Flux Cloning in Josephson Transmission Lines

D. R. Gulevich and F. V. Kusmartsev

Department of Physics, Loughborough University, Leicestershire LE11 3TU, United Kingdom

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We describe a novel effect related to the controlled birth of a single Josephson vortex. In this phenomenon, the vortex is created in a Josephson transmission line at a T-shaped junction. The "baby" vortex arises at the moment when a "mother" vortex propagating in the adjacent transmission line passes the T-shaped junction. In order to give birth to a new vortex, the mother vortex must have enough kinetic energy. Its motion can also be supported by an externally applied driving current. We determine the critical velocity and the critical driving current for the creation of the baby vortices and briefly discuss the potential applications of the found effect.

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Superconducting vortices have great potential to be used in superconducting electronics. A great achievement of superconducting physics today is that there are many ways to control them [1]. However, the problem of controlling the motion of a single Abrikosov vortex still remains [2]. The matter is that there are always correlations between many vortices existing in a sample as was demonstrated for Hall-bar-shaped superconductors [3]. These correlations form a very complex system that spoils such control. Similarly, there is an intensive search for control of Josephson vortices or fluxons. A long Josephson junction may be used as a Josephson transmission line (JTL) in which a Josephson vortex can propagate freely [4]. Because of their useful properties, Josephson systems have been under intensive attention from the scientific community [5–9]. A flux flow oscillator has been experimentally realized [5]. Different realizations of a vortex qubit have been proposed [6,7]. The quantum dynamics of a single fluxon trapped in an annular Josephson junction has been investigated [8]. Also, other useful applications include Josephson vortex ratchets, rectifiers, and diodes [9].

In this Letter, we show how a single Josephson vortex propagating in a long Josephson junction of a special T-shaped geometry may give birth to a new vortex. The geometry consists of two perpendicular JTLs forming a T junction: main line (MJTL) and additional line (AJTL); see Fig. 1. A new vortex is created when a mother vortex moving along the MJTL is passing the T junction. Just at the T junction, a new Josephson vortex begins its life and motion in the direction perpendicular to the MJTL. The process of creation of a new vortex depends on the energy of the original "mother" vortex. If the vortex is moving very slowly, it does not have enough kinetic energy to give birth to a new vortex. Then the T junction acts as a barrier and the mother vortex is just reflected from it. However, if the mother vortex has enough energy to overcome the barrier, the new baby vortex is born. In this case, the mother vortex continues its motion along the MJTL, while the "baby" vortex moves in the perpendicular direction along the AJTL. The novel effect described in this Letter may entail many promising applications.

Consider a fluxon propagating with a velocity u (normalized to the Swihart velocity $\bar{c}$) in a straight two-dimensional strip of width W_0 (here and further, we work with normalized units, the coordinates and distances are normalized to the Josephson penetration length λ_J, time is scaled by ω_p^{-1}, where ω_p is the plasma frequency, and the energy is normalized to $j_c\lambda_J^2\Phi_0/2\pi$, where $\Phi_0 = h/2e$ is the unitary flux quantum and j_c is the critical current density). The dynamics of a superconducting phase is described as a sine-Gordon soliton

$$\varphi(x, y, t) = 4\arctan\exp\left(\frac{x - x_0 - ut}{\sqrt{1-u^2}}\right) \tag{1}$$

with the energy

$$\int_{-\infty}^{\infty} dx \int_0^{W_0} dy\left[\frac{\varphi_t^2}{2} + \frac{(\nabla\varphi)^2}{2} + 1 - \cos\varphi\right] = \frac{8W_0}{\sqrt{1-u^2}}. \tag{2}$$

This is analogous to the energy of a propagating relativistic particle with a rest mass $8W_0$.

Let us state the problem of a fluxon approaching the T junction formed by two JTLs as shown in Fig. 1(a). How will the fluxon continue its motion after striking the obstacle associated with the T junction? An intuitive view is that the fluxon splits in two in such a way that each part of the incident fluxon travels independently in either direction. Such an "earthworm behavior" is confirmed by our numerical calculations. Using energy considerations and one-dimensional approximation, it is possible to extract analytical formulas for the critical velocity and the critical current.

The geometry consists of the MJTL of width W_0 laying along the X direction and the AJTL along the Y axis. The AJTL has a width W and is connected to the main waveguide at the center of coordinates forming a T junction [Fig. 1(a)]. When the fluxon approaches the fork, it experiences an energy barrier. Therefore, in order to split the

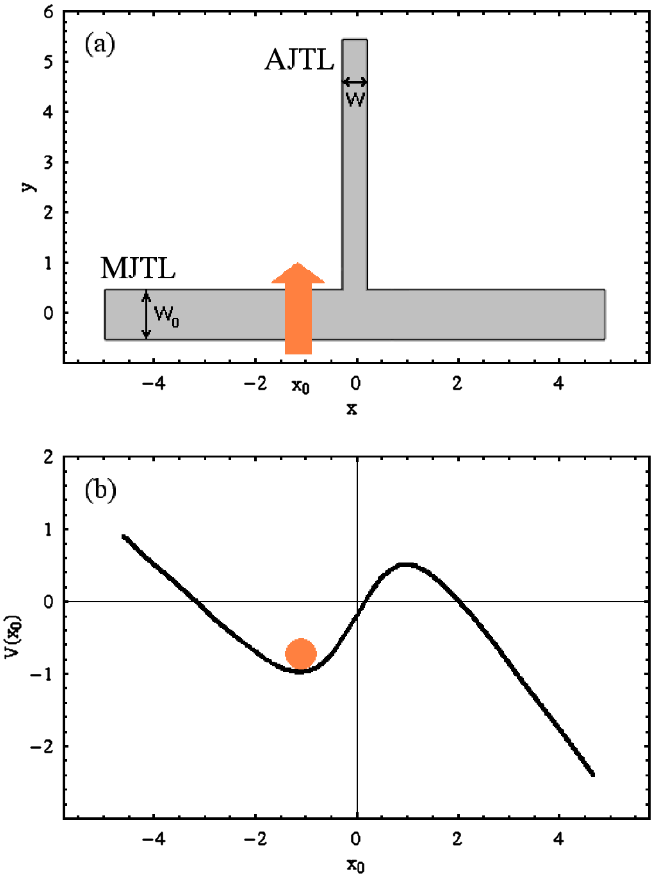

FIG. 1 (color online). (a) T junction formed by Josephson transmission lines (top view). Widths of the main (horizontal) and additional (vertical) transmission lines are $W_0 = 1$ and $W = 0.5$, correspondingly (normalized to the Josephson penetration length). The thick arrow represents a Josephson vortex approaching the T junction. (b) Effective potential energy normalized to $j_c\lambda_J^2\Phi_0/2\pi$ as a function of fluxon position x_0 in the MJTL. The driving current is $\gamma = 0.1$. The plot is due to the analytical formula (6). The solid ball represents the equilibrium position of the mother vortex at a finite driving current. With increasing driving current, the equilibrium state disappears and the creation of a baby vortex in the AJTL occurs.

fluxon, some minimal velocity is needed. In the case of low damping and absence of the driving current, the critical velocity can be estimated from the following energy considerations. The minimal energy for the fluxon splitting is equal to the rest mass of the fluxon $8W_0$ in the MJTL plus the rest mass of the fluxon $8W$ in the AJTL:

$$\frac{8W_0}{\sqrt{1-u_c^2}} = 8W_0 + 8W.$$

From this equation follows the critical velocity

$$u_c = \frac{\sqrt{W(W+2W_0)}}{W+W_0}. \tag{3}$$

The fluxon division can be supported by the external driving current as well. Assuming W, $W_0 \lesssim 1$, and employing the one-dimensional approximation, we may estimate the critical current analytically. We use the stationary collective coordinate approach with a single parameter x_0 playing a role of a fluxon position in the MJTL [Fig. 1(a)]. The single-variable variational functions describing stationary phase profiles in the main and additional JTLs are chosen as

$$\varphi(x) = 4\arctan e^{x-x0} \quad \text{and} \quad \varphi(y) = 4\arctan e^{y-x0}, \tag{4}$$

correspondingly. A fluxon in the MJTL described by $\varphi(x)$ contributes to the potential energy the term $-\gamma W_0 2\pi x_0$, where $\gamma = j/j_c$ is given by the ratio of the driving current density j and the critical current density j_c. The extra energy stored in the AJTL results in the energy barrier

$$\Delta V = W\int_0^\infty dy\left[\frac{\varphi_y^2}{2} + 1 - \cos\varphi + \gamma(\varphi - 2\pi)\right]. \tag{5}$$

Substituting the variational function $\varphi(y)$ (4) into (5) and adding the contribution of the MJTL, we obtain the effective potential energy as a function of the fluxon position x_0:

$$V(x_0) = 4W\int_{-\infty}^{x_0} d\xi[\mathrm{sech}^2\xi - \gamma\arctan e^{\xi}] - \gamma W_0 2\pi x_0, \tag{6}$$

where $\xi = y - x_0$ is a new integration variable. Suppose the fluxon approaching the T junction is stopped by the associated energy barrier. Its equilibrium position is defined by the minimum of the potential energy where $V'(x_0) = 0$ [see Fig. 1(b)]. On the other hand, from the condition $V''(x_0) = 0$ follows the critical point $x_0 = 0$, where instability of the fluxon equilibrium position occurs. After straightforward calculation, we obtain a very simple formula for the critical driving current

$$\gamma_c = \frac{4W}{\pi(2W_0 + W)}. \tag{7}$$

At this value of the driving current, the potential well in Fig. 1(b) disappears and a baby vortex is created.

We have used the finite element program package FEMLAB to analyze the dynamics of a fluxon in the T junction. Without damping, the 2D sine-Gordon equation reads

$$\varphi_{tt} - \nabla^2\varphi + \sin\varphi = 0 \tag{8}$$

with the boundary conditions (SI units) [10]

$$\mathbf{n}\cdot\nabla\varphi|_{\partial\Omega} = \frac{\mathbf{n}\cdot(\mathbf{H}\times\mathbf{e_z})}{\lambda_J j_c}, \tag{9}$$

where the normal vector $\mathbf{n}$ is defined on the boundary $\partial\Omega$ and points outward the junction domain Ω, $\mathbf{H}$ is the magnetic field generated by the driving current on the boundary $\partial\Omega$, and $\mathbf{e_z}$ is a normal to the plane of the junction. The magnetic field is proportional to the applied driving current γ and is estimated from Maxwell's equations.

Our numerical simulations of the time-dependent two-dimensional sine-Gordon equation with the geometry presented in Fig. 1 clearly show the cloning phenomenon when the fluxon approaches the T junction. The fluxon dynamics at zero driving current can be traced in Fig. 2. The color domains represent the value of the superconducting phase difference: blue (left dark area) stands for the phase φ equal to zero, and red (right dark area) represents the phase φ equal to 2π. The intermediate color (light) represents the Josephson vortices (see the color scale in Fig. 2). As one can see from Fig. 2, two different types of behavior are possible—reflection from the T junction [Fig. 2(a)] and flux cloning [Fig. 2(b)]. We have used initial conditions with the following form: $\varphi(x, y, t)|_{t=0}$ for the phase and $\partial\varphi(x, y, t)/\partial t|_{t=0}$ for its time derivative, where the soliton function $\varphi(x, y, t)$ is defined by (1). The initial soliton position was $x_0 = -3$, and the initial velocities are $u = 0.7$ for Fig. 2(a) and $u = 0.8$ for Fig. 2(b). The relative and absolute tolerances were 0.003–0.005 and 10^{-9}, correspondingly, and the time step was 0.001. We have used a mesh consisting of 1632 elements.

Further, we have investigated numerically a set of 2D geometries with varying JTL widths. The conditions were the same as described above; only the width of the AJTL was changing. Let us compare the obtained numerical dependence of the critical velocity on the width W of the AJTL to the theoretical prediction given by Eq. (3). This comparison is shown in Fig. 3(a). The numerical value for the critical velocity has been defined when the change of behavior of a fluxon from reflection [Fig. 2(a)] to splitting [Fig. 2(b)] occurred.

Finally, we have made a stationary analysis for a resting fluxon when the driving current is applied. The dependence of the critical current on the width W of the AJTL is compared to Eq. (7) and shown in Fig. 3(b). The coincidence of analytical and numerical results is very striking. A slight deviation arises only when the width of the AJTL is compared with the width of the main line.

It is important to note that this mechanism of vortex creation does not violate the conservation of vorticity. Indeed, consider a superconducting contour around the initial vortex line covering a single flux quantum. When the vortex line splits in two pieces, this contour appears around either the first or the second piece. Therefore, the contour again covers a single vorticity even after separation of the two vortices.

The proposed system can be implemented with Nb technology or using high temperature superconductors (HTSCs). For Nb junctions with parameters $\omega_p = 10^{12} s^{-1}$ and $\lambda_J = 10\ \mu$m, the typical fluxon velocity would be of the order 10^7 m/s, while the process of flux cloning [Fig. 2(b)] would take a few picoseconds. For a HTSC such as bismuth strontium copper oxide, with $\omega_p = 10^{11}\ s^{-1}$ and $\lambda_J = 0.5\ \mu$m, the typical velocities would be of the order 10^5 m/s. In this case, the typical cloning times would be 1 or 2 orders of magnitude longer compared to Nb junctions of the same dimensions.

In conclusion, we have demonstrated a novel effect constituting the controlled birth of a Josephson fluxon at the T junction of JTLs. We have studied this phenomenon analytically and have carried out a detailed numerical analysis of the dynamics of the superconducting phase. The numerical simulations with the use of the two-dimensional sine-Gordon equation appear in good agreement with our theoretical predictions. Although some two-dimensional Josephson junction geometries have been con-

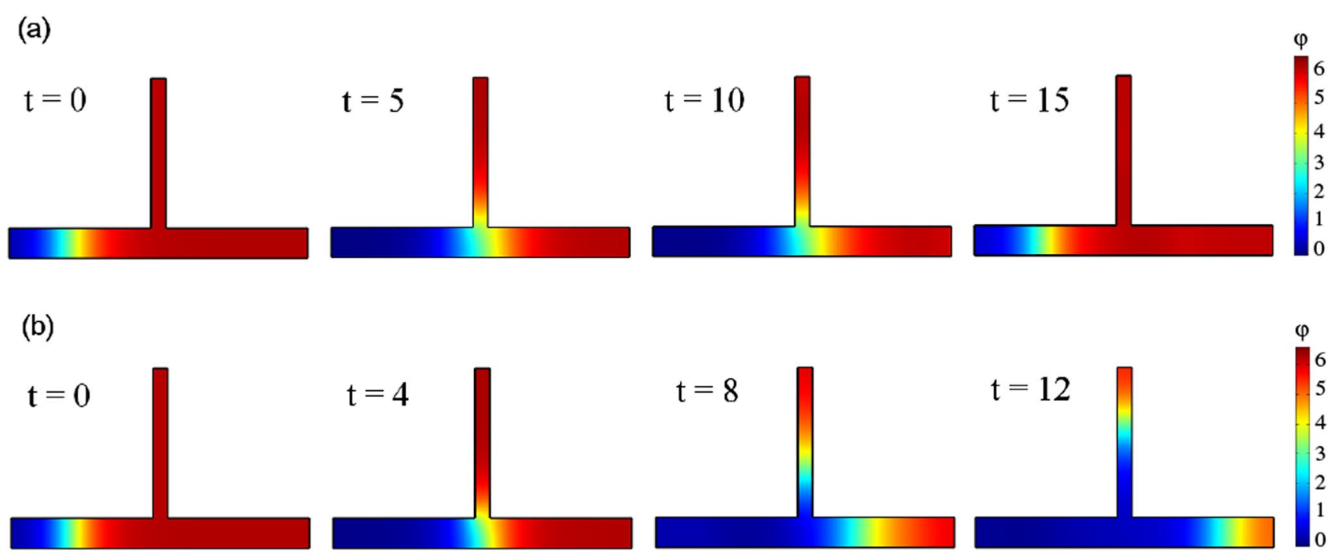

FIG. 2 (color online). (a) Reflection of an incident fluxon propagating with velocity $u = 0.7$ and (b) cloning of a fluxon propagating with velocity 0.8 higher than the critical $u_c = 0.76$ (normalized to the Swihart velocity $\bar{c}$). Both diagrams represent numerical simulations of the superconducting phase difference for the geometry in Fig. 1(a) with the use of the 2D sine-Gordon equation. The driving current is absent. The color scale represents the superconducting phase difference φ.

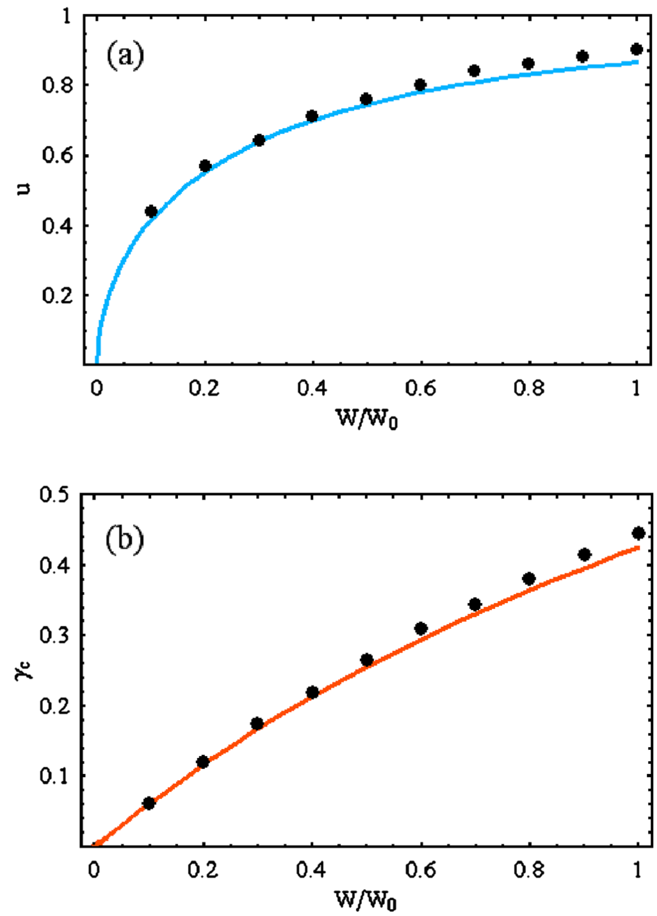

FIG. 3 (color online). (a) Dependence of the normalized critical velocity u on the ratio of widths of JTLs. The dots represent numerical simulation of the superconducting phase dynamics using the 2D sine-Gordon equation. The solid line represents the theoretical predictions (3). (b) Dependence of the normalized critical driving current γ_c on the ratio of widths of JTLs. The dots represent the results of numerical calculation of the stationary 2D sine-Gordon equation. The solid line is the theoretical prediction (7).

sidered earlier [11], the phenomenon may give rise to a special class of two-dimensional Josephson junctions where flux cloning is of a good use. Such systems can be used for generation of periodic fluxon chains, fluxon-antifluxon pairs, and continuous breathers. The flux cloning effect can also be helpful to implement fluxon-based logic gates [12], logic networks [13], and even terahertz oscillators. Interesting phenomena can be anticipated if the quantum regime is accessible. Indeed, the cloning of two identical fluxons can be helpful for quantum information processing. The authors hope that this Letter will stimulate further research in this area.

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