Unusual Microwave Response of Dirac Quasiparticles in Graphene

V. P. Gusynin,[1] S. G. Sharapov,[2] and J. P. Carbotte[2]

[1]*Bogolyubov Institute for Theoretical Physics, Metrologicheskaya Street 14-b, Kiev, 03143, Ukraine*
[2]*Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, L8S 4M1, Canada*
(Received 9 March 2006; published 30 June 2006)

Recent experiments have proven that the quasiparticles in graphene obey a Dirac equation. Here we show that microwaves are an excellent probe of their unusual dynamics. When the chemical potential is small, the intraband response can exhibit a cusp around zero frequency Ω and this unusual line shape changes to Drude-like by increasing the chemical potential $|\mu|$, with width linear in μ. The interband contribution at $T = 0$ is a constant independent of Ω with a lower cutoff at 2μ. Distinctly different behavior occurs if interaction-induced phenomena in graphene cause an opening of a gap Δ. At a large magnetic field B, the diagonal and Hall conductivities at small Ω become independent of B but remain nonzero and show a structure associated with the lowest Landau level. This occurs because in the Dirac theory the energy of this level, $E_0 = \pm\Delta$, is field independent in sharp contrast to the conventional case.

DOI: 10.1103/PhysRevLett.96.256802 PACS numbers: 73.50.Mx, 73.43.Qt, 81.05.Uw

The band structure of graphene (a single atomic layer of graphite) consists of two inequivalent pairs of the Dirac cones with an apex at the hexagonal Brillouin zone corners [1]. For a zero chemical potential μ one cone of each pair is full and the other empty. Through application of a gate voltage [2] the upper unoccupied cone can be populated with electrons and by reversing the sign of the bias, holes can be introduced in the lower energy cone. Since a 2 + 1 dimensional Dirac equation governs the dynamics of quasiparticles in graphene [3] many of its properties differ significantly from those of other materials. The most striking example is an unconventional quantization of the Hall conductivity predicted in Refs. [4,5], unaware of the experiments reported later in Refs. [6,7], which is related to the anomalous properties of the lowest Landau level in Dirac theory.

Until now, most experimental (see, e.g., Refs. [2,6–9]) and theoretical studies on graphene have been made for dc transport properties. In this Letter we study the microwave response of Dirac quasiparticles and show that it has several anomalous properties both in a strong magnetic field applied perpendicularly to the graphene plane and in its absence. We argue that an experimental investigation of these features can provide added support to the existing consensus that the carriers in graphene are Dirac-like, but also shed light on not yet fully understood questions such as the character of impurity scattering and the presence of interaction-induced phenomena. General expressions for the real part of the longitudinal $\sigma_{xx}(\Omega)$ and Hall $\sigma_{xy}(\Omega)$ conductivity for graphene at any temperature T and magnetic field B are given in Ref. [10]. They will not be repeated here although all numerical results presented are based on these exact expressions. To interpret these results we derive simple analytical expressions for $\sigma_{xx}(\Omega)$ and $\sigma_{xy}(\Omega)$ that allow us to elucidate the specifics of graphene.

We begin with the simplest case of *massless Dirac quasiparticles in zero field* which corresponds to the continuum limit of noninteracting quasiparticles on a hexagonal lattice. The corresponding band structure is shown in Fig. 1(a).

In the limit of small impurity scattering rate $\Gamma(\omega)$ neglecting the real part of the impurity self-energy, $\sigma_{xx}(\Omega, T)$ takes a particularly simple form

$$\sigma_{xx}(\Omega,T) = \frac{e^2 N_f}{2\pi^2\hbar}\int_{-\infty}^{\infty} d\omega \frac{[n_F(\omega) - n_F(\omega')]}{\Omega}\frac{\pi}{4\omega\omega'} \times \left[\frac{2\Gamma(\omega)}{\Omega^2 + 4\Gamma^2(\omega)} - \frac{2\Gamma(\omega)}{(\omega+\omega')^2 + 4\Gamma^2(\omega)}\right] \times (|\omega| + |\omega'|)(\omega^2 + \omega'^2), \qquad \omega' = \omega + \Omega, \tag{1}$$

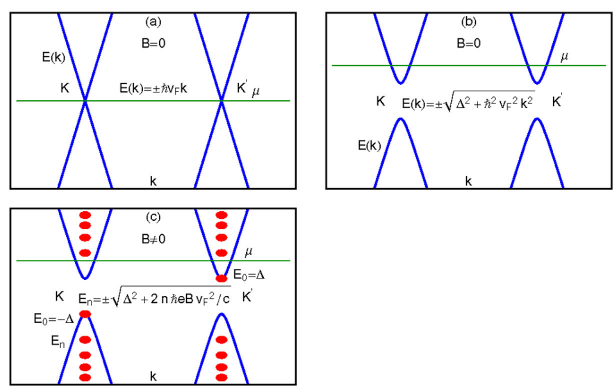

FIG. 1 (color online). Band structure of graphene. (a) The low-energy linear-dispersion $E(\mathbf{k})$ near the Dirac $\mathbf{K}$ and $\mathbf{K}'$ points for $B = 0$. (b) A possible modification of the quasiparticle spectrum by the finite gap (Dirac mass) Δ due to interaction-induced phenomena [19,20]. The chemical potential (indicated by the horizontal line) μ is shifted from zero by the gate voltage [2]. (c) Landau levels E_n in the Dirac theory of graphene. For a given direction of the magnetic field $\mathbf{B}$ applied perpendicularly to graphene's plane the lowest ($n = 0$) Landau level has the energy $E_0 = -\Delta$ at $\mathbf{K}$ and $E_0 = \Delta$ at $\mathbf{K}'$. The presence of a field independent $n = 0$ level causes an unconventional Hall effect and anomalies in the strong field microwave response.

where $n_F(\omega) = 1/[e^{(\omega-\mu)/T} + 1]$ is the Fermi distribution (we set $k_B = 1$) and $N_f = 2$ is the number of spin components. The first term in square brackets in the second line of Eq. (1) describes the intraband transitions and the second one interband. Similar expressions for the intraband contribution have already appeared in the literature on d-wave superconductivity [11,12] and for the interband case in the studies on d-density waves [13–15]. An essential feature of Eq. (1) is that we keep the energy dependence of $\Gamma(\omega)$. In deriving this equation we have assumed the small Ω limit, so that $\Gamma(\omega') \simeq \Gamma(\omega)$ and for $\Omega \ll T$ the difference $[n_F(\omega) - n_F(\omega')]/\Omega$ can be replaced by the derivative $-\partial n_F(\omega)/\partial\omega$.

Let us consider for a moment only the intraband term of Eq. (1)

$$\sigma_{xx}(\Omega, T) = \sigma_{00}\int_{-\infty}^{\infty} d\omega\left(-\frac{\partial n_F(\omega)}{\partial\omega}\right)\frac{2\pi|\omega|\Gamma(\omega)}{\Omega^2 + 4\Gamma^2(\omega)}, \tag{2}$$

with $\sigma_{00} = e^2 N_f/(2\pi^2\hbar)$. If we take $\mu = 0$ and assume $\Gamma(\omega) = \gamma_{00} + \alpha|\omega|$ with a small value of γ_{00} as expected in Born approximation in the weak impurity scattering limit, we can get an approximate analytical expression

$$\sigma_{xx}(\Omega, T) \simeq \frac{\pi\sigma_{00}}{2\alpha}\left[1 - \frac{\pi}{8\alpha}\frac{\Omega}{T}\right], \qquad \gamma_{00} < \Omega \ll T. \tag{3}$$

Such an equation has been recently used in Ref. [12] to explain a cusplike behavior of the microwave conductivity observed [16] in high purity samples of ortho II $YBCO_{6.5}$ and some numerical results for graphene are presented in Ref. [17]. This shows that if a cusplike behavior is observed in graphene, this would indicate that Born approximation is relevant. If, however, the chemical potential is increased so that $|\mu| > T$ Eq. (3) is replaced by

$$\sigma_{xx}(\Omega, T) = \sigma_{00}2\pi|\mu|\frac{(\gamma_{00} + \alpha|\mu|)}{\Omega^2 + 4(\gamma_{00} + \alpha|\mu|)^2}, \tag{4}$$

which has a Drude form. Here the factor of $|\mu|$ gives the amount of spectral weight which is distributed according to a Drude form with scattering rate $\gamma_{00} + \alpha|\mu|$. The width of the Drude peak can be increased simply by changing the gate voltage, $\mu \propto \mathrm{sgn}V_g\sqrt{|V_g|}$ with impurity scattering left the same. What has happened is that $\Gamma(\omega)$ near $\omega = \mu$ rather than near $\omega = 0$ has now become the relevant quantity and it increases with $|\mu|$. It is important to note that the frequency dependence of $\Gamma(\omega)$ in the more general case could be mapped out in this way, because the shape of $\Gamma(\omega)$ gets reflected in the shape of $\sigma_{xx}(\Omega)$. For example, in the unitary limit of a T-matrix approximation the dependence of $\Gamma(\omega)$ is completely different from Born limit and is inversely proportional to the electron density of states [5]. This dependence in turn modulates the Ω dependence of the intraband term.

Next we return to Eq. (1) and consider the strict $\Gamma = 0$ limit and $T = 0$ when it reduces to (see Ref. [13])

$$\sigma_{xx}(\Omega) = \frac{\pi e^2 N_f}{h}|\mu|\delta(\Omega) + \frac{\pi e^2 N_f}{4h}\theta\left(\frac{|\Omega|}{2} - |\mu|\right). \tag{5}$$

For $\mu = 0$ there is no intraband Drude contribution proportional to $\delta(\Omega)$. Also the interband contribution extends from zero to the band edge (assumed very large). It is independent of energy Ω and in the bare bubble approximation used to derive Eqs. (1) and (5), is equal to $\pi e^2 N_f/(4h)$ in height [17]. The flatness of this response can be traced to the topology of the Dirac cone and the linear in momentum dependence of the energy of the Dirac quasiparticles. As μ increases, the interband transitions become gapped at $\Omega = 2|\mu|$ and the missing spectral weight reappears in the $\delta(\Omega)$-function contribution.

A nonlinear behavior of the diagonal magnetoresistivity, ρ_{xx} in Kish graphite in high magnetic field has been observed and interpreted in Ref. [18] in terms of the formation of the new charge-density-wave phase. Also as predicted in Refs. [19,20], an interaction-induced phenomenon may cause an opening of an excitonic gap [see Fig. 1(b)] in the quasiparticle spectrum. Depending on the model parameters, the gap opens in zero or in a finite magnetic field. Recent measurements [21] of the Hall resistivity made on highly oriented pyrolytic graphite indicate that there is a gap at $B = 0.03$ T. In our opinion, if the value of the gap is expected to be the order of a few Kelvin, microwaves are the best technique to detect its existence. In this instance, for *massive Dirac quasiparticles in zero field* Eq. (5) acquires the form

$$\sigma_{xx}(\Omega) = \frac{\pi e^2 N_f}{h}\delta(\Omega)\frac{(\mu^2 - \Delta^2)\theta(\mu^2 - \Delta^2)}{|\mu|} + \frac{\pi e^2 N_f}{4h}\frac{\Omega^2 + 4\Delta^2}{\Omega^2}\theta\left(\frac{|\Omega|}{2} - \max(|\mu|, \Delta)\right). \tag{6}$$

Here the theta function $\theta(|\Omega|/2 - \max(|\mu|, \Delta))$ cuts off the low Ω part of the interband contribution at $2|\mu|$ or 2Δ, whichever is the largest. The additional factor $(\Omega^2 + 4\Delta^2)/\Omega^2$ is to be noted and allows one to distinguish in $\sigma(\Omega)$ the case of finite Δ from finite μ. As we saw in relation to Eq. (5) when $\Delta = 0$ the interband contribution remains flat although it cuts off at $2|\mu|$. For finite Δ this is no longer the case and for $|\mu| < \Delta$ the interband spectral weight at the gap has been increased by a factor of 2. This is a signature of a gap opening and effectively represents a change in the geometry of the Dirac cones as illustrated in Fig. 1 [from Fig. 1(a) to 1(b)].

In Fig. 2 we show results for the microwave conductivity $\sigma_{xx}(\Omega, T)$ vs Ω at finite temperature. The long dashed curve for $T = 1$ K shows a Drude peak at small Ω (intraband) superimposed on a interband contribution of $\pi e^2/(2h)$ constant independent of Ω except for some depletion below 200 GHz due to finite temperature with optical spectral weight transferred to the Drude peak. When T is increased to 5 K (solid curve), the thermal depletion increased considerably with increased weight in the Drude peak. On the other hand, if instead of increasing T we open a gap $\Delta = 5$ K (the dash-dotted curve), the Drude peak is almost completely depleted as is the region below $2\Delta \simeq 200$ GHz with extra optical weight accumulating at and above 2Δ in the interband region, where it is distributed over the region of $\Omega \gtrsim (2 \div 4)\Delta$. The peak at $\Omega = 2\Delta$ in $\sigma_{xx}(\Omega)$ is characteristic of the opening of a gap as we have already described.

Massless and massive Dirac quasiparticles in a strong field.—The latest measurements in graphene samples [22] show that at $B = 25$ T near $\mu = 0$ the diagonal and Hall dc magneto-resistances do not conform to the standard quantum Hall observation. Here we target the microwave response in the same region. When a magnetic field B is applied, the expressions for the conductivity become very complicated as given in Ref. [10]. In the very high field limit the Landau levels with $n \neq 0$ can all be shifted to high energy, but as we have already illustrated in Fig. 1(c) this is not the case for $n = 0$ level. This level may only depend on the field indirectly if the gap $\Delta(B, \mu)$ is induced by the field [20] (the phenomenon of magnetic catalysis [23]), so that it remains well below the rest of the levels. If we take B very large, only the contribution $\sim 1/B$ associated with the transitions from $n = 0$ to $n = 1$ levels survives and after multiplying it by the degeneracy factor of the Landau levels $\sim B$, we arrive at a very simple formula

$$\sigma_{xx}(\Omega, T) = \frac{e^2 N_f \Gamma}{2\pi h \Omega} \mathrm{Im}\left[\Psi\left(\frac{\Gamma + i(\mu + \Omega + \Delta)}{2\pi T} + \frac{1}{2}\right) - \Psi\left(\frac{\Gamma + i(\mu - \Omega + \Delta)}{2\pi T} + \frac{1}{2}\right) + (\Delta \to -\Delta)\right], \tag{7}$$

where ψ is the digamma function. Here for simplicity we neglected the Zeeman splitting in the strong field. Equation (7), which is applicable for an arbitrary relationship between Ω, T, and Γ, shows that the diagonal conductivity $\sigma_{xx}(\Omega, T)$ is independent of B, but the position in energy of the two inequivalent $n = 0$ Landau orbitals, namely $\mu - \Delta(B, \mu)$ and $\mu + \Delta(B, \mu)$, measured relative to the chemical potential [see Fig. 1(c)] remains in the problem. Results based on calculation of the full expression for $\sigma_{xx}(\Omega, T)$ from Ref. [10] are shown in Fig. 3 and we have verified that they are in a good agreement with the approximate expression (7).

There are four curves. Comparison of solid and long dashed curves shows that the large field limit is already attained for $B = 0.05$ T. Here the gap $\Delta = 0$ and the chemical potential equal to 6 K is clearly identifiable at 120 GHz, where a large rise smeared by temperature (0.5 K) and impurities $\Gamma = 0.5$ K is seen in σ_{xx}. When a gap develops, however, the shape of the curve is completely changed. In both dash-dotted and short dashed curves, structure at $\mu - \Delta$ and $\mu + \Delta$ is clearly visible reflecting that the energy of the $n = 0$ Landau level at **K** point is Δ and at **K**$'$ point is $-\Delta$, respectively. This is yet another clear signature of the Dirac character of the quasiparticles in graphene which can be used to detect the gap. Indeed, since in large fields all changes in the shape of $\sigma_{xx}(\Omega)$ are caused by the gap, one may consider that if the gap $\Delta(B)$ develops as the field increases, the curves with $\Delta = 0$, $\Delta = 3$ K, and $\Delta = 7$ K shown in Fig. 3 would actually show up as the field increases.

Finally we consider the microwave Hall conductivity which can provide more insight into the properties of graphene. Under the same conditions as were used to derive Eq. (7), the microwave Hall conductivity is given by

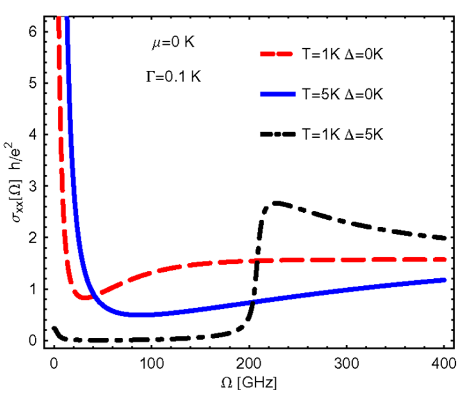

FIG. 2 (color online). The microwave conductivity $\sigma_{xx}(\Omega, T)$ in units e^2/h vs microwave frequency Ω in GHz. The long dashed line is for $T = 1$ K and the solid for $T = 5$ K. In both cases $\mu = \Delta = 0$ and $\Gamma = 0.1$ K. The dash-dotted curve has a gap $\Delta = 5$ K and $T = 1$ K.

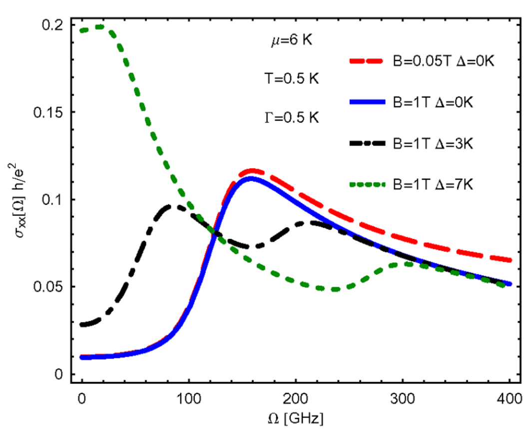

FIG. 3 (color online). Conductivity $\sigma_{xx}(\Omega, T)$ in units e^2/h vs microwave frequency Ω in GHz. The chemical potential is set at 6 K, the temperature at 0.5 K, and the impurity scattering rate is assumed constant set at 0.5 K. Long dashed curve $B = 0.05$ T and $\Delta = 0$, solid curve $\Delta = 0$, dash-dotted curve $\Delta = 3$ K, and short dashed $\Delta = 7$ K. The three last curves are all for $B = 1$ T.

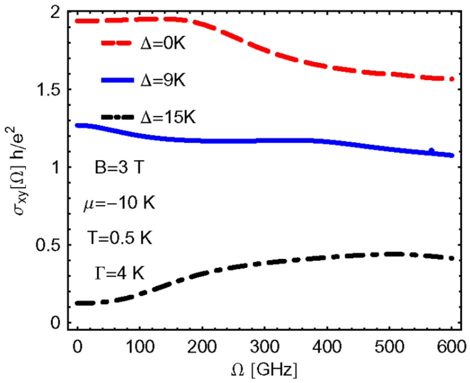

FIG. 4 (color online). Conductivity $\sigma_{xy}(\Omega, T)$ in units e^2/h vs microwave frequency Ω in GHz. The chemical potential is set at -10 K, the temperature at 0.5 K and the impurity scattering rate assumed constant set at 4 K. The gap Δ is also indicated in the figure.

$$\sigma_{xy}(\Omega) = -\frac{e^2 N_f \mathrm{sgn}(eB)}{\pi h}\left\{\mathrm{Im}\left[\Psi\left(\frac{\Gamma + i(\mu+\Delta)}{2\pi T}+\frac{1}{2}\right) + (\Delta \to -\Delta)\right] + \frac{\Gamma}{2\Omega}\mathrm{Re}\left[\Psi\left(\frac{\Gamma + i(\mu+\Omega+\Delta)}{2\pi T}+\frac{1}{2}\right) - \Psi\left(\frac{\Gamma + i(\mu-\Omega+\Delta)}{2\pi T}+\frac{1}{2}\right) + (\Delta \to -\Delta)\right]\right\}. \quad (8)$$

In Fig. 4 we show results for the frequency dependence of the Hall conductivity in units of e^2/h. As found in Ref. [10] the opening of a gap decreases the dc value of the Hall conductivity. For the case $\Delta = 0$ (dashed line) there is additionally structure at $\Omega \approx |\mu|$, while for Δ finite the equivalent structure is moved to energies $|\mu| + \Delta$ and $|\mu| - \Delta$ [see Fig. 1(c)] as we have seen in the case of diagonal conductivity.

Up to now we have considered the influence of opening a gap on the conductivity when it is set constant. A more interesting possibility is to explore the dependence of the gap Δ on μ and B by changing the gate voltage or the magnetic field. It is expected that a decrease in μ or an increase of B favors the opening of the gap when $|\mu| < \mu_c$ or $B > B_c$, as was predicted in Ref. [20]. The important conclusion of our studies is that the generation of an excitonic gap should lead to a new insulator phase around the point $\mu = 0$ or in very large magnetic fields and this would be one more striking difference of quantum Hall effect in graphene from standard semiconductors where the fractional quantum Hall effect is instead observed when entering the lowest Landau level.

In summary, the microwave response of graphene shows many distinct characteristics which reflect the Dirac nature of the quasiparticles. These include intraband and interband contributions to the conductivity in zero magnetic field and microwave response at large fields. While in conventional case large B would shift all Landau levels to high energies above the microwave region for the Dirac case the lowest Landau level contribution remains unshifted and affects $\sigma_{xx}(\Omega)$ and $\sigma_{xy}(\Omega)$ in the small Ω region in a distinct way which can be used to study the effect of a gap opening induced by the magnetic field.

We thank Yu. G. Pogorelov for useful discussion. The work of V. P. G. was supported by the SCOPES Project No. IB7320-110848 of the Swiss NSF. J. P. C. and S. G. Sh. were supported by the Natural Science and Engineering Research Council of Canada (NSERC) and by the Canadian Institute for Advanced Research (CIAR).

[1] P. R. Wallace, Phys. Rev. **71**, 622 (1947).
[2] K. S. Novoselov *et al.*, Science **306**, 666 (2004).
[3] G. W. Semenoff, Phys. Rev. Lett. **53**, 2449 (1984).
[4] V. P. Gusynin and S. G. Sharapov, Phys. Rev. Lett. **95**, 146801 (2005).
[5] N. M. R. Peres, F. Guinea, A. H. Castro Neto, Phys. Rev. B **73**, 125411 (2006).
[6] K. S. Novoselov *et al.*, Nature (London) **438**, 197 (2005).
[7] Y. Zhang *et al.*, Nature (London) **438**, 201 (2005).
[8] C. Berger *et al.*, J. Phys. Chem. B **108**, 19 912 (2004).
[9] J. S. Bunch *et al.*, Nano Lett. **5**, 287 (2005).
[10] V. P. Gusynin and S. G. Sharapov, Phys. Rev. B **73**, 245411 (2006).
[11] P. J. Hirschfeld, W. O. Putikka, and D. J. Scalapino, Phys. Rev. B **50**, 10 250 (1994).
[12] W. Kim, F. Marsiglio, and J. P. Carbotte, Phys. Rev. B **70**, 060505(R) (2004).
[13] X. Yang and C. Nayak, Phys. Rev. B **65**, 064523 (2002).
[14] B. Valenzuela, E. J. Nicol, and J. P. Carbotte, Phys. Rev. B **71**, 134503 (2005).
[15] L. Benfatto *et al.*, Phys. Rev. B **71**, 104511 (2005).
[16] P. J. Turner *et al.*, Phys. Rev. Lett. **90**, 237005 (2003).
[17] T. Ando, Y. Zheng, and H. Suzuura, J. Phys. Soc. Jpn. **71**, 1318 (2002).
[18] Y. Iye and G. Dresselhaus, Phys. Rev. Lett. **54**, 1182 (1985).
[19] D. V. Khveshchenko, Phys. Rev. Lett. **87**, 206401 (2001); **87**, 246802 (2001).
[20] E. V. Gorbar, V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Phys. Rev. B **66**, 045108 (2002).
[21] Y. Kopelevich *et al.*, Phys. Lett. A **355**, 233 (2006).
[22] Y. Zhang *et al.*, Phys. Rev. Lett. **96**, 136806 (2006).
[23] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Phys. Rev. Lett. **73**, 3499 (1994); Phys. Rev. D **52**, 4718 (1995).