Coexistence of Triplet Superconductivity and Spin Density Waves

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We discuss the possibility of the coexistence of spin density waves (antiferromagnetism) and triplet superconductivity as a particular example of a broad class of systems where the interplay of magnetism and superconductivity is important. We focus on the case of quasi-one-dimensional metals, where it is known that antiferromagnetism is in close proximity to triplet superconductivity in the pressure versus temperature phase diagram. Over a range of pressures, we propose an intermediate nonuniform phase consisting of antiferromagnetic and triplet superconducting orders. In the coexistence region, we propose a flop transition in the spin density wave order parameter vector, which affects the nature of the superconducting state and leads to the appearance of several new phases.

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The competition or coexistence of magnetic order and superconductivity is a very important problem in condensed matter physics. There is a broad class of systems that presents magnetic order and superconductivity in close vicinity. One of the most important systems is copper oxides, where singlet superconductivity (SSC) is found next to antiferromagnetism [1]. Another interesting system is strontium ruthenate Sr_2RuO_4, where the proximity to ferromagnetism (FM) has been argued as being important to the existence of possible triplet superconductivity (TSC) in these materials [2]. Furthermore, the ferromagnetic superconductors $ZrZn_2$ and UGe_2 have stimulated a debate on the coexistence of ferromagnetism and triplet or singlet superconductivity [3,4]. However, unlike any of these previous examples, we discuss in this Letter a three-dimensional but highly anisotropic organic superconductor, the Bechgaard salt $(TMTSF)_2PF_6$, which has a phase diagram of neighboring antiferromagnetism with well defined spin density wave (SDW) order and triplet superconductivity [5]. At first glance, the TSC and SDW orders would avoid coexistence since the two orders are competing to correlate electrons in the triplet and singlet spin sectors, respectively. For instance, the presence of SDW order would disrupt TSC in a more dramatic way than it would SSC, while the FM order would disrupt SSC more than TSC. Therefore, it is easier to find in nature examples of coexistence of SSC and SDW or TSC and FM, while the conditions to find the coexistence of TSC and SDW are much more stringent, as discussed in this Letter.

The antiferromagnetic state of $(TMTSF)_2PF_6$ is present at temperatures $T < 12$ K and pressures $P < 6$ kbar and is characterized by a spin density wave [5]. The SDW order parameter $\mathbf{N}$ (Néel vector) has a small anisotropy with the easy axis along the crystallographic $\mathbf{b}'$ axis [6], which is also the intermediate direction for conductivity. This antiferromagnetic state is suppressed at pressures higher than 6 kbar, where a superconducting instability takes over at low temperatures ($T < T_c \approx 1.2$ K). This superconducting state is very likely to be triplet, as suggested by upper critical fields [7] and NMR [8] measurements. Recent experiments [9–11] suggest a region of macroscopic coexistence of TSC and SDW, where both orders are nonuniform. This coexistence region can be related to existing theoretical proposals. For instance, strictly one-dimensional theories invoking SO(4) symmetry [12] or negative interface energies [13] have allowed for coexisting TSC and SDW. However, these previous theories are not directly applicable to three-dimensional but highly anisotropic superconductors such as the Bechgaard salts, where the SO(4) symmetry is absent, and negative interface energies are not necessary conditions for the coexistence.

The main results described in this Letter are as follows. First, we derive microscopically the pressure versus temperature phase diagram indicating the TSC, the SDW, and the TSC-SDW phases and show that the TSC and SDW order parameters are both nonuniform in the coexistence region. The suggestion of SDW and TSC coexistence is consistent with recent experimental results [11]. Second, we show explicitly that the SO(4) theories [12] cannot describe these highly anisotropic three-dimensional systems. Finally, we propose that external magnetic fields cause a canting transition of the SDW order parameter which alters the nature of the TSC state in the coexistence region since SDW and TSC are coupled. In addition to the normal phase, we find new phases in the magnetic field versus temperature phase diagram near the critical pressure.

The compound $(TMTSF)_2PF_6$ can be described approximately by an orthorhombic lattice with dispersion

$$\epsilon_{\mathbf{k}} = -|t_x|\cos(k_x a) - |t_y|\cos(k_y b) - |t_z|\cos(k_z c), \quad (1)$$

where transfer integrals $|t_x|$, $|t_y|$, and $|t_z|$ satisfy the relations $|t_x| \gg |t_y| \gg |t_z|$ representing the quasi-one-dimensionality. Here a, b, and c correspond to unit cell lengths along the crystallographic axes $\mathbf{a}(x)$, $\mathbf{b}'(y)$, and $\mathbf{c}^*(z)$, respectively.

We use natural units ($\hbar = k_B = c = 1$) and work with Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm int}$, where the noninteracting part is $\mathcal{H}_0 = \sum_{\mathbf{k},\alpha}(\epsilon_{\mathbf{k}} - \mu)c^\dagger_{\mathbf{k},\alpha}c_{\mathbf{k},\alpha}$, with μ the chemical potential. The interaction part is

$$\mathcal{H}_{\rm int} = \sum_{\mathbf{k}\mathbf{k}'\mathbf{p}}\sum_{\alpha\beta\gamma\delta} V(\mathbf{k},\mathbf{k}')\mathbf{d}^\dagger_{\alpha\beta}(\mathbf{k},\mathbf{p})\cdot\mathbf{d}_{\gamma\delta}(\mathbf{k}',\mathbf{p}) + \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}}\sum_{\alpha\beta\gamma\delta} J(\mathbf{q})\mathbf{s}^\dagger_{\alpha\beta}(\mathbf{k},\mathbf{q})\cdot\mathbf{s}_{\gamma\delta}(\mathbf{k}',\mathbf{q}), \quad (2)$$

where the first and second terms describe interactions in TSC and SDW channels, respectively. These interactions allow for the possibility of competition or coexistence of TSC and SDW instabilities at low temperatures. Here α, β, γ, and δ are spin indices and $\mathbf{k}$, $\mathbf{k}'$, $\mathbf{p}$, and $\mathbf{q}$ represent linear momenta. The vector operator $\mathbf{d}^\dagger_{\alpha\beta}(\mathbf{k},\mathbf{p}) \equiv c^\dagger_{\mathbf{k}+\mathbf{p}/2,\alpha}\mathbf{v}_{\alpha\beta}c^\dagger_{-\mathbf{k}+\mathbf{p}/2,\beta}$, and $\mathbf{s}^\dagger_{\alpha\beta}(\mathbf{k},\mathbf{q}) \equiv c^\dagger_{\mathbf{k}-\mathbf{q}/2,\alpha}\boldsymbol{\sigma}_{\alpha\beta}c_{\mathbf{k}+\mathbf{q}/2,\beta}$. The matrix $\mathbf{v}_{\alpha\beta} = (i\boldsymbol{\sigma}\sigma_y)_{\alpha\beta}$, and σ_i are Pauli matrices. In the case of weak spin-orbit coupling, the TSC interaction $V(\mathbf{k},\mathbf{k}')$ can be chosen as $V(\mathbf{k},\mathbf{k}') = Vh_\Gamma(\mathbf{k},\mathbf{k}')\phi_\Gamma(\mathbf{k})\phi_\Gamma(\mathbf{k}')$, where V is a prefactor with a dimension of energy, and $h_\Gamma(\mathbf{k},\mathbf{k}')$ $[\phi_\Gamma(\mathbf{k})]$ characterizes the momentum dependence [symmetry basis function] for an irreducible representation Γ of the orthorhombic D_{2h} group [14,15]. Without loss of the generality regarding symmetry properties, we take $h_\Gamma(\mathbf{k},\mathbf{k}') = 1$ and consider only unitary triplet states corresponding to p_x symmetry.

The order parameter for TSC can be defined as $\mathbf{D}(\mathbf{p}) = \langle\sum_{\mathbf{k},\alpha\beta}V\phi_\Gamma(\mathbf{k})\mathbf{d}_{\alpha\beta}(\mathbf{k},\mathbf{p})\rangle$, while the SDW order parameter can be defined as $\mathbf{N}(\mathbf{q}) = J(\mathbf{q})\langle\sum_{\mathbf{k},\alpha\beta}\mathbf{s}_{\alpha\beta}(\mathbf{k},\mathbf{q})\rangle$. With these definitions, the effective Hamiltonian is

$$\mathcal{H}_{\rm eff} = \mathcal{H}_0 + \mathcal{H}_{\rm TSC} + \mathcal{H}_{\rm SDW}, \quad (3)$$

where the TSC contribution is $\mathcal{H}_{\rm TSC} = \sum_{\mathbf{p}}[\mathbf{D}^\dagger(\mathbf{p})\cdot\sum_{\mathbf{k},\alpha\beta}\phi_\Gamma(\mathbf{k})\mathbf{d}_{\alpha\beta}(\mathbf{k},\mathbf{p}) + \text{H.c.}] - \sum_{\mathbf{p}}\mathbf{D}^\dagger(\mathbf{p})\cdot\mathbf{D}(\mathbf{p})/V$, and the TSC term is $\mathcal{H}_{\rm SDW} = \sum_{\mathbf{q}}[\mathbf{N}(-\mathbf{q})\cdot\sum_{\mathbf{k},\alpha\beta}\mathbf{S}_{\alpha\beta}(\mathbf{k},\mathbf{q}) + \text{H.c.}] - \sum_{\mathbf{q}}\mathbf{N}(-\mathbf{q})\cdot\mathbf{N}(\mathbf{q})/J(\mathbf{q})$. The effective action of this Hamiltonian is obtained by integrating out the fermions, where the quadratic terms are

$$S_2^{\rm TSC} = \sum_{\mathbf{p}} A(\mathbf{p})\mathbf{D}^\dagger(\mathbf{p})\cdot\mathbf{D}(\mathbf{p}),$$
$$S_2^{\rm SDW} = \sum_{\mathbf{q}} B(\mathbf{q})\mathbf{N}(-\mathbf{q})\cdot\mathbf{N}(\mathbf{q}), \quad (4)$$

where coefficients $A(\mathbf{p})$ and $B(\mathbf{q})$ can be obtained from their corresponding diagrams. Notice that the two order parameters $\mathbf{D}(\mathbf{p})$ and $\mathbf{N}(\mathbf{q})$ do not couple to quadratic order, because TSC and SDW are instabilities in the particle-particle and particle-hole channels, respectively. In addition, notice that we have neglected the small anisotropy in the SDW order parameter [6], but we will include this small anisotropy due to spin-orbit coupling later when discussing the effects of a magnetic field.

Next, we make the following assumptions. First, we assume that the saddle point TSC order parameter is dominated by the zero center-of-mass momentum component $\mathbf{D}_0 \equiv \mathbf{D}(\mathbf{p} = 0)$. Second, we assume that the saddle point SDW order parameter $\mathbf{N}$ is a real vector in coordinate space and that it has Fourier components determined by Fermi surface nesting vectors $\mathbf{q} = \mathbf{Q}_i = (\pm Q_a, \pm Q_b, \pm Q_c)$ [5]. In this case, the coefficients $B(\mathbf{Q}_i)$ are identical for all $\mathbf{Q}_i$'s, since the lattice dispersion is invariant under reflections and inversions compatible with the D_{2h} group. In addition, the coefficients of all higher order terms involving $\mathbf{N}(\mathbf{Q}_i)$ share the same symmetric property. Given that $\mathbf{N}(\mathbf{r})$ is real, and that we have periodic boundary conditions, we can choose a specific reference phase where $\mathbf{N}(\mathbf{Q}_i)$ are real and identical. Thus, we define $\mathbf{N}_0 \equiv \mathbf{N}(\mathbf{Q}_i)$ for all i, and the quadratic terms are dominated in the long wavelength limit by $S_2^{\rm TSC} \approx A(0)|\mathbf{D}_0|^2$ and $S_2^{\rm SDW} \approx (m/2)B(\mathbf{Q}_1)|\mathbf{N}_0|^2$, respectively. Here m is the number of nesting vectors, and $\mathbf{Q}_1 = (Q_a, Q_b, Q_c)$ is chosen for definiteness.

Although the order parameters for TSC and SDW do not couple to quadratic order, the coupling between $\mathbf{D}$ and $\mathbf{N}$ in fourth order is given by

$$S_4^C = (C_1 + C_2/2)|\mathbf{D}_0|^2|\mathbf{N}_0|^2 - C_2|\mathbf{D}_0\cdot\mathbf{N}_0|^2, \quad (5)$$

where coefficients C_1 and C_2 can be obtained from their corresponding diagrams. Notice that, because there are several distinct nesting vectors, the diagram corresponding to the coupling term is not unique. However, the contribution of these distinct diagrams is identical, due to symmetry of the dispersion relation compatible with the D_{2h} group. The second term in Eq. (5) can be parametrized as $C_2\cos^2(\theta)|\mathbf{D}_0|^2|\mathbf{N}_0|^2$, where $\cos^2\theta \equiv |\mathbf{D}_0\cdot\mathbf{N}_0|^2/|\mathbf{D}_0|^2|\mathbf{N}_0|^2 \le 1$ is independent of $|\mathbf{D}_0|$ and $|\mathbf{N}_0|$. Since we consider only a unitary state $\mathbf{D}_0$ for TSC, its global phase can be eliminated, and θ can be regarded as the angle between $\mathbf{D}_0$ and $\mathbf{N}_0$. The coefficient C_2 for $(\text{TMTSF})_2\text{PF}_6$ is positive, indicating that $\mathbf{D}_0$ and $\mathbf{N}_0$ are not free to rotate independently but are tending to be aligned ($\theta = 0$) or antialigned ($\theta = \pi$).

Additional fourth order terms are

$$S_4^{\rm TSC} = D_1|\mathbf{D}_0|^4; \qquad S_4^{\rm SDW} = D_2|\mathbf{N}_0|^4, \quad (6)$$

where coefficients D_1 and D_2 are obtained diagrammatically. However, it should be emphasized that there are several distinct diagrams that contribute to $S_4^{\rm SDW}$, due to different combination of nesting vectors. Unlike the case for S_4^C, the contributions of distinct diagrams for $S_4^{\rm SDW}$ are not identical. The effective action is

$$S_{\rm eff} = S_0 + S_2^{\rm TSC} + S_2^{\rm SDW} + S_4, \quad (7)$$

where S_0 is the normal state contribution, and $S_4 = D_1|\mathbf{D}_0|^4 + D_2|\mathbf{N}_0|^4 + C(\theta)|\mathbf{D}_0|^2|\mathbf{N}_0|^2$, with $C(\theta) = C_1 + C_2/2 - C_2\cos^2\theta$. The phase diagram that emerges from this action leads to either bicritical or tetracritical points as illustrated in Fig. 1. When $R = C^2(0)/(4D_1D_2) > 1$, the

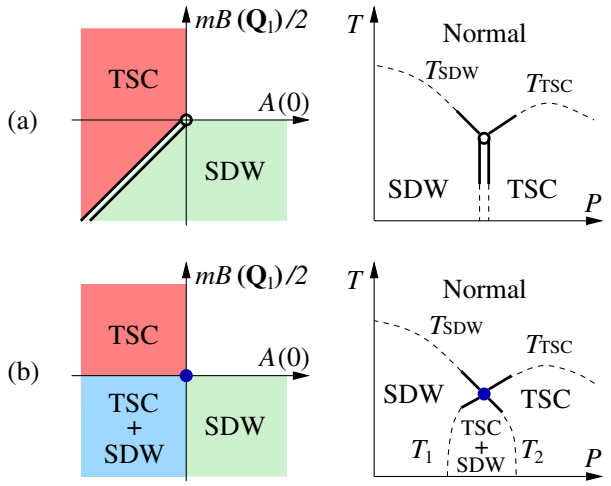

FIG. 1 (color online). Phase diagrams indicating (a) first order transition line with a no-coexistence phase and (b) two second order lines with a coexistence region between the TSC and SDW phases. The P-T phase diagrams are obtained by assigning the standard linear temperature and pressure dependence on the GL coefficients around the critical point, i.e., $A(0) \propto [T - T_{\mathrm{TSC}}(P)]$, with $T_{\mathrm{TSC}} = T_c + \nu_{\mathrm{TSC}}(P - P_c)$, and $B(\mathbf{Q}_1) \propto [T - T_{\mathrm{SDW}}(P)]$, with $T_{\mathrm{SDW}} = T_c + \nu_{\mathrm{SDW}}(P_c - P)$. Here ν_{TSC} and ν_{SDW} are both positive.

critical point (P_c, T_c) is bicritical and there is a first order transition line at $(m/2)B(\mathbf{Q}_1) = A(0)$ when both $B(\mathbf{Q}_1) < 0$ and $A(0) < 0$, as seen in Fig. 1(a). However, when $R < 1$, (P_c, T_c) is tetracritical and a coexistence region of TSC and SDW occurs when both $B(\mathbf{Q}_1) < 0$ and $A(0) < 0$, as shown in Fig. 1(b). Notice that the action S_{eff} obtained in three dimensions is not SO(4) invariant, and SO(4) symmetry based theories [12] can be applied only to one-dimensional systems but not to the highly anisotropic but three-dimensional Bechgaard salts.

The ratio $R \approx 0.12$ for the Bechgaard salt $(\mathrm{TMTSF})_2\mathrm{PF}_6$ around (P_c, T_c), when the interaction strengths V, J are chosen to give the same $T_c = 1.2$ K at quarter filling for parameters $|t_x| = 5800$ K, $|t_y| = 1226$ K, $|t_z| = 58$ K, used in combination with $\phi_\Gamma(\mathbf{k}) = \sin(k_x a)$ (p_x symmetry for TSC) and the nesting vectors $\mathbf{Q} = (\pm\pi/2a, \pm\pi/2b, 0)$. Our analysis shows that $(\mathrm{TMTSF})_2\mathrm{PF}_6$ has a TSC-SDW coexistence region as suggested by experiments [9–11]. However, our results are in contrast with an SO(4) based theory, which predicted a no-coexistence region [12].

To investigate the TSC-SDW coexistence region, the effective action (7) is Fourier transformed into real space to give the Ginzburg-Laudau (GL) free energy density

$$\mathcal{F} = \mathcal{F}_n + \mathcal{F}_{\mathrm{TSC}} + \mathcal{F}_{\mathrm{SDW}} + \mathcal{F}_C, \tag{8}$$

where $\mathcal{F}_n$ is the normal state contribution, and $\mathcal{F}_C = C(\theta)|\mathbf{N}(\mathbf{r})|^2|\mathbf{D}(\mathbf{r})|^2$ is the coupling term of the two order parameters. For the Bechgaard salt parameters, the prefactor $C(0)$ of the coupling term $\mathcal{F}_C$ is positive and, hence, represents a local repulsive interaction between the TSC and SDW order parameters. As a consequence, the TSC order parameter is nonuniform in the TSC-SDW coexistence region and has a modulation induced by the SDW order parameter. Since $R \ll 1$ for $(\mathrm{TMTSF})_2\mathrm{PF}_6$, the coupling term $\mathcal{F}_C$ is small in comparison with the other fourth order terms, and a perturbative solution is possible for $|\mathbf{D}(\mathbf{r})|$ and $|\mathbf{N}(\mathbf{r})|$. At assumed zero TSC-SDW coupling $\mathcal{F}_C = 0$, the saddle point modulation for the SDW order parameter is $\mathbf{N}(\mathbf{r}) = m\mathbf{N}_0 \cos(\mathbf{Q}_1 \cdot \mathbf{r})$, with $|\mathbf{N}_0| = [-mB(\mathbf{Q}_1)/3D_2]^{1/2}$, while the saddle point magnitude for the TSC order parameter is $|\mathbf{D}(\mathbf{r})| = |\mathbf{D}_0| = [-A(0)/2D_1]^{1/2}$. Including the coupling $\mathcal{F}_C$, the new solution for the magnitude of TSC order parameter is

$$|\mathbf{D}(\mathbf{r})| - |\mathbf{D}_0| = -\upsilon \frac{|N_0|^2}{|D_0|} R^{1/2} \Big[\frac{\cos(2Q_a x)}{4 + 8\xi_x^2 Q_a^2} + \frac{\cos(2Q_b y)}{4 + 8\xi_y^2 Q_b^2} + \frac{\cos(2Q_a x)\cos(2Q_b y)}{4 + 8\xi_x^2 Q_a^2 + 8\xi_y^2 Q_b^2} + \frac{1}{4} \Big], \tag{9}$$

which shows explicitly $2Q_a$ and $2Q_b$ modulations along the $\mathbf{a}$ and $\mathbf{b}'$ axes, respectively. Here $\xi_i = [|\gamma^{ii}_{\mathrm{TSC}}/A(0)|]^{1/2}$ represents the TSC coherence length along the i direction, and $\upsilon = (6D_2/D_1)^{1/2}$. The qualitative behavior of $|\mathbf{D}(\mathbf{r})|$ is shown in Fig. 2(a). The solution for the magnitude of SDW order parameter to the first order correction is shown in Fig. 2(b). Notice that the maxima of $|\mathbf{D}(\mathbf{r})|$ coincide with the minima of $|\mathbf{N}(\mathbf{r})|$ indicating that the two orders try to be locally excluded. Since the TSC and SDW modulations are out of phase, experiments that are sensitive to the spatial distribution of the spin density or Cooper pair charge density may reveal the coexistence of these inhomogeneous phases.

Next, we analyze the effect of magnetic fields on this coexistence region. A uniform magnetic field $\mathbf{H}$ couples with charge via the Peierls substitution $\mathbf{k} \rightarrow \mathbf{k} - |e|\mathbf{A}$ in the dispersion relation given in Eq. (1), where $\mathbf{A}$ is the vector potential, and couples with spin via the paramagnetic term $\mathcal{H}_{\mathrm{P}} = -\mu_0 \mathbf{H} \cdot \sum_{\mathbf{k},\alpha\beta} c^\dagger_{\mathbf{k}\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{\mathbf{k}\beta}$, where μ_0 is the effective magnetic moment. Upon integrating out the fermions, the corresponding effective action is

$$S_{\mathrm{eff}}(\mathbf{H}) = S_0(\mathbf{H}) + S_2^{\mathrm{TSC}}(\mathbf{H}) + S_2^{\mathrm{SDW}}(\mathbf{H}) + S_4(\mathbf{H}), \tag{10}$$

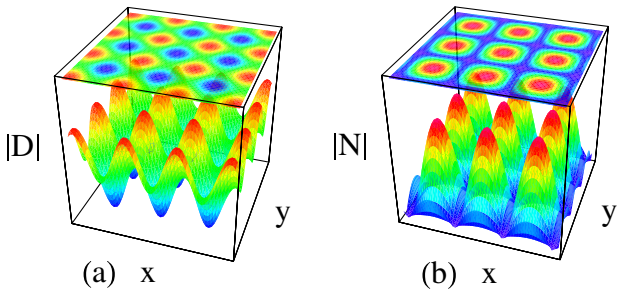

FIG. 2 (color online). Magnitude of (a) TSC and (b) SDW order parameters in the x-y plane, within the coexistence region.

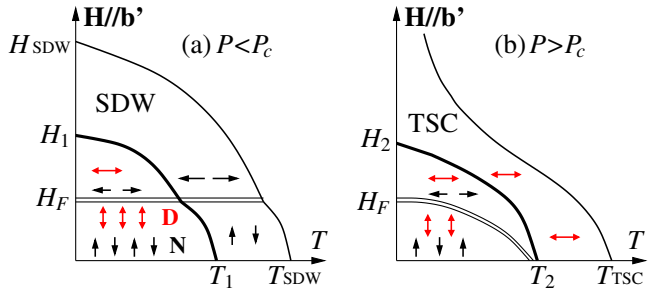

FIG. 3 (color online). H-T phase diagrams showing the TSC-SDW coexistence region (thick solid line) and canting transitions (double line) for (a) $P < P_c$ and (b) $P > P_c$.

where $S_0(\mathbf{H}) = S_0 + |\mathbf{H}|^2/8\pi - \chi_n|\mathbf{H}|^2/2$, χ_n is the uniform electronic spin susceptibility of the normal state, $S_2^{\mathrm{TSC(SDW)}}(\mathbf{H})$ is obtained from $S_2^{\mathrm{TSC(SDW)}}$ by the Peierls substitution, and $S_4(\mathbf{H}) = S_4 + (E_1 + E_2/2)|\mathbf{H}|^2|\mathbf{D}_0|^2 - E_2|\mathbf{H}\cdot\mathbf{D}_0|^2 + (F_1 - F_2/2)|\mathbf{H}|^2|\mathbf{N}_0|^2 + F_2|\mathbf{H}\cdot\mathbf{N}_0|^2$. A detailed calculation shows that the coefficient $E_1 = -E_2/2$; hence, the coupling of $\mathbf{H}$ to $\mathbf{D}$ can be described in the more familiar form $-\sum_{ij}H_i\chi_{ij}H_j/2$, where $\chi_{ij} = \chi_n\delta_{ij} + E_2D_i^*D_j$.

For Bechgaard salts, the coefficients $E_2 < 0$ and $F_2 > 0$, indicating that $\mathbf{D}$ and $\mathbf{N}$ prefer to be perpendicular to the magnetic field $\mathbf{H}$. These conditions, when combined with $C_2 > 0$ in Eq. (5), indicate that $\mathbf{D}$ and $\mathbf{N}$ prefer to be parallel to each other but perpendicular to $\mathbf{H}$. However, the relative orientation of these vectors in magnetic fields is affected by small spin anisotropy effects which were already observed in $(TMTSF)_2PF_6$, where the easy axis for $\mathbf{N}$ is the $\mathbf{b}'$ direction [6]. Such an anisotropy effect can be described by adding a quadratic term $-u_N N_{b'}^2$, with $u_N > 0$, which favors $\mathbf{N} \parallel \mathbf{b}'$. Similarly, the $\mathbf{D}$ vector also has anisotropic effect caused by spin-orbit coupling and can be described by adding a quadratic term $-u_D D_i^2$, where i is the easy axis for TSC. (Quartic TSC and SDW terms also become weakly anisotropic.)

However, a sufficiently large $\mathbf{H} \parallel \mathbf{b}'$ can overcome spin anisotropy effects and drive the $\mathbf{N}$ vector to flop onto the $\mathbf{a}$-$\mathbf{c}^*$ plane. This canting (flop) transition was reported [6] in $(TMTSF)_2PF_6$ for $H \approx 1$T at zero pressure and $T = 8$ K. If such a spin-flop transition persists near the TSC-SDW critical point (P_c, T_c), then the flop transition of the $\mathbf{N}$ vector forces the $\mathbf{D}$ vector to flop as well and has potentially serious consequences to the superconducting state. For $P < P_c$, if a flop transition occurs for $H_F < H_1(0)$ [see Fig. 3(a)], then $\mathbf{N}$ flops both in the pure SDW and in the TSC-SDW coexistence phases, in which case it forces $\mathbf{D}$ vector to flop as well. If the flop transition occurs for $H_{\mathrm{SDW}}(0) < H_F < H_1(0)$ (not shown), then only the pure SDW phase is affected. This situation is qualitatively different for $P > P_c$. In the zero (weak) spin-orbit coupling limit, the $\mathbf{D}$ vector is free to rotate in a magnetic field and tends to be perpendicular to $\mathbf{H}$. Thus, for $\mathbf{H} \parallel \mathbf{b}'$ and $|\mathbf{H}| > H_2(T)$, the $\mathbf{D}$ vector lies in the $\mathbf{a}$-$\mathbf{c}^*$ plane since there is no SDW order. However, at lower temperatures and small magnetic fields when TSC and SDW orders coexist, the spin anisotropy field forces $\mathbf{N}$ to be along $\mathbf{b}'$ and $\mathbf{N}$ forces $\mathbf{D}$ to flop from the $\mathbf{a}$-$\mathbf{c}^*$ plane to the $\mathbf{b}'$ direction. This canting transition occurs at $H_F < H_2(T)$ [see Fig. 3(b)]. However, in the case of coexistence of SDW and singlet superconductivity, there would be no vector coupling between SDW and SSC, and thus the canting transition of the SDW has no effect on the SSC order parameter.

In summary, we showed that the TSC and SDW order parameters can coexist in the P-T phase diagram of quasi-one-dimensional organic conductors in agreement with recent experiments. We find that the TSC order parameter is nonuniform in the coexistence region and that its modulation is induced via the SDW order parameter. We also showed that theories based on SO(4) symmetry cannot be applied to these highly anisotropic three-dimensional systems, since they are strictly valid only in the one-dimensional limit. Furthermore, we discussed qualitatively magnetic field effects on the coexistence region. We proposed that a magnetic field induced canting transition of the SDW order parameter affects dramatically the phase diagram of the coexistence region, both below and above the critical pressure.

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