

Sigma Coupling to Photons: Hidden Scalar in $\gamma\gamma \rightarrow \pi^0\pi^0$

M. R. Pennington

Institute for Particle Physics Phenomenology, Durham University, Durham, DH1 3LE, United Kingdom
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There has long been speculation about the nature of the σ resonance. For three decades Jaffe has argued for a tetraquark composition, while others have claimed it is largely a glueball. A key pointer to its nature is its coupling to two photons. Consequently, there have been recent proposals to observe this important scalar hiding in $\gamma\gamma \rightarrow \pi^0\pi^0$. We show here that the σ is already crouching in this cross section exactly as measured 20 years ago. What is new is that precise knowledge of the position of the σ pole, provided by the analysis of the Roy equations, now allows its two photon coupling to be accurately fixed. Its two photon width is found to be (4.1 ± 0.3) keV, a value far too large for a wholly gluonic, or even a tetraquark, state.

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New information on the σ .—The σ resonance has for long been a mnemonic for the highly correlated two pion exchange that generates the longest range isoscalar force revealed in nuclear binding. It is also the name of the scalar field, the nonzero vacuum expectation value of which breaks chiral symmetry, giving mass to all light hadrons [1]. While isoscalar $\pi\pi$ interactions grow rapidly above threshold, they have none of the features readily identified as a textbook resonance, quite unlike the ρ for instance. If $\pi\pi$ mass distributions, whether from classic meson-meson reactions or from final state interactions of decay products, are fitted with Breit-Wigner forms, then inevitably one finds a pole in the complex energy plane. However, fits give a position varying wildly from one analysis to another with both masses and widths from 350 MeV to 1 GeV [2,3].

Renewed interest in using the Roy equations, which encode the analyticity provable in axiomatic field theory with the three channel crossing symmetry of $\pi\pi$ scattering, has, when combined with chiral constraints and new experimental information, allowed a narrow corridor of possible amplitudes from 800 MeV down to threshold, as found by Colangelo, Gasser, and Leutwyler [4]. Recent recognition that the Roy equations can be evaluated not just on the real axis but in the complex energy plane has determined the position of the lightest resonance in QCD, the σ , to be at $E_R = 441 - i272$ MeV within small uncertainties [5]. But what is the nature of this state in the spectrum of hadrons? Is it a conventional $\bar{q}q$ state of the quark model [6]? Is it a tetraquark meson [7], composed of $\bar{q}q\bar{q}q$, with the expected $\bar{q}q$ nonet still higher in mass [8,9], or is it largely glue [10,11]? The coupling to photons is a key guide to a state's composition.

Now the same precise information on $\pi\pi$ amplitudes that determines the existence and the position of the pole allows the amplitude for $\gamma\gamma \rightarrow \pi\pi$ to be accurately determined. Exploiting this is most readily done by the use of partial wave dispersion relations. While the σ appears in the $I = 0$ channel, we will also need the $I = 2$ amplitude. The calculation follows the philosophy of [12,13].

Two photon amplitude.—Let us begin by considering the S -wave $\gamma\gamma \rightarrow \pi\pi$ amplitudes with isospin I , $\mathcal{F}^I(s)$, where s is the square of the $\pi\pi$ invariant mass. Each of these amplitudes, with $I = 0, 2$, being complex has a phase $\phi^I(s)$ along the right-hand cut, when s is above the two pion threshold, i.e., $s > s_{\text{th}} = 4m_\pi^2$. Unitarity, through Watson's theorem, requires the phase of each of these partial waves to be the same as the phase of the corresponding $\pi\pi$ partial wave amplitude in the elastic region [14]. To implement this constraint we define the Omnès function, $\Omega^I(s)$,

$$\Omega^I(s) = \exp\left[\frac{s}{\pi} \int_{s_{\text{th}}}^{\infty} ds' \frac{\phi^I(s')}{s'(s' - s)}\right], \quad (1)$$

which by construction has phase $\phi^I(s)$ for $s > s_{\text{th}}$. Thus the $\gamma\gamma \rightarrow \pi\pi$ S -wave amplitudes, $\mathcal{F}^I(s)$, can be written as $P^I(s)\Omega^I(s)$, where $P^I(s)$ is a function which is real along the right-hand cut with $s > s_{\text{th}}$. The phase, ϕ^I , is simply the phase shift in the region of elastic unitarity [14], which applies up to $\bar{K}K$ threshold, since multipion channels are negligible below 1.2 GeV. Moreover, in the low energy region of interest, the differences in phase above 1 GeV affect the results little as has been checked by replacing the $\pi\pi \rightarrow \pi\pi$ phase with that for $\pi\pi \rightarrow \bar{K}K$. Such a change is equivalent to assuming that the $\pi\pi$ final state in the two photon process is only accessed through a $\bar{K}K$ intermediate state. Outside the narrow confines of the $f_0(980)$ region, this would be an extreme possibility. Nevertheless, the effect is small and included in the uncertainties we quote. Representative input $\pi\pi$ S -wave phases, $\phi^I(s)$, for $I = 0, 2$ and the resulting Omnès functions are shown in Fig. 1.

Now Low's low energy theorem [15] requires that as $s \rightarrow 0$, and $t, u \rightarrow m_\pi^2$, at the threshold for Compton scattering $\gamma\pi \rightarrow \gamma\pi$, the full scattering amplitude is equal to its one pion exchange Born term. It is such crossed channel exchanges that generate the left-hand cut contribution to the $\gamma\gamma \rightarrow \pi\pi$ partial wave amplitudes, which we denote collectively by $\mathcal{L}^I(s)$. Because the pion is so much lighter than any other hadron, pion exchange determines the dis-

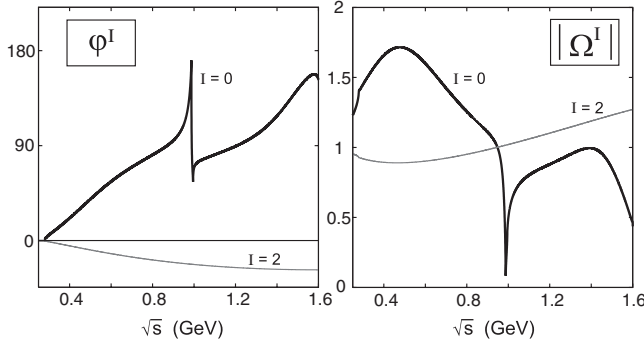


FIG. 1. Representative $I = 0, 2$ $\gamma\gamma \rightarrow \pi\pi$ S -wave phases and moduli of the Omnès functions, $\Omega^I(s)$, related by Eq. (1).

continuity across this left-hand cut not just at $s = 0$ but in the whole region $0 > s > -M_V^2$, beyond which other exchanges like ρ , ω start to contribute [16]. While the Born term assumes pointlike couplings for the pion, any form-factor dependence only affects the left-hand cut for $s < -M_V^2$, since it is vector masses that set the scale for such charged radii. Consequently, the left-hand cut from $s = 0$ to $s \approx -0.5 \text{ GeV}^2$ is precisely known and that is all we require to fix the amplitude in the region of $s = s_R = E_R^2$ shown in Fig. 2.

To see how, let us construct the function $G^I(s) \equiv (\mathcal{F}^I(s) - \mathcal{L}^I(s))\Omega^I(s)^{-1}$, which only has a right-hand cut. Its discontinuity is $\mathcal{L}^I(s) \sin\phi^I(s)/|\Omega^I(s)|$, which is accurately known at low energies. This information is embodied in a dispersion relation for the function $G^I(s)$ using a contour like that in Fig. 2. While the behavior of $G^I(s)$ means the integral at infinity converges with just one

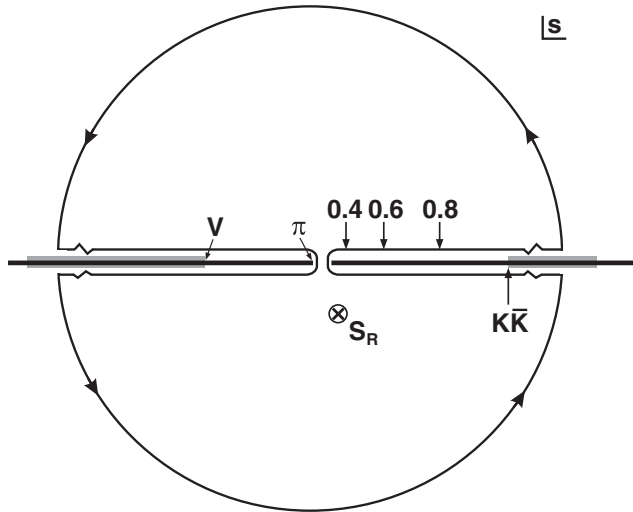


FIG. 2. The complex s -plane structure of the $\gamma\gamma \rightarrow \pi\pi$ amplitudes, $\mathcal{F}^I(s)$. π labels the start of the left-hand cut controlled by the pion exchange Born term, while V denotes where the vector exchanges ρ , ω start to contribute to the discontinuity. The right-hand cut is elastic effectively up to $\bar{K}K$ threshold. The point $s = s_R$ is the position of the σ pole [5]. The plot is drawn to scale so 0.4, 0.6, 0.8 are the c.m. energy in GeV.

subtraction, it is more convenient for our purpose to ensure that the integrals are dominated by the known low energy regime of $|s'| < M_\rho^2$. This is achieved by making two subtractions:

$$\mathcal{F}^I(s) = \mathcal{L}^I(s) + c_I s \Omega^I(s) + \frac{s^2}{\pi} \Omega^I(s) \times \int_{s_{\text{th}}}^{\infty} ds' \frac{\mathcal{L}^I(s') \sin\phi^I(s')}{s'^2 (s' - s) |\Omega^I(s')|}, \quad (2)$$

The constants c_I are specified by the QED low energy theorem and chiral dynamics. These two conditions apply to amplitudes with pions of definite charge (which are combinations of those with $I = 0, 2$). Low's theorem requires [17] that the S -wave amplitude for $\gamma\gamma \rightarrow \pi^+ \pi^-$, $\mathcal{F}^{+-}(s) \rightarrow B(s) + \mathcal{O}(s^2)$ as $s \rightarrow 0$, where $B(s)$ is the Born S wave, while chiral dynamics demands that the S -wave amplitude for $\gamma\gamma \rightarrow \pi^0 \pi^0$ $\mathcal{F}^{00}(s) = 0$ at $s = \mathcal{O}(m_\pi^2)$. At one loop level in Chiral Perturbation Theory [18], $\mathcal{F}^{00}(s) \propto \mathcal{T}(\pi^+ \pi^- \rightarrow \pi^0 \pi^0)$, fixing the Adler zero exactly at $s = m_\pi^2$ at this order. However, its precise position hardly affects our results.

These relations allow us to determine the $\gamma\gamma \rightarrow \pi\pi$ cross section in the low energy region. Precision comes from the more accurate determination of the $\pi\pi$ S -wave amplitudes obtained by combining new results from decays like K_{e4} , $J/\psi \rightarrow \phi X$, and $D \rightarrow \pi X$ [19] with the Roy equations, which fixes the pole at $s = s_R$. This calculation reproduces the cross section for the production of charged and neutral pions as measured by Mark II [20] and Crystal Ball [21] collaborations, respectively, in the low energy region with no free parameters. The predictions for the neutral cross section are shown in Fig. 3. The range shown delineates the uncertainties due to (i) different $\gamma\gamma$ phases $\phi^I(s)$ above $\bar{K}K$ threshold and (ii) different positions of the Adler zero in the $\pi^0 \pi^0$ channel. Notice that the cross section is very nearly unique up to 450 MeV.

Two photon coupling of the σ .—Of course, the $I = 0$ $\pi\pi$ phase and Omnès function, shown in Fig. 1, know about the σ pole at $s = s_R$ deep in the complex plane close to both the right- and left-hand cuts of Fig. 2. Not only can we determine the $\gamma\gamma$ amplitudes $\mathcal{F}^I(s)$ along the upper side of the right-hand cut on the physical sheet where experiments are performed, but everywhere on this first sheet. In particular, we can determine the $I = 0$ amplitude at $s = s_R$, marked in Fig. 2. The right-hand cut structure of the $\gamma\gamma \rightarrow \pi\pi$ amplitude mirrors that of the corresponding hadronic amplitude, \mathcal{T}^I , for $\pi\pi \rightarrow \pi\pi$ in the region of elastic unitarity [22]:

$$\mathcal{F}^I(s) = \alpha^I(s) \mathcal{T}^I(s), \quad (3)$$

where the function $\alpha^I(s)$ represents the intrinsic coupling of $\gamma\gamma \rightarrow \pi\pi$, while \mathcal{T} describes the final state interactions, which color and shape the electromagnetic process.

At $s = s_R$ on the first sheet, the amplitude $\mathcal{T}^{I=0}(s) = i/2\rho(s)$, since the S -matrix element vanishes at this point.

$\rho(s)$ is, as usual, the phase-space factor $\rho(s) = \sqrt{1 - s_{\text{th}}/s}$. The dispersion relation on the first sheet then determines the coupling function $\alpha(s_R)$, which, not having a right-hand cut, has the same value on the second sheet. We introduce subscripts to label the sheets *I* and *II*, while the superscripts continue to denote isospin.

In the neighborhood of the pole on the second sheet, the $\gamma\gamma \rightarrow \pi\pi$ *S*-wave amplitude is given by

$$\mathcal{F}_{II}^0(s) \simeq \frac{g_\gamma g_\pi}{s_R - s}, \quad \text{while} \quad \mathcal{T}_{II}^0(s) \simeq \frac{g_\pi^2}{s_R - s}. \quad (4)$$

Even if factorized residues are not strictly appropriate for such a very short-lived state, Eqs. (6) and (7) below provide a physically meaningful and unambiguously defined [3] $\gamma\gamma$ width. $\alpha^0(s_R)$ determines the ratio of g_γ/g_π for the isoscalar resonance. Now the hadronic amplitude on sheet *I* is related to that on sheet *II* by

$$\frac{1}{\mathcal{T}_{II}(s)} = \frac{1}{\mathcal{T}_I(s)} + 2i\rho, \quad (5)$$

so that

$$g_\gamma^2 = \lim_{s \rightarrow s_R} \frac{(s - s_R) \mathcal{F}_I^0(s)^2}{(\mathcal{T}_I^0(s) - i/2\rho)}. \quad (6)$$

Combining the representation cited above [19] for the hadronic amplitude, \mathcal{T}^0 , on sheet *I* with the present dispersive calculation then gives the two photon coupling of the σ , which specifies its radiative width to be [23]

$$\Gamma(\sigma \rightarrow \gamma\gamma) = \frac{\alpha^2 |\rho(s_R) g_\gamma^2|}{4M_\sigma} = (4.09 \pm 0.29) \text{ keV}. \quad (7)$$

That this is 10 times larger than the signal seen in $\gamma\gamma \rightarrow \pi^0\pi^0$ cross section requires some explanation, particularly in the light of proposals, e.g., [24], to search for the σ in this channel. If we consider this process and for the moment simply ignore the requirement that final state interactions shape the $\pi\pi$ distribution in a well-defined way, then one would say that, with no Born contribution, the cross section should reflect the appearance of resonant structures if they exist. If this is the σ , then one can read off from the observed cross section in Fig. 3 of 10–12 nb a $\gamma\gamma$ width an order of magnitude smaller than we have deduced. However, this is too naïve.

In hadronic channels, $I = 2$ amplitudes, which are *exotic* in the quark model, are much smaller than those with $I = 0$. In contrast in this two photon process both $I = 0, 2$ are equally important. The σ appears in the $I = 0$ amplitude, and this can only be separated from data by analyzing $\gamma\gamma \rightarrow \pi^+\pi^-$ and $\pi^0\pi^0$ together [23].

As we have seen what is really happening is that the Born amplitude is modified by final state interactions to ensure Watson's theorem is satisfied. As a result, the $I = 0$ and 2 amplitudes are no longer real and exactly canceling in the neutral channel. A vectorial representation of this is shown at 400 MeV in Fig. 4. The $I = 0$ component has the phase of $I = 0$ *S*-wave $\pi\pi$ scattering, while that with $I =$

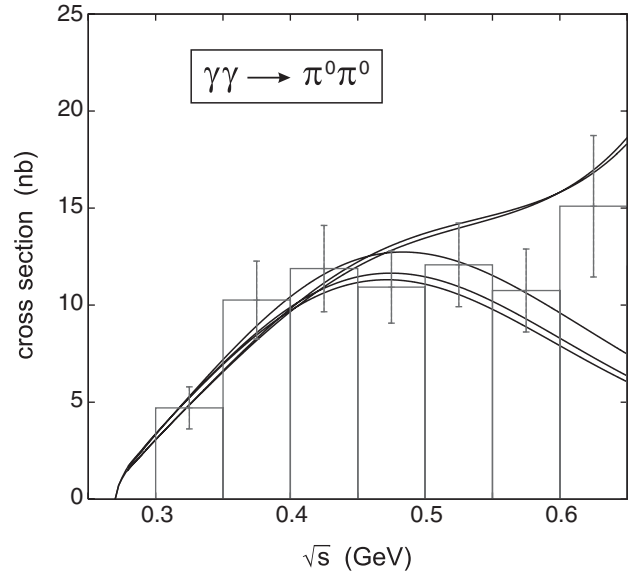


FIG. 3. Results of the dispersive calculation for the low energy $\gamma\gamma \rightarrow \pi^0\pi^0$ cross section for different input phases ϕ^l above $\bar{K}K$ threshold, each with 3 different positions of the Adler zero at $s = 1/2, 1, 2m_\pi^2$, compared with the Crystal Ball data [21] scaled to the whole angular range.

2 has the phase of the corresponding isotensor *S* wave. In Fig. 4 the vector OC ($\mathcal{F}^0 + \mathcal{F}^2/\sqrt{2}$) is $\sqrt{3}/2$ times the charged channel *S* wave, while the vector ON ($\mathcal{F}^0 - \sqrt{2}\mathcal{F}^2$) is $-\sqrt{3}$ times the neutral one. One sees that the square of the neutral channel *S* wave (which dominates its cross section) is a factor of 12 smaller than the modulus squared of the $I = 0$ *S* wave. It is in this amplitude that the σ is to be found. This delivers an $I = 0$ cross section averaged across the σ consistent with a 4 keV width determined from the pole residue.

How such a two photon width translates into ideas about the composition of the σ relies on models, which assume a

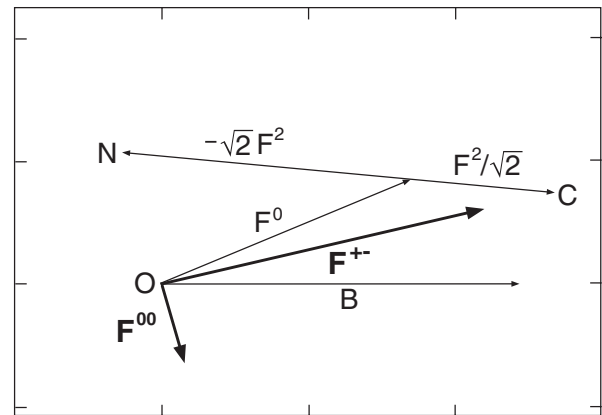


FIG. 4. $\gamma\gamma \rightarrow \pi\pi$ *S*-wave amplitudes at 400 MeV with definite isospin and with definite charges as indicated by the superscripts. *B* is the Born amplitude *S*-wave for comparison. *OC* and *ON* define the directions of the charged and neutral pion amplitudes as given by the vector sums described in the text.

single component. Its width is too large for a low lying glueball. Despite crouching in the cross section $\gamma\gamma \rightarrow \pi^0\pi^0$, the “red dragon” of Minkowski and Ochs [11] with low mass is unlikely to be gluonic. Indeed it would appear unlikely to be a $\bar{q}q$ state according to the older work of [25]. However, 4 keV is just what Chanowitz [26] and Barnes [27] have predicted for such a quark state. If it is the scalar companion of the $f_2(1275)$ with a $(\bar{u}u + \bar{d}d)/\sqrt{2}$ composition, adapting a positronium result to the nonrelativistic quark model gives

$$\Gamma(\sigma \rightarrow \gamma\gamma)/\Gamma(f_2 \rightarrow \gamma\gamma) = 15/4 \times (m_\sigma/m_{f_2})^n, \quad (8)$$

with relativistic corrections estimated in [28] to be ~ 0.5 . The power n depends on the shape of the potential, being $n = 3$ for a Coulomb form. With $\Gamma(f_2 \rightarrow \gamma\gamma) \simeq 3$ keV [2], our calculated σ width is reproduced by $n \simeq 0.3-1$, perhaps reflecting the long range nature of the binding needed for this scalar. Such a state, which is very short lived (its total width is ~ 550 MeV), inevitably has multi-quark components in its Fock space. However, these may well be more diffuse than any $\bar{q}q$ component and so less able to annihilate readily into photons. In keeping with this, Achasov [29] and Narison [30] predict tetraquark states to have tenths of keV as radiative widths. Thus in terms of simple components a 4 keV width is difficult to reconcile with either a tetraquark or glueball composition, and points to a conventional $\bar{u}u, \bar{d}d$ structure. Hopefully the present result will motivate dynamical calculations in strong coupling QCD to detail the Fock space decomposition of the σ : $\bar{q}q, \bar{q}q\bar{q}q, gg, \pi\pi$, etc. Comparison with the 4 keV width deduced here will teach us whether this enigmatic scalar does indeed couple to photons through its $\bar{q}q$ components.

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