Velocity Locking of Incoherent Nonlinear Wave Packets

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We show both theoretically and experimentally in an optical fiber system that a set of incoherent nonlinear waves irreversibly evolves to a specific equilibrium state, in which the individual wave packets propagate with identical group velocities. This intriguing process of velocity locking can be explained in detail by simple thermodynamic arguments based on the kinetic wave theory. Accordingly, the selection of the velocity-locked state is shown to result from the natural tendency of the isolated wave system to approach the state that maximizes the nonequilibrium entropy.

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The vector nature of nonlinear wave phenomena is known to arise in a plethora of physical contexts, such as optics, plasmas, or hydrodynamics. Most recently, vector phenomena have attracted growing interest because of their experimental realization in multicomponent Bose-Einstein condensed gases [1]. Vector phenomena play a natural important role in optics, because of the extra degrees of freedom associated with the polarization of the electromagnetic field [2]. The study of nonlinear polarization effects revealed a multitude of novel behaviors not found in the scalar case, for instance, by introducing the concept of a vector soliton [2].

Our purpose here is to analyze the properties of coherence of coupled nonlinear waves. A significant advance has been achieved in this framework since the first experimental demonstration of incoherent solitons, i.e., self-trapping of incoherent light, in photorefractive crystals [3]. The concept of a vector soliton was shown to be at the heart of the process of incoherent self-trapping [4]. Incoherent solitons and remarkable dynamical features inherent to stochastic nonlinear fields have also been recently investigated in instantaneous response nonlinear media [5,6].

We consider here the vector nonlinear Schrödinger (NLS) equation, which is known to provide a canonical description of weak nonlinear coupled dispersive waves [1–4,7,8]. We show both theoretically and experimentally in an optical fiber system that a set of incoherent nonlinear waves evolves, as a rule, to a specific equilibrium state, in which the individual wave packets propagate with identical group velocities. The unexpected existence of this process of velocity locking can be explained by simple thermodynamic arguments: The selection of the velocity-locked state is shown to result from the natural tendency of the isolated wave system (conservative and Hamiltonian) to approach an equilibrium state. In this respect, we emphasize the fundamental different nature between the process of velocity locking and the process of incoherent soliton generation [3,4], where each wave packet is intrinsically a coherent component of the corresponding vector soliton [4]. Along the same line, we also note the difference between the reported phenomenon and the velocity locking of coherent ultrashort pulses, known to occur through a process of soliton trapping, in which pulses of different velocities bind together under the form of a temporal vector soliton [9].

We analyze the process of thermal wave relaxation to equilibrium on the basis of the kinetic wave theory [10–13]. In this approach, phase information is averaged out through the random-phase approximation, which leads to a weak turbulence description of the interaction based on irreversible kinetic equations. It results that, in spite of the formal reversibility of the vector NLS equation, the kinetic equation describes an irreversible evolution of the fields to thermodynamic equilibrium. The mathematical statement of this irreversible behavior relies on the H theorem of entropy growth [12,13], whose origin is analogous to Boltzmann's H theorem relevant for gas kinetics [14]. In this respect, we show that the process of velocity locking results from the irreversible evolution of the wave system towards the "most disordered state," i.e., the state that realizes the maximum of entropy. The theory then reveals that it is thermodynamically advantageous for the coupled incoherent wave packets to propagate with identical group velocities.

Although the process of thermal relaxation to equilibrium is undeniably considered as a basic generic property of a molecular gas system [14], its role in the nonlinear evolution of a pure wave system has not been precisely established experimentally. This situation is mainly due to the fact that the irreversible relaxation process is predicted in a lossless (conservative and reversible) wave system, while any practical system unavoidably exhibits dissipation. Nonetheless, nonlinear optics appears to be a promising field of investigation of thermal wave relaxation because of the availability of low-loss silica optical fibers in which light propagation is accurately ruled by NLS-like equations over long distances [2]. Our aim in the present Letter is to provide the experimental demonstration and the theoretical description of a novel phenomenon of velocity locking of incoherent wave packets, which is fundamentally related to the process of thermal relaxation to equilibrium of an isolated wave system.

We consider M coupled NLS Manakov-like equations, which are known to be relevant for the description of vector phenomena in optics [2], plasma [7], hydrodynamics [8], or Bose-Einstein condensates [1]

$$iD_jA_j = -\alpha_j\partial_{tt}A_j + \gamma_j\left(|A_j|^2 + \kappa\sum_{i\neq j}|A_i|^2\right)A_j, \qquad (1)$$

where $D_j = \partial_z + u_j^{-1}\partial_t$. Note that, as usual in optics, the distance z of propagation in the nonlinear medium plays the role of an evolution "time" variable [2]. The parameters u_j and γ_j represent the group velocity and the nonlinear coefficient of the wave packet A_j. The dispersion parameters are given by $\alpha_j = (\partial^2 k_j/\partial\omega^2)/2$, where $k_j(\omega) = \alpha_j\omega^2 + \omega/u_j$ refers to the parabolic linear dispersion relation of A_j. The last term in Eq. (1) describes a cross-interaction between the fields, i.e., a phase modulation between pairs of distinct components A_j and $A_{i\neq j}$. The dimensionless constant κ then denotes the ratio between the cross- and self-interaction coefficients. Note that Eq. (1) conserves the "mass" $N_j = \int |A_j|^2\, dt$ of each component A_j, the Hamiltonian H [2], and the total momentum $P = \sum_i P_i$, with $P_i = \mathrm{Im}\int A_i^*\partial_t A_i dt$. In the following, we consider the "fully incoherent" regime of interaction in which $\sigma_j = \tau_{c,j}/\tau_{0,j} \ll 1$; i.e., the rapid fluctuations of A_j make its time correlation $\tau_{c,j}$ much smaller than the characteristic nonlinear time $\tau_{0,j} = \sqrt{\alpha_j\Lambda_j}$, $\Lambda_j = 1/\gamma_j\langle|A_j|^2\rangle$ being the corresponding nonlinear length.

The process of velocity locking may conveniently be analyzed in the frequency domain. Figure 1(a) illustrates a typical evolution of the mean frequencies $\bar\omega_j(z) = \int\omega n_j d\omega/\int n_j d\omega$ of $M = 3$ incoherent wave packets [15], $n_j(z, \omega)$ being the corresponding spectra. As an initial condition, we took three stochastic amplitudes $A_j(z = 0, t)$ of zero mean, characterized by a stationary statistics. In the linear limit of their evolutions ($\gamma_j = 0$), the components A_j would propagate with three distinct group velocities u_j. Figure 1(a) remarkably shows that, due to the nonlinear interaction, the mean frequencies $\bar\omega_j$ are rapidly attracted towards specific values $\bar\omega_j^{\mathrm{eq}}$. According to the group-velocity dispersion law $v_j^{-1}(\omega) = \partial k_j/\partial\omega = u_j^{-1} + 2\alpha_j\omega$, such a frequency shift is naturally accompanied by a shift of the group velocity, as schematically explained in Fig. 1(e). The remarkable and unexpected result is that the frequencies $\bar\omega_j^{\mathrm{eq}}$ are selected in such a way that the three wave packets propagate with identical group velocities $v_j(\bar\omega_j^{\mathrm{eq}}) = v^{\mathrm{eq}}$ for $j = 1, 2, 3$ [see Fig. 1(b)].

To grasp this intriguing process of velocity locking, let us remark that the vector NLS equation is completely integrable if $\gamma_j = \alpha_j = \kappa = 1$ [2]. The velocity-locking process is shown to occur in the nonintegrable case, in which the nonlinear dynamics is known to lead to an irreversible process of diffusion in phase space [12]. The essential properties of this irreversible evolution to equilibrium are described by the kinetic wave theory, which derives a set of irreversible kinetic equations describing the coupled evolutions of the spectra $n_j(z, \omega)$, defined by $\langle a_j(z, \omega_1)a_j^*(z, \omega_2)\rangle = n_j(z, \omega_1)\delta(\omega_1 - \omega_2)$, where a_j is the Fourier transform of A_j [$a_j(z, \omega) = \frac{1}{2\pi}\int A_j(z, t)\times e^{-i\omega t}dt$], and $\langle.\rangle$ denotes an ensemble average [13]:

$$\partial_z n_j(z, \omega_1) = \kappa^2\sum_{i\neq j}\mathrm{Coll}[n_i, n_j]. \qquad (2)$$

The cross-collision term

$$\mathrm{Coll}[n_i, n_j] = \int d\omega_2 d\omega_3 d\omega_4 W n_j(\omega_1)n_i(\omega_2)n_i(\omega_3)n_j(\omega_4)[n_j^{-1}(\omega_1) + n_i^{-1}(\omega_2) - n_i^{-1}(\omega_3) - n_j^{-1}(\omega_4)] \qquad (3)$$

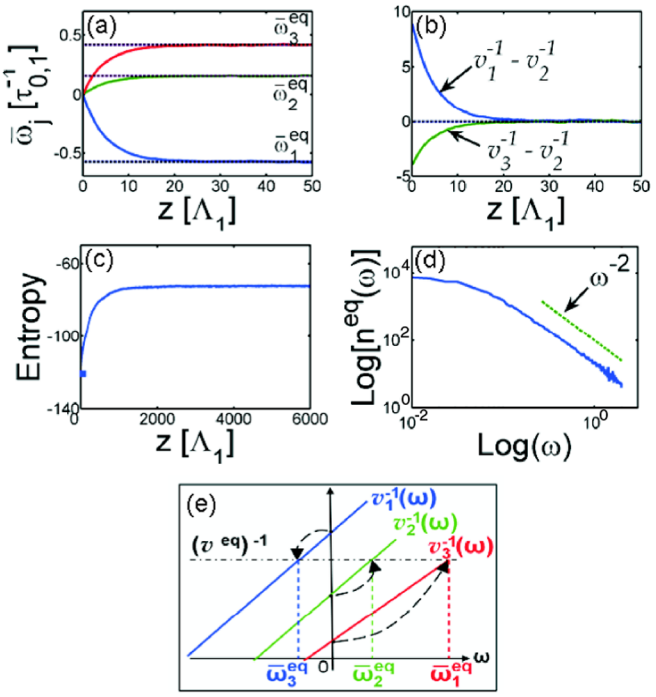

FIG. 1 (color online). Numerical simulations representing the evolution of the mean frequencies $\bar\omega_j(z)$ of $M = 3$ incoherent wave-packets (a) and corresponding group-velocity differences (b). The dashed lines in (a) refer to the theoretical values $\bar\omega_j^{\mathrm{eq}}$ calculated from Eq. (5) [$u_1 = 1/14$, $u_2 = 1/5$, $u_3 = 1$ in units of $\Lambda_1/\tau_{0,1}$]. Simulations showing thermal wave relaxation to equilibrium at large z: The entropy growth saturates to a constant value $d_z\mathcal{S} \simeq 0$ (c), and the spectrum of A_j follows a power law $n_j(\omega) \sim \omega^{-2}$ at large ω (d), in agreement with the equilibrium distribution (4). An average over 200 numerical realizations has been taken [$\kappa = 2$, $N_1 = N_2 = N_3$, $\alpha_2/\alpha_1 = 0.9$, $\alpha_3/\alpha_1 = 1.1$, $\alpha_1 = 20$ ps^2/km, $\tau_{0,1} = 0.26$ ps, $\Lambda_j = 3.33$ m, $\sigma_j = 0.15$ ($j = 1, 2, 3$)]. Schematic illustration of the velocity locking induced by the frequency shift of A_j (e): The arrows indicate the evolution from $z = 0$ to equilibrium [$v_j^{-1}(\omega = 0) = u_j^{-1}$].

provides a kinetic description of the cross-interaction term of Eq. (1). Basically, the kinetic approach models the four-wave interaction as a collisional gas of quasiparticles satisfying the resonant conditions of energy and momentum conservation at each elementary collision, as expressed through the presence of Dirac's δ functions in $W = (\gamma_i^2/\pi)\delta(\omega_1 + \omega_2 - \omega_3 - \omega_4)\delta[k_j(\omega_1) + k_i(\omega_2) - k_i(\omega_3) - k_j(\omega_4)]$. Note that the self-interaction term of Eq. (1) does not contribute to the kinetic equation (2), because the conservations of energy and momentum are trivially satisfied in the pure one-dimensional problem considered here.

As for the usual Boltzmann's equation [14], the set of M coupled kinetic equations (2) conserves the mass (quasiparticle number) of each wave packet A_j, $N_j = \int n_j(z, \omega)d\omega$, the total "kinetic energy" $E = \sum_i E_i$, $E_i = \int k_i(\omega)n_i(z, \omega)d\omega$, and the total "momentum" $P = \sum_i P_i$, $P_i(z) = \int \omega n_i(z, \omega)d\omega$. The irreversible character of Eq. (2) is expressed through the H theorem of entropy growth $d\mathcal{S}/dz \geq 0$, where the nonequilibrium entropy reads $\mathcal{S} = \sum_i \mathcal{S}_i$, $\mathcal{S}_i = \int \log[n_i(z, \omega)]d\omega$. From the postulate of maximum entropy [14], one may calculate the equilibrium spectra $n_j^{\mathrm{eq}}(\omega)$ realizing the maximum of $\mathcal{S}[n_j]$, subject to the constraints of conservation of E, P, and N_j. By introducing the corresponding Lagrange multipliers $1/T$, λ/T, and $-\mu_j/T$, one obtains

$$n_j^{\mathrm{eq}}(\omega) = \frac{T}{|\alpha_j|\omega^2 + s_j(\lambda + u_j^{-1})\omega - \mu_j}, \qquad (4)$$

where T denotes the temperature, μ_j the chemical potential of A_j [6], and $s_j = \mathrm{sgn}(\alpha_j)$. The distribution (4) yields an exactly vanishing collision term (3), $\mathrm{Coll}[n_i^{\mathrm{eq}}, n_j^{\mathrm{eq}}] = 0$. This means that once the spectra have reached the equilibrium distribution (4), they no longer evolve during the propagation, $\partial_z n_j = 0$. Equation (4) is a Lorentzian in which the $M + 2$ constants T, λ, and μ_j can be determined from the $M + 2$ conserved quantities E, P, and N_j. In particular, from the distribution (4) one readily finds $P_j^{\mathrm{eq}} = -(\lambda + u_j^{-1})N_j/2\alpha_j$, so that $\lambda = -(2P + \sum_i N_i/u_i\alpha_i)/\sum_i N_i/\alpha_i$.

The influence of momentum conservation on the thermodynamic properties of an ensemble of incoherent wave packets has not been the subject of a detailed investigation. Important to note is that the multiplier λ leads to a frequency shift of the equilibrium spectrum (4), so that the selected equilibrium frequency reads [15]

$$\bar{\omega}_j^{\mathrm{eq}} = P_j^{\mathrm{eq}}/N_j = -(\lambda + u_j^{-1})/2\alpha_j, \qquad (5)$$

where the expression of λ was given above. According to the group-velocity dispersion law $v_j^{-1}(\omega) = u_j^{-1} + 2\alpha_j\omega$, one readily obtains $v_j(\omega = \bar{\omega}_j^{\mathrm{eq}}) = -1/\lambda$ [15], which turns out to be the equilibrium velocity v^{eq} of the fields. This significant result reveals that, regardless of their initial group velocities u_j, each wave packet A_j irreversibly evolves towards an equilibrium state, in which it propagates with the common group velocity

$$v^{\mathrm{eq}} = \frac{\sum_i N_i/\alpha_i}{2P + \sum_i N_i/(u_i\alpha_i)}, \qquad (6)$$

where the conserved momentum is fixed from the initial condition $P = P(z = 0)$. The theoretical predictions of $\bar{\omega}_j^{\mathrm{eq}}$ and v^{eq} [Eqs. (5) and (6)] have been found to be in excellent agreement with the numerical simulations of the vector NLS equation, as shown in Figs. 1(a) and 1(b). Moreover, our numerical study confirms the existence of the irreversible process of thermal wave relaxation to equilibrium [Figs. 1(c) and 1(d)]: The growth of entropy saturates to a constant equilibrium value ($d\mathcal{S}/dz \simeq 0$), while the spectra $n_j(z, \omega)$ eventually reach the Lorentzian distributions (4).

The experimental observation of this velocity-locking effect has been performed in 5 m of high-birefringence optical fiber. In such a system, A_1 and A_2 represent the slowly varying optical envelopes along the orthogonal axes of the fiber ($\kappa = 2/3$) [2]. A scheme of the experimental setup is shown in Fig. 2(a). Incoherent light was obtained from the amplified spontaneous emission (ASE) of a dye amplifier pumped by a frequency-doubled

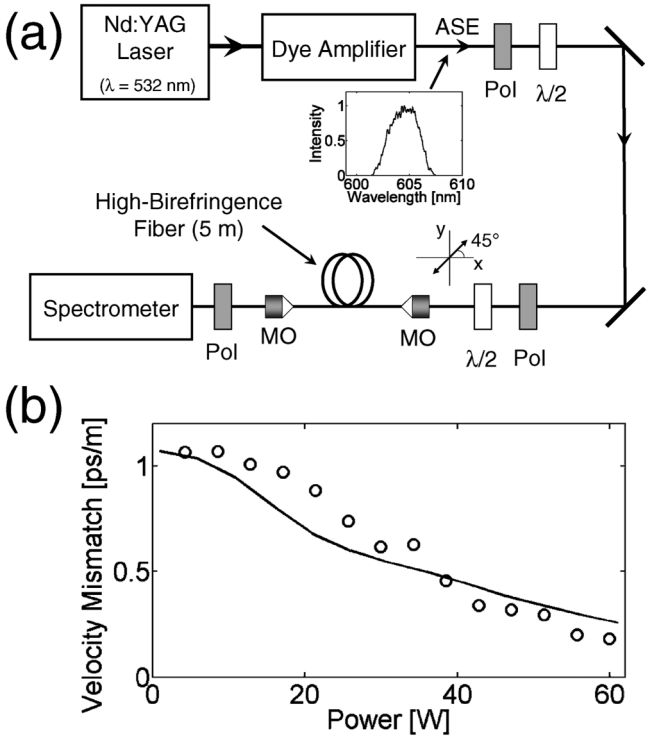

FIG. 2. (a) Experimental setup: ASE, amplified spontaneous emission; Pol: Glan polarizer; $\lambda/2$: half-wave plate; MO: microscope objective (inset: averaged ASE spectrum). (b) Evolution of the group-velocity mismatch as a function of the optical power of each wave packet. Circles: Experimental measurements; solid line: numerical simulations ($P = 20$ W corresponds to $L = 5\Lambda_1$).

Nd:YAG laser emitting 5-ns pulses at a repetition rate of 25 Hz. The mean carrier frequency of the ASE was $\nu_0 = 496$ THz ($\lambda_0 = 605$ nm) and its spectral width $\Delta\nu \simeq 3$ THz (i.e., $\tau_c \simeq 300$ fs). A set of half-wave plates and Glan polarizers was placed in front of the input microscope objective to polarize linearly the incoherent wave and to control the orientation of its polarization. The polarized beam was injected at 45° with respect to the fiber axes, so as to ensure that the two incoherent waves $A_{1,2}$ propagate with the same power along the slow and fast axes of the fiber ($N_1 = N_2$). The spectrum of light emerging from the fiber was recorded by means of a spectrometer, preceded by an analyzer which allows one to record separately the orthogonal polarization components. The values of fiber dispersion and nonlinear interaction coefficients were found to be $\alpha_1 \simeq \alpha_2 \simeq 0.03$ ps^2/m and $\gamma_{1,2} = 0.05$ W^{-1} m^{-1}, while the group-velocity mismatch (GVM) due to the natural fiber birefringence, $\delta = u_1^{-1} - u_2^{-1}$, was measured to be $\delta = 1.07$ ps/m.

The GVM at the fiber output $\delta_{\text{out}} = \delta(z = L)$ was calculated from the difference of the mean frequencies of the incoherent waves $\bar{\omega}_{1,2}(z = L)$, where $\bar{\omega}_j$ represent the frequency detunings with respect to the mean carrier frequency $2\pi\nu_0$. Figure 2(b) shows the experimental evolution of δ_{out} vs the optical power injected along each axis (circles) together with the theoretical predictions obtained from the numerical integration of Eq. (1) (solid line) [2]. At low power, the two incoherent waves do not interact, so that the polarization components propagate with the same mean carrier frequency $2\pi\nu_0$, and the initial GVM remains invariant, i.e., $\delta_{\text{out}} = \delta = 1.07$ ps/m. As the optical power is increased, the two beams are coupled through the nonlinear polarization effect, and, as predicted by the theory, the incoherent wave packets tend to propagate with identical group velocities. At $P = 60$ W (corresponding to $L = 15\Lambda_1$), the GVM drops to 0.21 ps/m, which represents only 20% of its initial value. The main limiting effect was the Raman scattering, which induces pump depletion and therefore limits the maximal pump power effectively involved in the observed phenomenon. Finally, the good agreement between the experimental results and the numerical simulations confirms the observation of a velocity-locking effect between the two incoherent polarization components.

In summary, we have identified theoretically and demonstrated experimentally a novel phenomenon of velocity locking of incoherent nonlinear wave packets. A kinetic approach of the problem revealed that the locking of the group velocities results from the natural irreversible evolution of the fields to thermal equilibrium. According to thermodynamic considerations, it is shown that an increase of entropy requires the matching of the group velocities of the incoherent fields. Note that, when one considers the interaction of spatially localized incoherent envelopes, the velocity locking manifests itself as a spatial trapping of distinct wave packets. Such a fusion of incoherent wave packets takes place irrespective of the nature of the interaction, which may be repulsive ("defocusing," $\alpha_j > 0$) or attractive ("focusing," $\alpha_j < 0$). Given the universality of the vector NLS equation in nonlinear physics, the reported phenomenon has potential ramifications in other physical contexts. For instance, our results could be relevant for the study of finite temperature multicomponent Bose-Einstein condensates. Considering that partially condensed gases could be described classically in terms of incoherent matter waves satisfying Gross-Pitaevskii-like equations [16], the process of velocity locking could be exploited to achieve an effective spatial trapping of a species owing to its nonlinear interaction with the other atomic species.

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