Anomalous Viscosity of an Expanding Quark-Gluon Plasma

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We argue that an expanding quark-gluon plasma has an anomalous viscosity, which arises from interactions with dynamically generated color fields. We derive an expression for the anomalous viscosity in the turbulent plasma domain and apply it to the hydrodynamic expansion phase, when the quark-gluon plasma is near equilibrium. The anomalous viscosity dominates over the collisional viscosity for weak coupling and not too late times. This effect may provide an explanation for the apparent ''nearly perfect'' liquidity of the matter produced in nuclear collisions at the Relativistic Heavy Ion Collider without the assumption that it is a strongly coupled state.

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Measurements of the anisotropic collective flow of hadrons emitted in noncentral collisions of heavy nuclei at the Relativistic Heavy Ion Collider (RHIC) are in remarkably good agreement with the predictions of ideal relativistic fluid dynamics [1]. In order to describe the data, calculations need to assume that the matter formed in the nuclear collision reaches thermal equilibrium within a time $\tau_{\rm i} < 1\ {\rm fm}/c$ [2] and then expands with a very small shear viscosity $\eta \ll s$, where s is the entropy density [3]. The comparison between data and calculations indicates that the viscosity of the matter cannot be much larger than the postulated lower bound $\eta_{\rm min} = s/4\pi$ [4], which is reached in certain strongly coupled supersymmetric gauge theories [5].

This result is nontrivial because the shear viscosity of a weakly coupled, perturbative quark-gluon plasma is not small. In fact, the perturbative result for the shear viscosity, in leading logarithmic approximation, is [6]

$$\eta_{\rm C} = \frac{d_f T^3}{g^4 \ln g^{-1}}, \tag{1}$$

where $d_f \sim O(100)$ is a numerically determined constant that weakly depends on the number of quark flavors n_f. The result (1), as well as the finding that numerical solutions of the Boltzmann equation exhibit fluid dynamical behavior only when the cross section between gluons is artificially increased by a large factor [7], have invited speculations that the matter produced at RHIC is a strongly coupled quark-gluon plasma. The possible microscopic structure of such a state is not well understood at present [8–10].

Here we present an alternative mechanism that may be responsible for a small viscosity of a weakly coupled but expanding quark-gluon plasma. The new mechanism is based on the theory of particle transport in turbulent plasmas [11,12]. Such plasmas are characterized by strongly excited random field modes in certain regimes of instability, which coherently scatter the charged particles and, thus, reduce the rate of momentum transport. The scattering by turbulent fields in electromagnetic plasmas is known to greatly increase the energy loss of charged particles [13] and reduce the heat conductivity [14,15] and the viscosity [16,17] of the plasma. Following Abe and Niu [17], we call the contribution from turbulent fields to transport coefficients ''anomalous.''

The sufficient condition for the spontaneous formation of turbulent, partially coherent fields is the presence of instabilities in the gauge field due to the presence of charged particles. This condition is met in electromagnetic plasmas with an anisotropic momentum distribution of the charged particles [18], and it is known to be satisfied in quark-gluon plasmas with an anisotropic momentum distribution of thermal partons [19–21].

Most of the work exploring the consequences of the instabilities [22–24] has been focused on the early stage of the collision, when the momentum distribution is highly anisotropic and far from equilibrium. It was pointed out that the fields generated by the instabilities will drive the parton distribution rapidly toward local isotropy and, thus, into the hydrodynamical regime [25]. Here we are concerned with the later stage of the reaction, when the matter is nearly equilibrated and evolves by hydrodynamical expansion. Because the partonic plasma expands rapidly, the momentum distribution of the partons remains anisotropic even at late times, with the size of the anisotropy being proportional to the viscosity.

As we will show, the turbulent plasma fields induce an additional, anomalous contribution to the viscosity, which we denote as $\eta_{\rm A}$. This anomalous viscosity decreases with increasing strength of the turbulent fields. Since the amplitude of the turbulent fields grows with the magnitude of the momentum anisotropy, a large anisotropy will lead to a small value of $\eta_{\rm A}$. Because the relaxation rates due to different processes are additive, the total viscosity is given by

$$\eta^{-1} = \eta_{\rm A}^{-1} + \eta_{\rm C}^{-1}. \tag{2}$$

This equation implies that $\eta_{\rm A}$ dominates the total shear viscosity, if it is smaller than $\eta_{\rm C}$. In that limit, the anomalous mechanism exhibits a stable equilibrium in which the viscosity regulates itself: The anisotropy grows with η, but an increased anisotropy tends to suppress η_A and, thus, $\eta \approx \eta_A$. We derive the resulting self-consistency condition for η_A below.

The fireballs formed in relativistic heavy-ion collisions exhibit collective expansion in both the longitudinal and the transverse directions with respect to the beam axis. Here we focus on the longitudinal expansion, but our arguments apply as well to the transverse expansion component. We assume the longitudinal flow profile during the hydrodynamic expansion phase of a relativistic heavy-ion collision to be approximately boost invariant [26] and of the form $u_z(z, t) = z/t$, where z and t are measured from the collision point in the center-of-mass frame. The velocity gradient $\partial u_z/\partial z$ leads to an anisotropy in the local momentum distribution [3,27]

$$\frac{2T_{zz}}{T_{xx} + T_{yy}} - 1 = -\frac{8}{T\tau}\frac{\eta}{s}, \tag{3}$$

where T denotes the temperature, s is the entropy density of the matter, and $\tau = \sqrt{t^2 - z^2}$ is the time in local comoving coordinates. For simplicity, we have assumed that the equation of state of the matter is that of free massless partons, $\epsilon = 3P = 3sT/4$. As (3) shows, the anisotropy is linearly dependent on the viscosity of the matter.

As already mentioned, any anisotropy of the local momentum distribution of quasithermal partons engenders instabilities of soft gluon modes in the momentum regime $k < gT$, whose growth rate increases in proportion to the anisotropy [21]. In electromagnetic plasmas, the growth of the instability eventually saturates when the increasing field modifies the particle distribution in such a way that the instability is eliminated. In non-Abelian plasmas, the nonlinear self-interactions of the gauge field restrict the growth of the unstable modes; this mechanism dominates at weak coupling [28–30]. In the quasistationary state that is reached in the nonlinear domain, energy absorbed from the thermal partons cascades from the most unstable gauge field modes into modes of increasingly shorter wavelengths. The power spectrum of this energy cascade has the form $P(k) \sim k^{-2}$, analogous to the Kolmogorov cascade in a turbulent fluid [31].

As already stated, we are concerned here with the effect of the unstable field modes on the transport properties of the medium when it has reached the collective expansion phase. Following the standard Chapman-Enskog theory of transport coefficients, we assume that the plasma is driven only slightly out of equilibrium by the collective flow and that the local phase-space distribution can be written as

$$f(\mathbf{p}, \mathbf{r}) = f_0(\mathbf{p})\{1 + f_1(\mathbf{p}, \mathbf{r})[1 \pm f_0(\mathbf{p})]\}, \tag{4}$$

where $f_0(\mathbf{p})$ is the local equilibrium distribution, $+$ $(-)$ applies to bosons (fermions), and f_1 has the form

$$f_1(\mathbf{p}, \mathbf{r}) = -\frac{\bar{\Delta}}{2ET^2} p_i p_j \Delta_{ij}(u). \tag{5}$$

Here $\Delta_{ij}(u) = (\nabla_i u_j + \nabla_j u_i - \frac{2}{3}\delta_{ij}\boldsymbol{\nabla}\cdot\mathbf{u})$ is the traceless part of the flow gradient related to the shear viscosity, and $\bar{\Delta}$ parametrizes the strength of the anisotropy. For massless particles, $\bar{\Delta}$ is related to the macroscopic transport coefficient of shear viscosity by $\bar{\Delta} = 5\eta/s$. The boost invariant longitudinal expansion corresponds to the perturbation

$$f_1(\mathbf{p}) = -\frac{\bar{\Delta}}{3ET^2\tau}(3p_z^2 - p^2). \tag{6}$$

In order to explore the response of the plasma to this perturbation, we need to determine the influence of the saturated field modes on the propagation of thermal plasma particles. The relevant transport theory was developed by Dupree [11,12] for an electromagnetic plasma in the limit of strong turbulence and weak coupling. We now generalize this formalism to a non-Abelian plasma. Our starting point is the Vlasov-Boltzmann equation for the phase-space distribution of color charges Q^a in a color-magnetic field $\mathbf{B}^a$:

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}_r + \mathbf{F}\cdot\boldsymbol{\nabla}_p\right] f(\mathbf{r}, \mathbf{p}, t) = C[f], \tag{7}$$

where $\mathbf{v} = \mathbf{p}/E$ is the velocity of a thermal parton with momentum $\mathbf{p}$ and energy E, $\mathbf{F} = gQ^a(\mathbf{E}^a + \mathbf{v}\times\mathbf{B}^a)$ is the color Lorentz force, and $C[f]$ denotes the collision term. We focus here on the effects of the Vlasov term and refer to the results of Arnold, Moore, and Yaffe [6] for the viscosity due to incoherent collisions. Because transverse field modes have the highest growth rates in the linear regime and transverse color-electric fields are less effective in restoring the particle distribution to isotropy, we concentrate here on the effects due to coherent color-magnetic field modes. The effects of longitudinal electric field modes, which are strongly excited in the nonlinear domain and can also lead to isotropy, will be discussed in a forthcoming longer publication [32].

In order to isolate the dissipative effects of the color field, one averages the particle trajectories over an ensemble of color-magnetic fields. Assuming that $\langle B^a\rangle = 0$ and factorizing higher than second moments of the field distribution, one can then show that the ensemble averaged phase-space distribution $\bar{f}$ satisfies an equation of the Fokker-Planck type [11]:

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}_r - \boldsymbol{\nabla}_p D(\mathbf{p}, t)\boldsymbol{\nabla}_p\right]\bar{f} = C[\bar{f}], \tag{8}$$

with the diffusion tensor

$$D_{ij} = \int_{-\infty}^{t} dt' \langle F_i(\bar{r}(t'), t') F_j(\bar{r}(t), t)\rangle. \tag{9}$$

Dupree's treatment [11] is based on the argument that the Fourier components of the coherent field $B(k, \omega_k)$ are slowly varying functions, and the autocorrelation function of the Lorentz force in (9) is determined by the action of the magnetic field on the particle trajectories. This leads to a self-consistency condition for the mean deviation $\langle \Delta \bar{r}^2 \rangle$ of the particle trajectories from straight lines. Because the growth and coherence of the non-Abelian gauge field (at weak coupling) is not controlled by the backreaction of the distorted particle trajectories on the gauge field, but by the inherent nonlinearities of the gauge field itself [23], we adopt here a different approach. The velocity of propagation of the collective modes of the non-Abelian plasma is less than the speed of light, while the thermal particles move (nearly) at the speed of light. The autocorrelation function of the color-magnetic field along the path of a particle will, thus, be controlled by the spatial correlation length of the fields created by the growth of the unstable modes. Assuming that the correlation length for the color-magnetic fields is short in comparison with the curvature radius of the trajectory of a plasma particle, we can then take $\bar{v}(t) = \bar{v}(t') = v$ out of the average in (9) and are left with the autocorrelation function of the magnetic field along a typical particle trajectory:

$$\int_{-\infty}^{t} dt' \langle B_i^a(t') B_j^b(t) \rangle \equiv \langle B_i^a B_j^b \rangle \tau_{\mathrm{m}}. \qquad (10)$$

In our case, the color-magnetic fields generated by the plasma instability point in a transverse direction with respect to the beam. Assuming that the ensemble average is diagonal in color and employing the notation $\mathbf{L}^{(p)} = -i\mathbf{p} \times \nabla_p$, we can write the diffusive term as

$$\nabla_p D(\mathbf{p}) \nabla_p = -\frac{g^2 Q^2}{2(N_c^2 - 1)E^2} \langle B^2 \rangle \tau_{\mathrm{m}} (L_\perp^{(p)})^2, \qquad (11)$$

where $N_c = 3$ is the number of colors and the index $\perp$ denotes the components transverse to the beam axis.

The action of the diffusion operator on $\bar{f}$ is evaluated easily by noting that the perturbation $f_1(\mathbf{p})$ has quadrupole form. In order to derive the anomalous viscosity due to the diffusion term, we follow Abe and Niu [17] and take moments of the drift and diffusion terms in (8) with p_z^2. Using massless quarks and gluons in the momentum integrals, we obtain:

$$\int \frac{d^3p}{(2\pi)^3 E} p_z^2 \mathbf{v} \cdot \nabla_r \bar{f}(\mathbf{p}) = \frac{1}{T\tau} \frac{16\nu_4 \zeta(4)}{15\pi^2} T^5, \qquad (12)$$

$$\int \frac{d^3p}{(2\pi)^3 E} p_z^2 \nabla_p D(\mathbf{p}) \nabla_p \bar{f}(\mathbf{p}) = \frac{1}{T\tau} \frac{\bar{\Delta} g^2 \langle B^2 \rangle \tau_{\mathrm{m}}}{(N_c^2 - 1)} \times \frac{4N_c \nu_2' \zeta(2)}{15\pi^2} T^2, \qquad (13)$$

where

$$\nu_N = 16 + 12(1 - 2^{-N}) n_f, \qquad (14)$$

$$\nu_N' = 16 + 6(1 - 2^{-N}) n_f (N_c^2 - 1)/N_c^2, \qquad (15)$$

and we have used $Q^2 = N_c$ for gluons and $Q^2 = (N_c^2 - 1)/(2N_c)$ for quarks. Equating the two results and using the relation $\bar{\Delta} = 5\eta/s$, we obtain the sought after expression for the anomalous shear viscosity due to the coherent color-magnetic fields:

$$\eta_{\mathrm{A}} = \frac{4(N_c^2 - 1)\nu_4 \zeta(4)}{5N_c \nu_2' \zeta(2)} \frac{sT^3}{g^2 \langle B^2 \rangle \tau_{\mathrm{m}}}. \qquad (16)$$

It is noteworthy that the right-hand side of (16) itself depends implicitly on the viscosity, because the intensity of the turbulent fields grows with increasing anisotropy of the momentum distribution in the plasma.

Next, we need to address the question how large $\langle B^2 \rangle$ and τ_{m} are. The coherent color-magnetic fields are only generated by the plasma instability when the momentum distribution of partons in the quark-gluon plasma is deformed due to the collective expansion. We know from analytical studies how the growth rate of the instability depends on the anisotropy of the momentum distribution, but there are no published systematic studies that show how the saturation level of the coherent field energy depends on the anisotropy.

The study by Romatschke and Strickland [21] expresses the anisotropy in terms of a parameter ξ and a unit vector $\hat{n}$. Choosing $\hat{n} = \hat{e}_z$, this ansatz corresponds to a perturbation of the equilibrium distribution of the form (6) with $\bar{\Delta} = \xi T\tau/2$. The average intensity of the coherent color-magnetic fields is a function of the momentum anisotropy. For lack of a precise knowledge of this function, we here parametrize it as a power law: $g^2 \langle B^2 \rangle = b_0 g^4 T^4 \xi^{b_1}$, and conjecture a linear dependence ($b_1 = 1$). We argued above that the scale for the memory time τ_{m} for the non-Abelian plasma will be set by the spatial coherence length of the coherent fields. This coherence length is given by the wavelength of the maximally unstable mode, which is of the order of the Debye length: $\tau_{\mathrm{m}} \sim \mu_D^{-1} \sim (gT)^{-1}$, where the last form assumes weak coupling. We currently lack a precise determination of τ_{m}, which could be obtained from numerical solutions of the Yang-Mills equations for the anisotropy discussed here.

Using the relations between ξ, $\bar{\Delta}$, and η, we can now state the dependence of the right-hand side of (16) on the viscosity:

$$g^2 \langle B^2 \rangle \tau_{\mathrm{m}} = 10 b_0 g^3 \frac{T^2 \eta}{s\tau}. \qquad (17)$$

Note that η here is the viscosity due to all sources of dissipation, including collisions. If we neglect the viscosity η_{C} due to particle collisions and set $\eta = \eta_{\mathrm{A}}$ on the right-hand side of (17), Eq. (16) yields a self-consistency condition for the anomalous viscosity η_{A}, which has the solution:

$$\eta_{\mathrm{A}} = \left(\frac{2(N_c^2 - 1)\nu_4 \zeta(4) T\tau}{25 b_0 N_c \nu_2' \zeta(2)} \right)^{1/2} \frac{s}{g^{3/2}}. \qquad (18)$$

Several things are notable about this result. First, if the memory time τ_m is longer than our estimate, the value of η_A decreases. Second, the dependence of η_A on the gauge coupling is parametrically much weaker than that of the collisional viscosity (1). Thus, for weak coupling $g \ll 1$ and not too late times τ, the anomalous viscosity will be much smaller than the collisional viscosity. According to (2), this implies that η_A is dominant at early times, and the collisional viscosity η_C may dominate at large times. The crossover time τ_c between the two regimes is given by the condition $\eta_A = \eta_C$. Because of the failure of the perturbative result for η_C in the domain $g \geq 1$, it is difficult to give a reliable estimate of the crossover time between the two regimes. Ignoring those limitations and assuming $b_0 \sim O(1)$, one can surmise that the anomalous viscosity dominate for a few fm$/c$ in heavy-ion collisions at RHIC or LHC energies. At late times, the transverse expansion of the medium also needs to be taken into account. Assuming a radial dependence of the form $u_r(r) \approx \beta_0 r/R$, where R is the nuclear radius, the anisotropy due to transverse flow is

$$p_i p_j \Delta_{ij}(u) = -\frac{2\beta_0}{3R}(3p_z^2 - p^2). \tag{19}$$

Comparing with (6), one sees that the effects from the radial expansion become comparable to those from the longitudinal expansion when $\beta_0 \sim R/\tau$. The anomalous contribution to the viscosity may, therefore, never be negligible during the lifetime of the plasma phase. We note that collisions among thermal particles may suppress the Weibel instability when the coupling constant α_s exceeds a threshold value [33]. It is presently unknown whether this occurs before or after the crossover between η_A and η_C for experimentally relevant conditions.

The approach outlined here can be used to derive other anomalous transport properties of an expanding, turbulent quark-gluon plasma. Maybe the most important among these are the coefficient $\hat{q}$ of radiative energy loss of an energetic parton [34], which might be increased by scattering on turbulent fields.

In summary, we have shown that an expanding quark-gluon plasma acquires an anomalous viscosity due to the interaction of thermal partons with chromomagnetic fields generated by instabilities of soft field modes. In the weak coupling limit, the anomalous viscosity is much smaller than the viscosity due to collisions among thermal partons. By reducing the shear viscosity of a weakly coupled but expanding quark-gluon plasma, this mechanism could possibly explain the observations of the RHIC experiments without the assumption of a strongly coupled plasma state. A definitive answer will require the numerical evaluation of the correlation function (10) as a function of the anisotropy parameter $\bar{\Delta}$.

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