Interferences in Parametric Interactions Driven by Quantized Fields

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We report interferences in the quantum fluctuations of the output of a parametric amplifier when the cavity is driven by a quantized field at the signal frequency. The interferences depend on the phase fluctuations of the input quantized field and result in splitting of the spectrum of the output, and thus the recent observation [H. Ma *et al.*, Phys. Rev. Lett. **95**, 233601 (2005)] of interferences in the classical domain have a very interesting counterpart in the quantum domain. The interferences can be manipulated, for example, by changing the amount of squeezing in the input field.

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In recent times quantum interferences have been central to many new applications of few level quantum systems driven by the coherent fields. In particular, the electromagnetically induced transparency (EIT) has led to the possibilities of enhanced nonlinear optical phenomena [1], ultraslow light [2], storage and stopping of light [3], an efficient method of laser cooling [4], and control of chiral anisotropies [5]. The EIT has also been demonstrated for quantized fields [6]. In this Letter we report the possibility of quantum interferences in totally different class of systems which are nonresonant and which are being extensively used in the context of quantum imaging and quantum information science [7,8]. We use parametric interactions driven by quantized fields at the signal or the idler frequency. The generated quantum fields in parametric interactions exhibit a variety of interferences depending on the phase fluctuations in the input quantized field [8]. We note that optical parametric interactions have been studied extensively since the classical work of Armstrong *et al.* [9]. These classical interactions are known to possess unusual properties. Kaplan [10] discovered that under certain conditions on the pump and signal amplitudes, there is no exchange of energy between the pump and signal. There is yet another very interesting situation where by a suitable choice of the phases of the input amplitudes can lead to either purely growing solutions or decaying solutions. To see this consider the classical Hamiltonian for the parametric process

$$H = \hbar g(a^2 b^\dagger + a^{\dagger 2} b), \tag{1}$$

which in the limit of undepleted pump $[b]$ leads to

$$a(t) \equiv \frac{1}{2} e^{2|g|bt}\left[a - \frac{igb}{|gb|}a^\dagger\right] + \frac{1}{2}e^{-2|g|bt}\left[a + \frac{igb}{|gb|}a^\dagger\right].$$

Thus by choosing $(a/a^\dagger) = igb/|gb|$ one can suppress the growing solution. Yet another very interesting aspect of classical parametric interactions was discovered in a recent letter by Ma *et al.* [11]. They have studied a variety of interference effects in parametric interactions in a cavity when the cavity is also driven by a field at the signal frequency, i.e., $a \neq 0$. The interferences arise from the nonzero signal field at the input and the signal field produced by the pump field. Ma *et al.* in particular reported mode splitting in transmission spectra. All the above remarkable developments refer to the behavior of parametric interactions when all the fields are treated classically. Clearly it is important to investigate the nature of the interference effects when fields are treated quantum mechanically [12–14]. It is well known that parametric interactions generate fields which have important quantum mechanical properties [15]. In this Letter we report on the interferences which are initiated due to input quantized fields at the signal frequency, and in particular, we report on interferences in the quantum fluctuations of the output of an optical parametric amplifier (OPA). We demonstrate how the fluctuations in the output can be manipulated by the choice of the input quantized field.

Let us consider an optical parametric amplifier in a cavity with right mirror that is 100% reflecting and with partially reflecting left mirror. The pump field b has large amplitude and is treated as undepleted as shown in Fig. 1. Let the frequency of the pump be $2\tilde{\omega}$ and that of the signal be $\tilde{\omega}$. Let ω_c be the frequency of the cavity. We assume that the field a is quantized and is driven by a quantum field $a_{\text{in}}(\omega)$. If the input quantum field is in vacuum state, then this reduces to the problem well studied in the literature. In

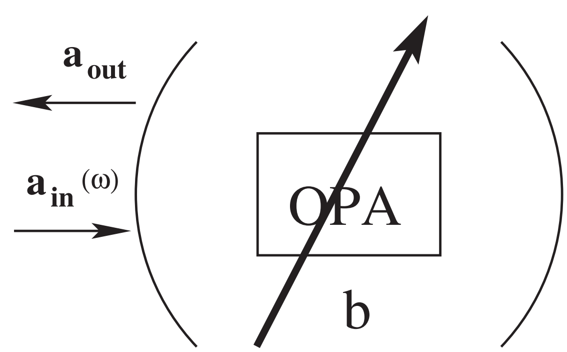

FIG. 1. Schematic diagram of an optical parametric amplifier in a cavity.

this Letter we are concerned with the new features arising from the quantum nature of $a_{\rm in}(\omega)$. In a frame rotating with the frequency of the signal field, we can use the connections between the input and output fields to obtain the fundamental equation for a quantized field inside the cavity,

$$\frac{da}{dt} = -i\Delta a - \kappa a + \sqrt{2\kappa}a_{\rm in} - 2igba^{\dagger}, \quad \Delta = \omega_c - \tilde{\omega}, \tag{2}$$

and where 2κ gives the rate of leakage of photons from the cavity. The output field is given by

$$a_{\rm out}(t) + a_{\rm in}(t) = \sqrt{2\kappa}a. \tag{3}$$

Note that a is the single mode cavity field with the communication relation $[a, a^{\dagger}] = 1$ and is dimensionless. The field $a_{\rm out}(t)$ includes contributions from all the vacuum modes and satisfies commutation relation $[a_{\rm out}(t), a^{\dagger}_{\rm out}(t')] = \delta(t - t')$. Thus the dimension of $a_{\rm out}(t)$ is $1/(\sqrt{\rm time})$ or $(\sqrt{\rm frequency})$. Using (2) and (3) we can eliminate the cavity field and obtain the output field in terms of the input field,

$$a_{\rm out}(\omega) = V(\omega)a^{\dagger}_{\rm in}(-\omega) + U(\omega)a_{\rm in}(\omega), \tag{4}$$

where $U(\omega) = 2\kappa I_{11} - 1$, $V(\omega) = 2\kappa I_{12}$, and where the matrix I is given by

$$I = \begin{pmatrix} -i\omega + i\Delta + \kappa & iG \\ -iG^{*} & -i\omega - i\Delta + \kappa \end{pmatrix}^{-1}; \quad G = 2gb. \tag{5}$$

Here the parameter G has the same dimensions as the coupling parameter g as b is dimensionless. For simplicity we set the phase of b zero. It should be noted that $a^{\dagger}(-\omega)(\equiv [a(-\omega)]^{\dagger})$ is the Fourier transform of $a^{\dagger}(t)$. The spectrum $S_{\rm out}(\omega)$ of the output field is defined by

$$\langle a^{\dagger}_{\rm out}(t)a_{\rm out}(t+\tau)\rangle = \frac{1}{2\pi}\int S_{\rm out}(\omega)e^{-i\omega\tau}d\omega, \quad \langle a^{\dagger}_{\rm out}(\omega_2)a_{\rm out}(\omega_1)\rangle = 2\pi\delta(\omega_2 - \omega_1)S_{\rm out}(\omega_1). \tag{6}$$

Let us now assume that the input field be a multimode squeezed field. Then using the basic definitions (6), the properties of the multimode squeezed field,

$$\langle a^{\dagger}_{\rm in}(\omega_2)a_{\rm in}(\omega_1)\rangle = 2\pi\delta(\omega_1 - \omega_2)\bar{n}, \quad \langle a_{\rm in}(-\omega_2)a_{\rm in}(\omega_1)\rangle = 2\pi\delta(\omega_1 - \omega_2)|m|e^{-i\theta}, \tag{7}$$

and Eq. (4) we can calculate $S_{\rm out}(\omega)$. The result can be written in the form

$$S_{\rm out}(\omega) = |m||U(\omega) + e^{i\theta}V(\omega)|^2 + (\bar{n} - |m|)|U(\omega)|^2 + (\bar{n} - |m| + 1)|V(\omega)|^2, \quad \bar{n} = \sinh^2\zeta, \quad |m| = \sinh\zeta\cosh\zeta, \tag{8}$$

and θ is the phase of m of the squeezed field. The parameter ζ characterizes squeezing in the field. We assume weak frequency dependence of ζ. Equation (8) gives our key result, which we would use to demonstrate a variety of quantum interference effects. We first note that if the squeezed input field is replaced by vacuum, then $S_{\rm out}(\omega) = |V(\omega)|^2$ and there are no interferences in fluctuations of the output field. Similarly for input thermal fields ($|m| = 0$) $\bar{n} \neq 0$, there are no interference effects. We also note from Eq. (4) that mean field $\langle a_{\rm out}(\omega)\rangle = 0$ if the initial quantum field is such that $\langle a_{\rm in}(\omega)\rangle = 0$. The phase fluctuations in the squeezed field at the signal frequency allow the possibility of two terms in Eq. (4) to beat, which in turn leads to interferences. These interference are functions of ω, detuning Δ, amplitude of the pump b, and the phase of the input squeezed light. We first examine the detuning dependence of the fluctuation spectrum at $\omega = 0$. We choose all parameters in units of the cavity decay rate κ. In Fig. 2 we show the behavior of $S_{\rm out}(0)$ for different values of θ. We choose the parameter G so that we are well below the threshold of parametric oscillation. In fact, G/κ is equal to $|b/b_t|$, where b_t is the value of the pump amplitude at the

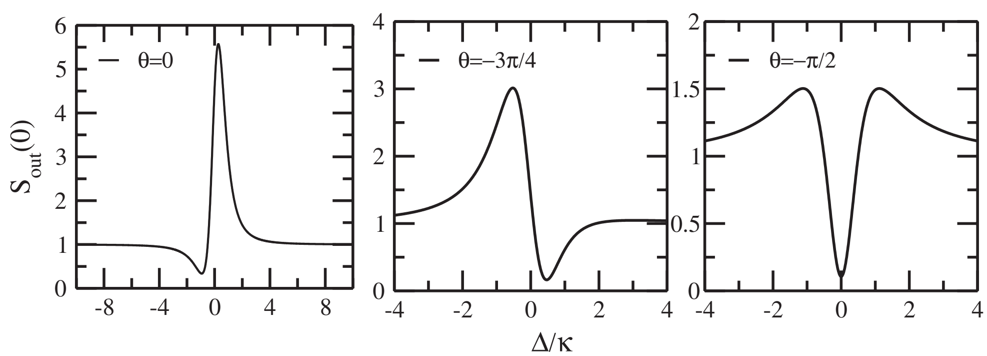

FIG. 2. The fluctuation spectrum of the output amplitude as a function of the detuning for different values of θ. The common parameters of the above three graphs are chosen as $G/\kappa = 0.5$, $\zeta = 3.0$, and $\omega = 0$. The output spectrum is normalized to the output spectrum for $G = 0$.

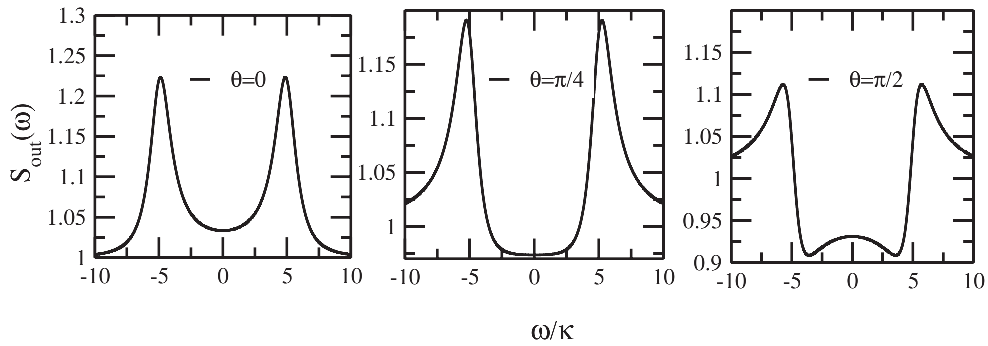

FIG. 3. The behavior of $S_{\mathrm{out}}(\omega)$ shown as a function of angular frequency ω at different values of θ. The common parameters of the above three plots are chosen as $G/\kappa = 0.5$, $\zeta = 3.0$, and $\Delta/\kappa = 5$.

threshold of oscillation. The chosen values of G are in the same range as used in the experiment [11]. Evidence of quantum interferences in fluctuations of the output is clearly seen. In order to understand the interferences in Fig. 2, we examine the result Eq. (8) in the limit of large ζ. For large ζ, Eq. (8) can be approximated by

$$S_{\mathrm{out}}(\omega) \approx |m||U(\omega) + e^{i\theta}V(\omega)|^2. \tag{9}$$

In the limit $\omega = 0$, we have

$$\begin{aligned} S_{\mathrm{out}}(0) &= |m|\frac{(G^2 + \kappa^2 - \Delta^2 + 2G\kappa\sin\theta)^2 + (2\kappa\Delta + 2G\kappa\cos\theta)^2}{(\kappa^2 + \Delta^2 - G^2)^2}, \\ &\rightarrow |m|\frac{[(G-\kappa)^2 - \Delta^2]^2 + 4\kappa^2\Delta^2}{(\Delta^2 + \kappa^2 - G^2)^2}, \qquad \theta = -\frac{\pi}{2}, \\ &\rightarrow |m|\frac{(G^2 + \kappa^2 - \Delta^2)^2 + 4\kappa^2(\Delta + G)^2}{(\Delta^2 + \kappa^2 - G^2)^2}, \qquad \theta = 0. \end{aligned} \tag{10}$$

Note that as $G \rightarrow \kappa$ (threshold of parametric oscillation), the numerator in (10) becomes much closer to Δ^2 than the denominator in (10) which would lead to splitting of the line.

We next consider the fluctuation spectrum of the output intensity as a function of frequency. This is shown in Fig. 3. The two well-defined resonances correspond to the two eigenvalues of (5) $\omega = \pm\sqrt{\Delta^2 - G^2}$ if $\Delta \gg G$. For $\Delta < G$ values we have a splitting of the line shape which can be associated with the two purely imaginary eigenvalues of (5) $\kappa \pm \sqrt{G^2 - \Delta^2}$. In Fig. 4 we display the effect of the amount of squeezing in the input quantized field, on quantum interferences in the output field. We also note that the quantum statistics of the output field can be easily calculated since the output field is related linearly to the input field via the relation (4).

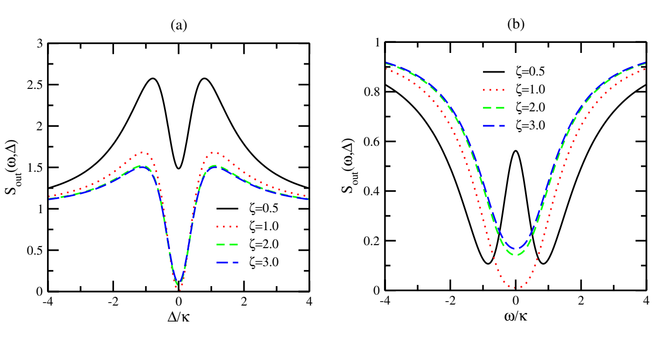

FIG. 4 (color online). (a) The fluctuation spectrum of the output field as a function of the detuning at $\theta = -\pi/2$. (b) The fluctuation spectrum of the output field as a function of the angular frequency for the value of $\Delta/\kappa = 0.5$ and $\theta = -3\pi/4$. The parameter G/κ is chosen as 0.5.

Thus, in conclusion, we have shown how quantum interferences, so widely studied in the context of multilevel systems, also occur for nonresonant systems like a parametric amplifier [16]. The interferences manifest in quantum fluctuations of the output field and can be manipulated by the phase fluctuations of the input quantized field at signal frequency. Similar interferences are expected to occur in the case of a nondegenerate parametric amplifier and in other quantum nonlinear optical processes like four-wave mixing.

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