Critical Opalescence in Baryonic QCD Matter

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We show that critical opalescence, a clear signature of second-order phase transition in conventional matter, manifests itself as critical intermittency in QCD matter produced in experiments with nuclei. This behavior is revealed in transverse momentum spectra as a pattern of power laws in factorial moments, to all orders, associated with baryon production. This phenomenon together with a similar effect in the isoscalar sector of pions (sigma mode) provide us with a set of observables associated with the search for the QCD critical point in experiments with nuclei at high energies.

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It has been suggested recently that the baryon-number fluctuations, in event-by-event studies of multiparticle production, may lead to new observables, in experiments with nuclei, associated with the existence and location of the critical point in the QCD phase diagram [1]. Theoretically, the significance of the baryon-number density $n_B(\vec{x})$ near the critical point comes from the fact that its fluctuations, expressed in terms of the density-density correlator $\langle n_B(\vec{x})n_B(0)\rangle$, follow the same power law as the sigma field correlator $\langle\sigma(\vec{x})\sigma(0)\rangle$ associated with the chiral order parameter $\sigma(\vec{x}) \sim \bar{\Psi}\Psi$ [1,2]. As a result, the critical thermodynamics of QCD matter in equilibrium can be formulated in terms of the baryon-number density $n_B(\vec{x})$, an alternative but equivalent order parameter related to the final state of nuclear collisions when they freeze out close to the critical point. The corresponding theory of critical power laws in the baryon sector, the observability of which we explore in this Letter, is based on the effective action belonging to 3D Ising universality class:

$$\Gamma_c[n_B] = T_c^{-5}g^2\int d^3\vec{x}\left[\frac{1}{2}|\nabla n_B|^2 + Gg^{\delta-1}T_c^8|T_c^{-3}n_B|^{\delta+1}\right], \tag{1}$$

where $\delta \simeq 5$ is the isotherm critical exponent and G a universal, dimensionless coupling in the effective potential, with a value in the range $G \simeq 1.5$–2 [3]. In writing Eq. (1) we have considered, following the above discussion, the order parameter $m(\vec{x}) = gT_c^{-2}n_B(\vec{x})$, associated with the fluctuations of baryon-number density at the critical point; the factor T_c^{-2} guarantees dimensional consistency and g is a nonuniversal dimensionless constant.

The free energy (1) in order to describe a critical system of QCD matter produced in nuclear collisions must be adapted to the relativistic geometry of the collision. To this end, the longitudinal coordinate $x_\parallel$ is replaced by the space-time rapidity y, and the corresponding measure of integration in (1) takes the form $dx_\parallel = \tau_c \cosh y dy$, where τ_c is the formation time of the critical point. In the central region of size δy the configurations $n_B(y, \vec{x}_\perp)$ contributing to the partition function are boost invariant quantities defining at the same time two-dimensional baryon-number density configurations $\rho_B(\vec{x}_\perp)$ in the transverse plane: $n_B(\vec{x}_\perp) = \rho_B(\vec{x}_\perp)[2\tau_c \sinh(\delta y/2)]^{-1}$. Integrating now in rapidity and rescaling the basic variables, $\vec{x}_\perp \to T_c\vec{x}_\perp$, $\rho_B \to T_c^{-2}\rho_B$, Eq. (1) is simplified as follows:

$$\Gamma_c[\rho_B] = Cg^2\int d^2\vec{x}_\perp\left[\frac{1}{2}|\nabla_\perp\rho_B|^2 + G(gC)^{\delta-1}\rho_B^{\delta+1}\right],$$
$$C \equiv (T_c\tau_{\text{eff}})^{-1}, \qquad \tau_{\text{eff}} \equiv 2\tau_c\sinh(\delta y/2). \tag{2}$$

The critical fluctuations of systems belonging to the class (2) of a 2D effective action can be consistently described in a scheme where the saturation of the partition function is obtained considering the contribution of the singular solutions $\rho_B^{(s)}$ of the Euler-Lagrange equation [4]:

$$Z \simeq \sum_s e^{-\Gamma_c[\rho_B^{(s)}]}; \qquad \nabla_\perp^2\rho_B^{(s)} - (\delta+1)G(gC)^{\delta-1}[\rho_B^{(s)}]^\delta = 0. \tag{3}$$

The description of the critical system in this scheme is optimal if $Cg^2 \gg 1$.

The main characteristics of critical QCD matter ($\delta \simeq 5$, $G \simeq 2$) produced in nuclear collisions and described by Eq. (3) are summarized as follows [4]: (a) The system is organized within critical domains (clusters) of a maximal size (correlation length): $\xi_\perp \simeq \frac{\pi\tau_c}{8}\sinh(\delta y/2)$. (b) For typical nuclear collisions ($\tau_c \simeq 10$ fm, $\delta y \geq 2$) the transverse correlation length becomes sufficiently large ($\xi_\perp \geq 6$ fm) allowing for critical fluctuations to develop in full strength when the system freezes out near the critical point. (c) Within a critical domain ($|\delta\vec{x}_\perp| \leq \xi_\perp$) the correlator obeys a power law corresponding to a fractal dimension $d_F = \frac{2\delta}{\delta+1}$ ($d_F \simeq \frac{5}{3}$): $\langle\rho_B(\vec{x}_\perp)\rho_B(0)\rangle \sim |\vec{x}_\perp|^{d_F-2}$. This power law is the origin of critical opalescence [5] in baryonic QCD matter produced in high-energy nuclear collisions.

In conventional matter (QED matter) the phenomenon of critical opalescence is revealed when light of long wavelength (comparable to the correlation length) scatters from a substance near criticality. The intensity of scattered light is proportional to the Fourier transform of the density-density correlator and becomes singular for small momentum transfer k giving rise to a macroscopic effect [5]. In QCD matter, correspondingly, the Fourier transform of the correlator $\langle \rho_B(\vec{x}_\perp)\rho_B(0)\rangle$ obeys also a power law:

$$\begin{aligned} \langle \rho_{\vec{k}}\rho^*_{\vec{k}}\rangle &\sim \int d^2\vec{x}_\perp e^{-i\vec{k}\cdot\vec{x}_\perp}\langle \rho_B(\vec{x}_\perp)\rho_B(0)\rangle, \\ \langle \rho_B(\vec{x}_\perp)\rho_B(0)\rangle &\sim |\vec{x}_\perp|^{d_F-2}; \quad |\vec{x}_\perp| \lesssim \xi_\perp, \\ \langle \rho_{\vec{k}}\rho^*_{\vec{k}}\rangle &\sim |\vec{k}|^{-d_F}; \quad |\vec{k}| \gtrsim \xi_\perp^{-1}, \end{aligned} \tag{4}$$

where $\rho_{\vec{k}}$ is the Fourier transform of $\rho_B(\vec{x}_\perp)$ and $\langle \rho_{\vec{k}}\rho^*_{\vec{k}}\rangle$ the two-particle correlator in transverse momentum plane, associated with baryon production in nuclear collisions. More precisely,

$$\langle \rho_{\vec{k}}\rho^*_{\vec{k}}\rangle = \sum_{\vec{p}}\langle \rho_B(\vec{p})\rho_B(\vec{p}+\vec{k})\rangle \sim \langle \rho_B(0)\rho_B(\vec{k})\rangle \tag{5}$$

under the assumption that the dependence of $\langle \rho_B(\vec{p})\rho_B(\vec{p}+\vec{k})\rangle$, on the reference momentum $\vec{p}$, is weak.

The power law (4) in momentum space can be generalized to a self-similarity property of multiparticle correlators, valid for the solution (3) of QCD matter at criticality [4]:

$$\langle \rho_B(\lambda\vec{k}_1)\cdots\rho_B(\lambda\vec{k}_{q-1})\rho_B(0)\rangle = \lambda^{-d_F(q-1)}\langle \rho_B(\vec{k}_1)\cdots\rho_B(\vec{k}_{q-1})\rho_B(0)\rangle \quad (q=2,3,\ldots). \tag{6}$$

In geometrical language, Eq. (6) defines a fractal dimension $\tilde{d}_F$ in transverse momentum space, independent of q ($\tilde{d}_F = 2 - d_F$), directly related to the isotherm critical exponent of QCD: $\tilde{d}_F = \frac{2}{\delta+1}$ [6]. The observable effect implied by (6) is a specific class of power laws, satisfied by the scaled factorial moments of all orders [7]:

$$F_q(M) = \left[\frac{\langle N_B(N_B-1)\cdots(N_B-q+1)\rangle}{\langle N_B\rangle^q}\right]_{\delta S_M} \sim M^{d_F(q-1)} \quad (M \gg 1) \tag{7}$$

($q = 2, 3, \ldots$) defined in small domains δS_M of transverse momentum plane, constructed by a subdivision of the available space in M^2 two-dimensional cells ($\delta S_M \sim M^{-2}$). The power laws (7) with a characteristic linear spectrum of indices $\alpha_q = (2 - \tilde{d}_F)(q-1)$ describe, in general, the fluctuations in a second-order phase transition [6,8], but here, in particular, they reveal the effect of critical intermittency associated with the production of baryonic QCD matter near the critical point. This phenomenon is the analogue of critical opalescence, observed in conventional matter near criticality. Both phenomena have their origin in the power laws of the correlators in configuration space, implying giant density fluctuations at the critical point as a result of the appropriate fractal geometry, valid within the universality class of the critical system under consideration.

On the basis of the prediction (7) one may proceed to a Monte Carlo simulation of critical events in the case of a typical process $A + A$ at high energies. We have chosen collisions of medium size nuclei producing net baryons in the central region with a constant density in rapidity $\frac{dN_B}{dy} \simeq 10$ (plateau), within a domain of size $\delta y \simeq 2$, at energies corresponding to CERN-SPS. A self-consistency requirement in this treatment is the constraint $Cg^2 \gg 1$, which may justify *a posteriori* the saddle point saturation of the partition function leading to Eq. (3). In order to clarify this issue for the above system under simulation, we consider the following generic form coming from the thermal average $\langle \rho_B\rangle$ with the aid of Eq. (3):

$$\langle N_B\rangle \simeq \frac{1}{g}\left(\frac{S_{\rm cr}}{CG^{1/\delta}}\right)^{\delta/(\delta+1)}\frac{\Gamma(\frac{2}{1+\delta})}{\Gamma(\frac{1}{1+\delta})} \quad (\delta \simeq 5, G \simeq 2), \tag{8}$$

where $S_{\rm cr}$ is the area of the critical cluster, $S_{\rm cr} \simeq \mathcal{O}(\xi_\perp^2)$. For the specifications of the system under study and for a typical time scale $\tau_c T_c \simeq 20$, we find $Cg^2 \simeq \mathcal{O}(10)$, a result which fulfils the consistency requirement ($Cg^2 \gg 1$).

The simulation of events involving critical fluctuations in the baryonic density requires the generation of baryon transverse momenta correlated according to the power law (4). One possibility to produce such a set of momenta is to use the method of Lévy random walks [9]. It is convenient to perform independent one-dimensional walks in each transverse momentum component separately and then form the Cartesian product in order to obtain the corresponding vectors. In this case the successive steps in each dimension are chosen according to the probability density:

$$\begin{aligned} \tilde{\rho}(p_i) &= \frac{\nu p_{\rm min}^\nu}{1-(\frac{p_{\rm min}}{p_{\rm max}})^\nu} p_i^{-1-\nu}; \\ p_{\rm min} &\le p_i \le p_{\rm max}, \quad i = x, y. \end{aligned} \tag{9}$$

To generate a single event, one performs $n-1$ random walk steps in each direction (p_x or p_y), where n is the multiplicity of the event. At each step the updated position of the walker determines the p_x (or p_y) coordinate of the baryon transverse momentum, respectively. The starting transverse momentum vector in each event is chosen uniformly in $[-p_{\rm max}, p_{\rm max}] \times [-p_{\rm max}, p_{\rm max}]$. The produced set of baryon transverse momenta possesses the correct fractal dimension $\tilde{d}_F = \frac{1}{3}$ provided that $\frac{p_{\min}}{p_{\max}} \approx 10^7$ and $\nu = \frac{1}{6}$. The described algorithm can be used to generate an ensemble

of events involving critical baryon density fluctuations. The corresponding fractal pattern can be revealed through factorial moment analysis [7]. In Fig. 1 we show, in a log-log plot, the results obtained for an ensemble of 600 critical events, each one having the multiplicity $n \equiv \langle N_B \rangle = 20$. The solid circles are the calculated moments up to the 4th order. The solid lines are the theoretical predictions according to Eq. (7). It is clearly seen that the factorial moments follow to a good approximation, and for a wide range of scales a power-law behavior $F_q \sim (M^2)^{s_q}$ with exponents s_q satisfying very well the theoretical prediction $s_q = (q-1)(1-\frac{\tilde{d}_F}{2})$. In the same plot we show for comparison the behavior of the second moment (gray triangles) in a conventional system corresponding to the formation of mixed events from the original ensemble of the critical events. As expected, the effect of critical fluctuations has disappeared in the system of mixed events (the slope $s_2^{(m)} \simeq 0$).

In order to reveal the observational content of the pattern in Fig. 1 we may extract the local multiplicity fluctuations occurring in a typical cell in transverse momentum plane corresponding to the partition number $M = 10$. In the simulated system under consideration we find in this cell $\langle N_B \rangle \approx 0.19$ and $\langle N_B^2 \rangle \approx 1.8$ corresponding to a value of the factorial moment $F_2 \approx 40$. These values lead to a sizeable local fluctuation $\frac{\delta N_B}{\langle N_B \rangle} \approx 7$, which increases with M, following the power law underlying the pattern in Fig. 1. In fact, the size of local multiplicity fluctuations is associated with the size of F_2 through the lower-bound

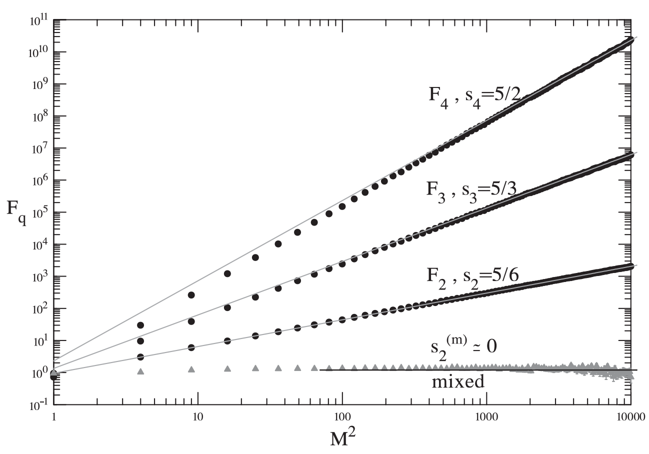

FIG. 1. The log-log plot of the factorial moments of order $q = 2, 3, 4$ (solid circles) for an ensemble of 600 critical events generated by the Monte Carlo algorithm described in the text. The corresponding theoretical power-law predictions are shown with solid lines. The gray triangles present the result for the second moment for a set of mixed events.

constraint: $\frac{\delta N_B}{\langle N_B \rangle} \geq (F_2 - 1)^{1/2}$. In the limit $M \gg 1$ the second moment in Fig. 1 is also very large ($F_2 \gg 1$), and as a result, giant local fluctuations are developed, probing the critical singularity of the system. In the actual measurements, however, experimental limitations, due to finite statistics (empty bin effect) as well as to finite momentum resolution $\frac{\delta p}{p}$, put an upper limit to the partition number M corresponding to a threshold of intermittency breaking and therefore to an interruption of the pattern of critical fluctuations at very small scales. In the simulated system under study the pattern in Fig. 1 corresponds to a data set of 6×10^2 events and the upper limit of the partition number M—assuming infinite momentum resolution—turns out to be of the order $M = \mathcal{O}(10^2)$ determined solely by finite statistics. However, in a real experiment, finite statistics may be less restrictive than the finite momentum resolution leading to an upper bound: $M \simeq \frac{p}{\delta p} \approx 100$ for a typical resolution scale $\frac{\delta p}{p} \approx 10^{-2}$ in current experiments.

We have shown that baryon-number density fluctuations, near the QCD critical point, develop a distinct pattern of power laws, in transverse momentum plane, associated with the effect of critical intermittency in QCD matter. The observability of this effect is enhanced by the conjecture that the same power laws (Fig. 1) are expected also in the case of net proton-number density fluctuations, avoiding therefore the observational ambiguities from the contribution of neutrons to the factorial moments (7). In fact, it has been argued by Hatta and Stephanov [1] that the singular parts of baryon susceptibility (χ_B) and proton-number fluctuations coincide, $\chi_B \sim \langle \delta N_{p-\bar{p}} \delta N_{p-\bar{p}} \rangle$. As a result, the power laws associated with these singularities are the same for both the baryon-number and proton-number density correlators. With these remarks we may conclude that proton-number measurements in the transverse momentum plane may reveal the effect of critical baryon-number intermittency in nuclear collisions, as a signal of QCD criticality. If one combines this observable effect with the corresponding phenomenon in the sigma mode of pion production [10], then a complete set of observables, associated with the existence and location of the QCD critical point, naturally emerges.

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