Experimental Entangled Entanglement

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All previous tests of local realism have studied correlations between single-particle measurements. In the present experiment, we have performed a Bell experiment on three particles in which one of the measurements corresponds to a projection onto a maximally entangled state. We show theoretically and experimentally that correlations between these entangled measurements and single-particle measurements are too strong for any local-realistic theory and are experimentally exploited to violate a Clauser-Horne-Shimony-Holt-Bell inequality by more than 5 standard deviations. We refer to this possibility as "entangled entanglement."

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Seventy years ago, Einstein, Podolsky, and Rosen (EPR) argued that quantum theory could not be a complete description of physical reality, based firmly on plausible assumptions about locality, realism, and theoretical completeness [1]. It was not until almost 30 years later that the EPR paradox was formulated in terms of an experimentally testable prediction, discovered by John Bell [2], where the assumptions of locality and realism put measurable limits on the strength of correlations between outcomes of remote measurements. Since Bell's discovery, these limits, known as Bell's inequalities, have been subject to a large number and diverse range of experimental tests [3]. All previous Bell experiments measure degrees of freedom corresponding to properties of *individual* systems. In these Bell experiments, the joint properties of two or more particles, which correspond to the specific type of their entanglement, could still be independent of the measurements performed.

Bell's argument can be applied to outcomes of any measurements. In the present work, we experimentally demonstrate the first example of a Bell inequality test in which one of the measurements is a projection onto a maximally entangled state. The measurement on a single particle by "Alice" defines a relational property between another two particles without defining their single-particle properties. These relational properties are measured by "Bob." Correlations between the measurement outcomes of the polarization state of a single photon and the entangled state of another two are experimentally demonstrated to violate the Clauser-Horne-Shimony-Holt (CHSH)-Bell inequality [4]. This shows that entanglement itself can be entangled [5].

We begin with a brief discussion of two-qubit entanglement. Consider the state $|\phi^-\rangle_{a,b} = \frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b - |V\rangle_a|V\rangle_b)$, which is one of the four Bell states $|\phi^\pm\rangle_{a,b} = \frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b \pm |V\rangle_a|V\rangle_b)$ and $|\psi^\pm\rangle_{a,b} = \frac{1}{\sqrt{2}} \times (|H\rangle_a|V\rangle_b \pm |V\rangle_a|H\rangle_b)$. The subscripts a and b label Alice and Bob's photons and the kets $|H\rangle$ and $|V\rangle$ and represent states of horizontal and vertical polarization. The entangled state $|\phi^-\rangle_{a,b}$ is special in that the individual photons have perfect polarization correlations at any angle in the y-z plane of the Bloch sphere. It is well known that any Bell state is capable of violating the CHSH-Bell inequality at Cirel'son's bound [6].

In our experiment, we need a quantum state in which the polarization state of one photon is nonclassically correlated to the entangled state of the other two in a manner that is directly analogous to that in $|\phi^-\rangle_{a,b}$. An example of such a state can be written down by replacing the polarization state of particle b with the Bell states, $|\phi^-\rangle_{b_1,b_2}$ and $|\psi^+\rangle_{b_1,b_2}$, resulting in

$$|\Phi^-\rangle_{a,b} = \frac{1}{\sqrt{2}}(|H\rangle_a|\phi^-\rangle_{b_1,b_2} - |V\rangle_a|\psi^+\rangle_{b_1,b_2}), \tag{1}$$

where now Bob possesses two photons instead of just one. Drawing on our understanding of the bipartite entanglement, we can immediately make the following statements. First, the perfect correlations between polarization of Alice's photon and joint properties of Bob's two photons imply, by EPR premises of locality and realism [1], that the entangled states of Bob's photons are elements of reality. Second, each of Bob's two photons have no well-defined individual properties; i.e., individual detection events at Bob's detectors are random and cannot be inferred by a linear polarization measurement by Alice. Therefore, the entangled state that Bob possesses is an element of physical reality in the EPR sense whereas his individual photons are not.

Interestingly, the state in Eq. (1) is equivalent to the Greenberger-Horne-Zeilinger (GHZ) state [7] $|\Phi^-\rangle_{a,b} = \frac{1}{\sqrt{2}}(|R\rangle_a|R\rangle_{b_1}|R\rangle_{b_2} + |L\rangle_a|L\rangle_{b_1}|L\rangle_{b_2})$, where $|R/L\rangle = 1/\sqrt{2}(|H\rangle \pm i|V\rangle)$ represent the different senses of circularly polarized light and the subscripts $b1$ and $b2$ each photon at Bob's side. This connection suggests practical preparation methods, since three-qubit GHZ states have been generated [8]. While such GHZ states have been used in tests of local realism through Mermin, Ardehali, and Klyshko [9] inequalities or quantum erasers [10], we stress that these experiments are based solely on the correlations between single-particle observables. Quantum state tomography [11] has been used to reconstruct quantum states of up to 4 photons [12] and up to 8 ions [13]; again, these experiments are based on measurements of correlations between single-particle observables. No previous experiment has directly measured correlations in violation of local realism using more complex two-particle observables.

In Fig. 1, we show a schematic for our experiment in which a source emits an entangled state of three particles. Alice receives a single photon and Bob receives the other two. Alice chooses a measurement setting in the form of an angle θ_1. She then makes linear polarization measurements parallel to that angle or perpendicular to it by orienting a linear polarizer. A value of $+1$ ($-$ 1) is assigned to those outcomes where the photon is measured with the linear polarizer parallel (perpendicular) to θ_1. Similarly, Bob performs a restricted Bell-state measurement in the subspace $|\phi^-\rangle_{b_1,b_2}$ or $|\psi^+\rangle_{b_1,b_2}$. These two Bell states can be coherently mixed using the half-wave plate (HWP) [14] in

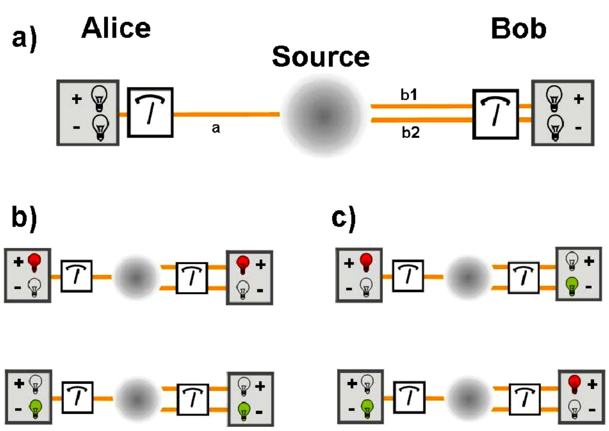

FIG. 1 (color online). Schematic cartoon for the Bell experiment based on an entangled entangled state. (a) A source emits the entangled three-photon state $|\Phi^-\rangle_{a,b} = \frac{1}{\sqrt{2}}(|H\rangle_a|\phi^-\rangle_{b_1,b_2} - |V\rangle_a|\psi^+\rangle_{b_1,b_2})$, where one photon is received by Alice and the two other photons by Bob. Alice makes polarization measurements on her photon. If the photon's polarization is measured to be parallel (perpendicular) to the orientation θ_1 of the analyzer, the measurement outcome is $+1$ ($-$ 1). In contrast, Bob makes projective measurements onto a two-particle entangled state, where the orientation of his analyzer is defined by the angle θ_2. Bob's outcomes are also defined as $+1$ or -1. (b) For the local settings $\theta_1 + \theta_2 = 0$, π, 2π, etc., they observe perfect correlations; i.e., the product of their local measurement outcomes yields $+1$. (c) Perfect anticorrelations will be obtained when $\theta_1 + \theta_2 = \pi/2, 3\pi/2$, etc., given by the product -1 of the local results.

mode $b2$ so that Bob's Bell-state analyzer makes projective measurements onto the maximally entangled state $\cos\theta_2|\phi^-\rangle_{b_1,b_2} + \sin\theta_2|\psi^+\rangle_{b_1,b_2}$. By analogy with the polarization measurements, Bob assigns a value of $+1$ or -1 for measurements when the HWP is set to $\theta_2/2$ or $(\theta_2 + \pi/2)/2$, respectively. For consistency throughout this Letter. we have adopted the convention for the angle θ to mean the rotation of a polarization in real space. Therefore, the same polarization rotation on the Bloch sphere is 2θ and that rotation is induced by a HWP which is itself rotated by $\theta/2$. When Alice and Bob choose the orientations θ_1 and θ_2, their shared entangled state transforms to

$$\begin{aligned}|\Phi^-\rangle_{a,b} = {}& \cos(\theta_1 + \theta_2)\frac{1}{\sqrt{2}}(|H\rangle_a|\phi^-\rangle_{b_1,b_2} - |V\rangle_a|\psi^+\rangle_{b_1,b_2}) \\ & + \sin(\theta_1 + \theta_2)\frac{1}{\sqrt{2}}(|V\rangle_a|\phi^-\rangle_{b_1,b_2} \\ & + |H\rangle_a|\psi^+\rangle_{b_1,b_2}), \end{aligned} \tag{2}$$

which entails perfect correlations for any local settings θ_1 and θ_2 such that $\theta_1 + \theta_2 = 0$, π, 2π, etc., and perfect anticorrelations when $\theta_1 + \theta_2 = \pi/2$, $3\pi/2$, etc. Imperfectly correlated and anticorrelated events will occur at angles away from these specific settings and form the basis of a test of local realism.

We generate our three-photon state using a pulsed ultraviolet laser (pulse duration 200 fs, repetition rate 76 MHz) which makes two passes through a type-II phase-matched β-barium borate (BBO) nonlinear crystal [15], in such a way that it emits highly polarization-entangled photon pairs into the modes $a1$ and $b1$ and $a2$ and $b2$ (Fig. 2). Transverse and longitudinal walk-off effects are compensated using a HWP and an extra BBO crystal in each mode. By additionally rotating the polarization of one photon in each pair with additional HWPs and tilting the compensation crystals, any of the four Bell states can be produced in the forward and backward direction. We align the source to produce the Bell state $|\phi^+\rangle$ on each pass of the pump. Photons are detected using fiber-coupled single-photon counting modules. We spectrally and spatially filter the photons using 3-nm bandwidth filters and single-mode optical fibers. We measured 26 000 polarization-entangled pairs into the modes $a1$ and $b1$ and 18 000 pairs into the modes $a2$ and $b2$. The visibilities of each pair were measured to exceed 95% in the $|H/V\rangle$ basis and 94% in the $|\pm\rangle = 1/\sqrt{2}(|H\rangle \pm |V\rangle)$ basis. Parametric down-conversion is a probabilistic emitter of photon pairs and as such can sometimes emit two pairs of photons from the same pump pulse into the same pair of modes. In our experiment, fourfold coincidence events from double-pair emission are highly suppressed by a quantum interference effect due to the polarization rotation incurred in the quarter-wave plate (QWP) and the polarizing beam splitter (PBS) [16].

To generate our target state, we superpose one photon from each pair, those in modes $a1$ and $a2$, on PBS1. The

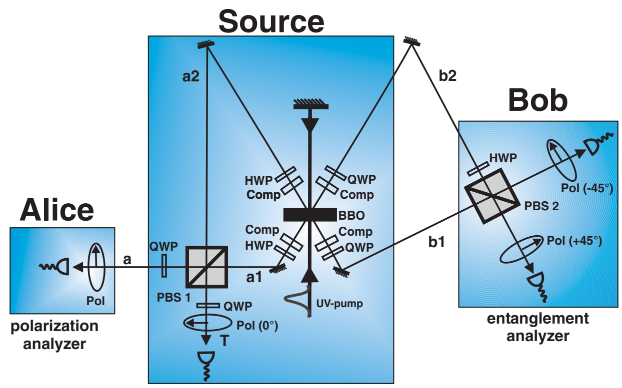

FIG. 2 (color online). Setup for the experimental realization. A spontaneous parametric down-conversion source emits $|\phi^+\rangle$ states, into both pair of modes $a1$ and $b1$ and $a2$ and $b2$. Comp is a 1 mm thick BBO crystal used to compensate the walk-off in the down-conversion crystal [15]. The modes $a1$ and $a2$ are superposed at the polarizing beam splitter PBS1. In our case, the PBS transmits horizontally polarized and reflects vertically polarized photons. Each mode T, a, $b1$, $b2$ passes through a QWP. Projecting the trigger qubit T onto the state $|H\rangle_T$, we generate the state $|\Phi^-\rangle_{a,b} = \frac{1}{\sqrt{2}}(|H\rangle_a|\phi^-\rangle_{b_1,b_2} - |V\rangle_a|\psi^+\rangle_{b_1,b_2})$. The photon in mode a is sent to Alice, who makes single-photon polarization measurements, determined by the orientation angle θ_1 of her linear polarizer. The photons in mode $b1$ and $b2$ belong to Bob, who uses a modified Bell-state analyzer to make projective measurements onto a coherent superposition of $|\phi^-\rangle_{b_1,b_2}$ and $|\psi^+\rangle_{b_1,b_2}$, where the mixing angle θ_2 is determined by the angle $\theta_2/2$, of the HWP in mode $b2$.

PBS implements a two-qubit parity check [17]: If two photons enter the PBS from different input ports, then they must have the same polarization in the $|H/V\rangle$ basis in order to pass to the two different output ports. Provided the photons overlap at the PBS, the initial state $|\phi^+\rangle_{a_1,b_1}|\phi^+\rangle_{a_2,b_2}$ is converted to the GHZ state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle_T|H\rangle_a|H\rangle_{b_1}|H\rangle_{b_2} + |V\rangle_T|V\rangle_a|V\rangle_{b_1}|V\rangle_{b_2})$ provided the photons emerge into different output spatial modes [18]. Rotations incurred in QWPs and the subsequent projection of the trigger photon in mode T onto $|H\rangle_T$ reduce the four-particle GHZ state to the desired three-photon entangled entangled state $|\Phi^-\rangle_{a,b}$.

The polarization of Alice's photon was measured with a polarizer oriented along the angle θ_1. Bob's measurements were made using a Bell-state analyzer based on a PBS [19]. By performing a check that the parity of Bob's photons is even, the PBS acts as a $|\phi^\pm\rangle_{b_1,b_2}$-subspace filter. The two Bell states in this subspace $|\phi^+\rangle_{b_1,b_2}$ and $|\phi^-\rangle_{b_1,b_2}$ have opposite correlations in the $|\pm\rangle$ basis and are distinguished using linear polarizers. One polarizer oriented along the $|+\rangle$ direction and the other along $|-\rangle$ completes a projective measurement onto $|\phi^-\rangle_{b_1,b_2}$. The setting of the HWP in mode $b2$ before PBS2 allows projection of any state of the form $\cos\theta_2|\phi^-\rangle_{b_1,b_2} + \sin\theta_2|\psi^+\rangle_{b_1,b_2}$.

Correlation measurements between Alice and Bob were made by rotating Alice's polarizer in 30° steps while Bob's HWP was kept fixed at $\theta_2/2 = 0°$ or 22.5°. Fourfold coincidence counts at each setting were measured for 1800 seconds (Fig. 3). The count rates follow the expected relation $N(\theta_1, \theta_2) \propto \cos^2(\theta_1 + \theta_2)$ with visibilities of $(78 \pm 2)\%$ in the $|H/V\rangle$ basis and $(83 \pm 2)\%$ in the $|\pm\rangle$ basis. Both surpass the crucial limit of ~71%, which, in the presence of white noise, is the threshold for demonstrating a violation of the CHSH-Bell inequality.

For our state $|\Phi^-\rangle_{a,b}$, the expectation value for the correlations between a polarization measurement at Alice and a maximally entangled state measurement at Bob is $E(\theta_1, \theta_2) = \cos[2(\theta_1 + \theta_2)]$. The correlation can be expressed in terms of experimentally measurable counting rates using the relation $E(\theta_1, \theta_2) = (N^{++} + N^{--} - N^{+-} - N^{-+})/(N^{++} + N^{--} + N^{+-} + N^{-+})$, where N is the number of coincidence detection events between Alice and Bob with respect to their set of analyzer angles θ_1 and θ_2, where +1 (− 1) outcomes are denoted as "+" ("−"). These correlations can be combined to give the CHSH-Bell parameter $S = |-E_1(\theta_1, \theta_2) + E_2(\tilde{\theta}_1, \theta_2) + E_3(\theta_1, \tilde{\theta}_2) + E_4(\tilde{\theta}_1, \tilde{\theta}_2)|$, which is maximized at $\{\theta_1, \tilde{\theta}_1, \theta_2, \tilde{\theta}_2\} = \{0°, 45°, 22.5°, 67.5°\}$ to $S = 2\sqrt{2}$. This violates the inequality $S \leq 2$ for local-realistic theories.

In Fig. 4, the count rates for the 16 required measurement settings to perform the CHSH inequality are shown and give the four correlations $E_1(\theta_1, \theta_2) = 0.69 \pm 0.05$, $E_2(\tilde{\theta}_1, \theta_2) = -0.61 \pm 0.04$, $E_3(\theta_1, \tilde{\theta}_2) = -0.58 \pm 0.04$,

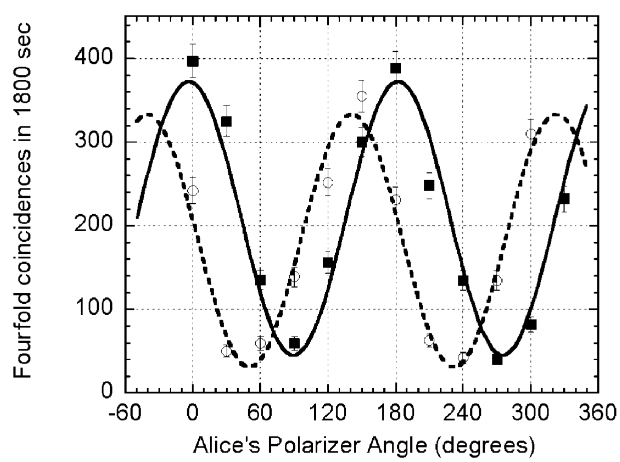

FIG. 3. Measured coincidence fringes for the entangled entangled state. Bob's half-wave plate was initially set to 0° and made projective measurements onto the state $|\phi^-\rangle_{b_1,b_2}$. The total number of fourfold coincidence counts measured in 1800 seconds as a function of the angle of Alice's polarizer is shown as solid squares. Fitting the curve to a sinusoid (solid line) yields a visibility of $(78 \pm 2)\%$. After changing Bob's measurement setting to project onto the state $\frac{1}{\sqrt{2}}(|\phi^-\rangle_{b_1,b_2} + |\psi^+\rangle_{b_1,b_2})$, the procedure was repeated. The data for these settings are shown as open circles. The sinusoidal fit (dotted line) yields a visibility of $(83 \pm 2)\%$.

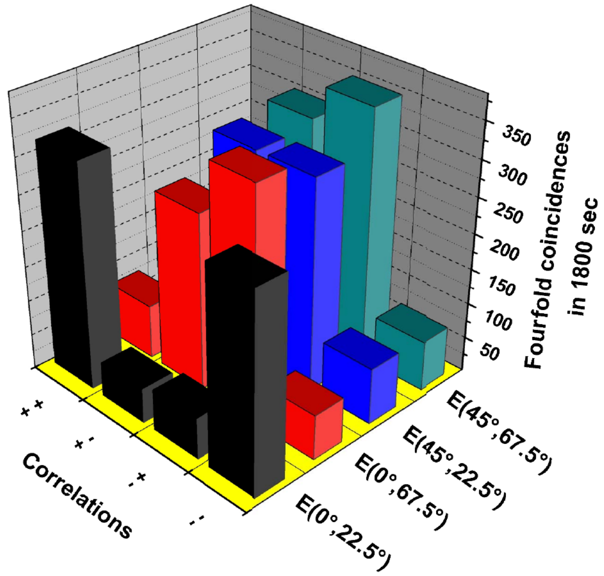

FIG. 4 (color online). Experimentally measured coincidence counting rates used to test the CHSH-Bell inequality. The requisite coincidence measurements for the 16 different measurement settings are shown. Each measurement was performed for 1800 seconds. For measurement settings θ_1 and θ_2, the axis labels $++$, $+-$, $-+$, and $--$ refer to the actual settings of (θ_1, θ_1), $(\theta_1, \theta_2 + \pi/2)$, $(\theta_1 + \pi/2, \theta_2)$, and $(\theta_1 + \pi/2, \theta_2 + \pi/2)$, respectively. These data yielded the Bell parameter $S = 2.48 \pm 0.09$, which is in conflict with local realism by over 5 standard deviations.

and $E_4(\tilde{\theta}_1, \tilde{\theta}_2) = -0.60 \pm 0.04$. These correlations yield the Bell parameter $S = 2.48 \pm 0.09$, which strongly violates the CHSH-Bell inequality by 5.6 standard deviations.

We have demonstrated a violation of the CHSH-Bell inequality using the correlations between a single-particle property, the polarization state of a photon, and a joint property of two particles, the entangled state of a photon pair. In doing so, we have experimentally demonstrated that two-particle correlations have the same ontological status as single-particle properties. Our result shows that it only makes sense to speak about measurement events (detector "clicks") whose statistical correlations may violate limitations imposed by local realism and thus indicate entanglement.

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