand side of Eq. (10) is similar to the surface integral in the identity (6). When this is the case, the flow reversal theorem allows one to express the noise correlations in terms of the deterministic Green's functions between the receivers' locations.

Noise from sources on an impedance surface.—Let noise be generated by sources located on an impedance surface S. This is a suitable model, for instance, of a noise in the atmosphere due to sources on the ground. We assume that $\mathrm{Im}\,(\mathbf{N}\cdot\nabla p_0 - i\omega\zeta)^{-1} \neq 0$. This condition excludes S being a surface with purely reactive impedance. The condition is imposed only on the surface where the sources are located.

If S is a closed surface, and the noise field is generated inside the surface, we choose $\partial\Omega$ to coincide with S. If either noise is generated outside of S or S is not closed, $\partial\Omega$ is a closed surface that consists of S and the surfaces at infinity. From the identity (6) and the boundary condition (3), we have

$$G(\mathbf{x}_2, \mathbf{x}_1, \omega) + G^*(\mathbf{x}_1, \mathbf{x}_2, \omega) = 2\omega \int_S ds(\mathbf{x}) G(\mathbf{x}, \mathbf{x}_1, \omega) \times G^*(\mathbf{x}, \mathbf{x}_2, \omega) \times \mathrm{Im}\,(i\omega\zeta - \mathbf{N}\cdot\nabla p_0)^{-1}. \tag{11}$$

Writing the identity (11) for a medium with reversed flow and then utilizing the symmetry property of the Green's functions, we obtain

$$G(\mathbf{x}_1, \mathbf{x}_2, \omega) + G^*(\mathbf{x}_2, \mathbf{x}_1, \omega) = 2\omega \int_S ds(\mathbf{x}) G(\mathbf{x}_1, \mathbf{x}, \omega) \times G^*(\mathbf{x}_2, \mathbf{x}, \omega) \times \mathrm{Im}\,(i\omega\zeta - \mathbf{N}\cdot\nabla p_0)^{-1}. \tag{12}$$

In Eq. (12), unlike (11), integration is carried over the position of a sound source rather than a receiver.

If the spectral intensity of the noise sources and the surface impedance are related by the equation $E(\mathbf{x}, \omega) = \theta(\omega)\,\mathrm{Im}\,(i\omega\zeta - \mathbf{N}\cdot\nabla p_0)^{-1}$, from Eqs. (10) and (12), we find

$$C(\mathbf{x}_1, \mathbf{x}_2, \omega) = 2\pi^2\omega^{-1}\theta(\omega)[G(\mathbf{x}_2, \mathbf{x}_1, \omega) + G^*(\mathbf{x}_1, \mathbf{x}_2, \omega)]. \tag{13}$$

This is an exact relation between the frequency spectrum of the cross-correlation function of noise and the deterministic Green's function between the two points where acoustic noise is being measured.

Noise from distant sources.—The points $\mathbf{x}_{1,2}$, where the noise is measured, are located within a volume of inhomogeneous, moving fluid. Let this volume be embedded into a much larger volume of a homogeneous, motionless fluid containing the surface S. The distance from points on S to receivers at $\mathbf{x}_{1,2}$ is large compared to the dimensions of the region where the medium is inhomogeneous and moving. Under these assumptions, $\nabla G(\mathbf{x}, \mathbf{x}_1, \omega) \approx i\omega c^{-1}(\mathbf{x}) \times G(\mathbf{x}, \mathbf{x}_1, \omega)\mathbf{x}/|\mathbf{x}|$ in the vicinity of S, and Eq. (6) becomes

$$G(\mathbf{x}_2, \mathbf{x}_1, \omega) + G^*(\mathbf{x}_1, \mathbf{x}_2, \omega) = 2\int_S \frac{ds}{\rho c}\left(\mathbf{N}\cdot\frac{\mathbf{x}}{|\mathbf{x}|}\right) \times G(\mathbf{x}, \mathbf{x}_1, \omega) G^*(\mathbf{x}, \mathbf{x}_2, \omega). \tag{14}$$

Let S be a sphere centered at a point within the inhomogeneous region. Then the dot product in the integrand in Eq. (14) equals unity. Suppose the random sources are uniformly distributed on S. Using the same reasoning as in the derivation of Eq. (12) from Eq. (11), from Eqs. (14) and (10), we find

$$C(\mathbf{x}_1, \mathbf{x}_2, \omega) = 2\pi^2\rho c E(\omega)[G(\mathbf{x}_2, \mathbf{x}_1, \omega) + G^*(\mathbf{x}_1, \mathbf{x}_2, \omega)], \tag{15}$$

where ρ and c are the mass density and sound speed in the vicinity of S. If the noise sources $e(\mathbf{x}, t)$ are δ correlated in time, E is independent of ω, and, in the time domain, the relation (15) becomes

$$G(\mathbf{x}_2, \mathbf{x}_1, t) + G(\mathbf{x}_1, \mathbf{x}_2, -t) = (2\pi^2\rho c E)^{-1} C(\mathbf{x}_1, \mathbf{x}_2, t). \tag{16}$$

Because of causality, $G(\mathbf{x}_2, \mathbf{x}_1, t) = 0$ at $t < 0$ and $G(\mathbf{x}_1, \mathbf{x}_2, -t) = 0$ at $t > 0$. This observation makes retrieval of $G(\mathbf{x}_2, \mathbf{x}_1, t)$ and $G(\mathbf{x}_1, \mathbf{x}_2, -t)$ from Eq. (16) a trivial matter.

High-frequency acoustic waves.—At sufficiently high frequencies, when the ray approximation becomes applicable, the Green's function $G(\mathbf{x}, \mathbf{y}, \omega)$ is a sum of terms $A(\mathbf{x}, \mathbf{y}) \exp[i\omega\phi(\mathbf{x}, \mathbf{y})]$ with a slowly varying complex amplitude A and a rapidly varying, real-valued eikonal ϕ, with eikonals being the same for $G(\mathbf{x}, \mathbf{y}, \omega)$ and $\mathbf{g}(\mathbf{x}, \mathbf{y}, \omega)$ [see, e.g., [11], Chap. 5]. The derivative of the eikonal $\mathbf{h}(\mathbf{x}, \mathbf{y}) \equiv -\partial\phi(\mathbf{x}, \mathbf{y})/\partial\mathbf{y}$ has the meaning of a slowness vector at point $\mathbf{y}$ on a ray which goes from $\mathbf{y}$ to point $\mathbf{x}$. The dominant terms of high-frequency asymptotic expansions of the surface integral in Eq. (10) originate from contributions of stationary points $\mathbf{x} = \mathbf{x}_s \in S$, which satisfy the equation

$$[\mathbf{h}(\mathbf{x}_2, \mathbf{x}_s) - \mathbf{h}(\mathbf{x}_1, \mathbf{x}_s)] \times \mathbf{N}(\mathbf{x}_s) = 0. \tag{17}$$

Let a ray that originates at some point $\mathbf{x} = \mathbf{x}_0$ on S go through both points $\mathbf{x}_1$ and $\mathbf{x}_2$. Then $\mathbf{h}(\mathbf{x}_2, \mathbf{x}_0) = \mathbf{h}(\mathbf{x}_1, \mathbf{x}_0)$, Eq. (17) is met, and, hence, $\mathbf{x}_0$ is a stationary point. Inversely, let $\mathbf{x} = \mathbf{x}_s$ be a stationary point on S. Using the dispersion equation $ch + \mathbf{u}\cdot\mathbf{h} = 1$ of sound in moving fluid, it can be shown that, for points $\mathbf{x}_1$ and $\mathbf{x}_2$ on the same side of S, the equality $\mathbf{h}(\mathbf{x}_2, \mathbf{x}_s) = \mathbf{h}(\mathbf{x}_1, \mathbf{x}_s)$ follows from Eq. (17). Thus, there is a one-to-one correspondence between the stationary points in the integral Eq. (10) and points on the surface S from which acoustic rays go through both $\mathbf{x}_1$ and to $\mathbf{x}_2$.

The asymptotic contribution of a stationary point $\mathbf{x}_s$ into the surface integral over S has eikonal $\Phi(\mathbf{x}_s) = \phi(\mathbf{x}_2, \mathbf{x}_s) - \phi(\mathbf{x}_1, \mathbf{x}_s)$. It follows from the additivity of an eikonal along a given ray and the correspondence between the stationary points and the rays through the points $\mathbf{x}_1$ and to $\mathbf{x}_2$ that either $\Phi(\mathbf{x}_s) = \phi(\mathbf{x}_2, \mathbf{x}_1)$ (which is the case for rays that go from $\mathbf{x}_s$ through $\mathbf{x}_1$ and then $\mathbf{x}_2$) or $\Phi(\mathbf{x}_s) = -\phi(\mathbf{x}_1, \mathbf{x}_2)$ (which is the case for rays that go from $\mathbf{x}_s$ through $\mathbf{x}_2$ and then $\mathbf{x}_1$). These are exactly the eikonals of the individual ray components of the Green's functions $G(\mathbf{x}_2, \mathbf{x}_1, \omega)$ and $G^*(\mathbf{x}_1, \mathbf{x}_2, \omega)$.

In the motionless case $\phi(\mathbf{x}_2, \mathbf{x}_1) + \phi(\mathbf{x}_1, \mathbf{x}_2) = 0$ because of reciprocity. In moving media, the sum is generally nonzero and quantifies flow-induced nonreciprocity. In ocean acoustic tomography [13], Chap. 3; [14], the two eikonals are measured in reciprocal transmission experiments using acoustic transceivers. Nonreciprocity in an eikonal (or, equivalently, travel time) can be inverted for flow velocity and allows one to measure currents that are small compared to uncertainties in the sound speed [13,14].

Let points $\mathbf{x}_1$ and $\mathbf{x}_2$ be situated within a closed surface S. Choosing $\partial\Omega$ in the identity (6) to coincide with S and using Eq. (1) to express $\mathbf{g}$ in terms of G, we obtain asymptotically

$$G(\mathbf{x}_2, \mathbf{x}_1, \omega) + G^*(\mathbf{x}_1, \mathbf{x}_2, \omega) = -2\int_S \frac{ds(\mathbf{x})}{\rho c} \times \mathbf{N}\cdot\mathbf{q}\, G^*(\mathbf{x}_1, \mathbf{x}, \omega) \times G(\mathbf{x}_2, \mathbf{x}, \omega), \tag{18}$$

$$\mathbf{q}\,(\mathbf{x}) = \frac{\omega}{2ch_1^2h_2^2}\left[h_2^2\mathbf{h}_1 + h_1^2\mathbf{h}_2 + (h_1 + h_2)(\mathbf{h}_1\cdot\mathbf{h}_2)\frac{\mathbf{u}}{c}\right], \tag{19}$$

where $\mathbf{h}_j \equiv \mathbf{h}(\mathbf{x}_j, \mathbf{x})$, $j = 1, 2$. In a derivation of Eq. (18), we again utilized the invariance of the Green's function with respect to interchange of its arguments with simultaneous flow reversal. When there are multiple rays from $\mathbf{x}$ to $\mathbf{x}_1$ and/or $\mathbf{x}_2$, a double sum over rays corresponding to the two Green's functions appears on the right-hand side of Eq. (18).

The integrands in Eqs. (10) and (18) contain the same rapidly oscillating factors and, hence, have the same stationary points. Therefore, the spectrum $C(\mathbf{x}_1, \mathbf{x}_2, \omega)$ of the cross-correlation function (9) of noise provides an estimate of the sum $G(\mathbf{x}_2, \mathbf{x}_1, \omega) + G^*(\mathbf{x}_1, \mathbf{x}_2, \omega)$ of the Green's functions such that the eikonals of all terms are reproduced exactly, while the amplitudes of individual ray terms are multiplied by factors $\alpha(\mathbf{x}_s, \omega)$, which are generally different for distinct stationary points. Here,

$$\alpha(\mathbf{x}, \omega) = -2\pi^2\rho(\mathbf{x})c(\mathbf{x})E(\mathbf{x}, \omega)/[\mathbf{N}(\mathbf{x})\cdot\mathbf{q}(\mathbf{x})]. \tag{20}$$

An inspection shows that these conclusions are consistent with the exact results (13) and (15).

If the points $\mathbf{x}_1$ and $\mathbf{x}_2$ are situated outside of S, or S is not a closed surface, $C(\mathbf{x}_1, \mathbf{x}_2, \omega)$ reproduces contributions only of those rays connecting $\mathbf{x}_1$ and $\mathbf{x}_2$, extensions of which in the direction opposite to the direction of sound propagation intersect S.

Conclusion.—We have demonstrated that one can retrieve the Green's function of an inhomogeneous, lossless, moving medium from two-point correlation of acoustic noise. Our results apply to arbitrary flow of inhomogeneous fluid. The fluid can be either unbounded or have free, rigid, and impedance boundaries. Fluid-solid boundaries may also exist. Acoustic noise was assumed to be generated by random sources δ correlated in space and distributed along a surface. The relation between two-point correlation of acoustic pressure in the noise field and the deterministic Green's function has been shown to be exact in some cases and approximate under more general conditions. For motionless acoustic media, our results are equivalent to the known relation between the Green's function and the noise correlation function proved by Wapenaar [6]. The extension to moving media has been made possible by use of the flow reversal theorem [10,12], which extends the reciprocity principle to waves in moving media. It has been shown that measurement of the two-point correlation function of noise allows one to quantify the flow-induced acoustic nonreciprocity and determine the travel times of waves propagating in opposite directions, which are normally obtained in reciprocal transmissions experiments employing acoustic transceivers located at the two points.

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