Regular Phantom Black Holes

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We study self-gravitating, static, spherically symmetric phantom scalar fields with arbitrary potentials (favored by cosmological observations) and single out 16 classes of possible regular configurations with flat, de Sitter, and anti–de Sitter asymptotics. Among them are traversable wormholes, bouncing Kantowski-Sachs (KS) cosmologies, and asymptotically flat black holes (BHs). A regular BH has a Schwarzschild-like causal structure, but the singularity is replaced by a de Sitter infinity, giving a hypothetic BH explorer a chance to survive. It also looks possible that our Universe has originated in a phantom-dominated collapse in another universe, with KS expansion and isotropization after crossing the horizon. Explicit examples of regular solutions are built and discussed. Possible generalizations include k-essence type scalar fields (with a potential) and scalar-tensor gravity.

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Observations provide more and more evidence that the modern accelerated expansion of our Universe is governed by a peculiar kind of matter, called dark energy (DE), characterized by negative values of the pressure to density ratio w. By current estimates, even $w < -1$ seems rather likely [1–6], though many estimates are model dependent. Thus, assuming a perfect-fluid DE with $w = \text{const}$ implies, using combined data from CMB, type Ia supernovae, and large-scale structure, $-1.39 < w < -0.79$ at 2σ level [4]. Similar values are obtained assuming a perfect fluid with variable w [1,2]. A model-independent study [5] of data sets from 172 SNIa showed a preferable range $-1.2 < w < -1$ for the recent epoch. An analysis of the Chandra telescope observations of hot gas in 26 x-ray luminous dynamically relaxed galaxy clusters [6] gives $w = -1.20^{+0.24}_{-0.28}$.

Moreover, a highly negative w makes negligible the undesirable DE contribution to the total energy density in the period of structure formation. Thus, even if the cosmological constant, giving precisely $w = -1$, is still admitted by observations as possible DE, there is a need for a more general framework allowing $w < -1$.

The perfect-fluid description of DE is plagued with instability at small scales due to imaginary velocity of sound; more consistent descriptions providing $w < -1$ use self-interacting scalar fields with negative kinetic energy (phantom scalars) or tachyonic fields [7–9] (see also references therein). To avoid the obvious quantum instability, a phantom scalar may perhaps be regarded as an effective field description following from an underlying theory with positive energies [10]. Curiously, a classical massless phantom field even shows a more stable behavior than its usual counterpart [11,12]. A fundamental origin of phantom fields is under discussion, but they naturally appear in some models of string theory [7], supergravities [13], and theories in more than 11 dimensions like F theory [14].

If a phantom scalar, be it basic or effective, is part of the real field content of our Universe, it is natural to seek its manifestations not only in cosmology but also in local phenomena, in particular, in black hole (BH) physics, as, e.g., in the recent works on DE accretion onto BHs [15,16] and BH interaction with a phantom shell [17].

We here try to find out which kinds of regular static, spherically symmetric configurations may be formed by a phantom scalar field itself. Since it violates the usual energy conditions, regular solutions of interest for BH physics and/or cosmology are expected. Our main finding is, in our view, the existence of regular asymptotically flat BH solutions with an expanding, asymptotically de Sitter Kantowski-Sachs (KS) cosmology beyond the event horizon. It is, to our knowledge, quite a new way of avoiding a BH singularity, alternative to the known type of regular BH solutions which possess a regular center [see, e.g., [18–20]].

After writing the field equations, we will mention some no-go theorems for both normal and phantom scalars (without proofs), making sure that they leave sufficient freedom for solutions of interest. Then follows a simple qualitative analysis which reveals 16 classes of possible nonsingular solutions and the properties of potentials needed for their existence. Using the inverse problem method, we construct simple explicit examples, including, among others, regular bouncing KS cosmologies and regular black holes, asymptotically flat or anti–de Sitter (AdS) in the static (R) region and asymptotically de Sitter in the nonstatic (T) region. Some generalizations of these results are indicated in conclusion.

We start with the action for a self-gravitating scalar field with an arbitrary potential $V(\phi)$

$$S = \int \sqrt{g} d^4x [R + \varepsilon g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2V(\phi)], \quad (1)$$

where R is the scalar curvature, $\varepsilon = +1$ corresponds to a usual scalar field with positive kinetic energy, and $\varepsilon = -1$ to a phantom field. For the general static, spherically symmetric metric

$$ds^2 = A(\rho)dt^2 - \frac{d\rho^2}{A(\rho)} - r^2(\rho)(d\theta^2 + \sin^2 d\varphi^2), \quad (2)$$

and $\phi = \phi(\rho)$, the scalar field equation and three independent combinations of the Einstein equations read

$$(Ar^2\phi')' = \varepsilon r^2 dV/d\phi, \quad (3)$$

$$(A'r^2)' = -2r^2 V; \quad (4)$$

$$2r''/r = -\varepsilon \phi'^2; \quad (5)$$

$$A(r^2)'' - r^2 A'' = 2, \quad (6)$$

where the prime denotes $d/d\rho$. Equation (3) follows from (4)–(6), which, given a potential $V(\phi)$, form a determined set of equations for the unknowns $r(\rho)$, $A(\rho)$, $\phi(\rho)$. Equation (6) can be integrated giving

$$B' \equiv (A/r^2)' = 2(\rho_0 - \rho)/r^4, \quad (7)$$

where $B(\rho) = A/r^2$ and ρ_0 is an integration constant.

The coordinate ρ is chosen so that Killing horizons, if any, correspond to regular zeros of the function $A(\rho)$: $A(\rho) \approx (\rho - \rho_h)^p$, where $p \in \mathbb{N}$ is the order of the horizon [11]. The metric is static where $A(\rho) > 0$ (in R regions), while where $A < 0$ (in T regions) ρ is a time coordinate, and (2) describes a homogeneous anisotropic KS cosmology.

Some general consequences of Eqs. (3)–(5) (no-go theorems) constrain the nature of possible solutions. (A) For $\varepsilon = +1$ one cannot obtain wormholes or configurations ending with a regular 3-cylinder of finite radius r [21]. This result follows from Eq. (5) (giving $r'' \leq 0$) and is valid independently of the large r behavior of the metric—flat, AdS, or any other. For phantom fields Eq. (5) gives $r'' \geq 0$, and such a restriction is absent. Thus, for a free massless phantom field wormhole solutions are well known since the ’70s [22,23]. (B) Particlelike (or starlike) solutions (PLS), i.e., asymptotically flat solutions with a regular center, are not excluded for both kinds of scalar fields but under certain constraints on the potential. Thus, for $\varepsilon = +1$, PLS cannot be obtained with $V(\phi) \geq 0$ [24]. For $\varepsilon = -1$, on the contrary, no PLS exist if $V(\phi) \leq 0$. (C) Numerous no-hair theorems for $\varepsilon = +1$ [see, e.g., the reviews [25–27] and references therein] restrict the shape and sign of $V(\phi)$ with which BH metrics may appear as solutions to Eqs. (4)–(6) but do not entirely rule out their existence. The same is true for phantom fields. (D) Equation (7) severely restricts the possible dispositions of Killing horizons in the resulting metric and consequently the global causal structure of space-time [21].

Indeed, horizons are regular zeros of $A(\rho)$ and hence $B(\rho)$. By (7), $B(\rho)$ increases at $\rho < \rho_0$, has a maximum at $\rho = \rho_0$, and decreases at $\rho > \rho_0$. It can have *at most* two simple zeros, bounding a range $B > 0$ (R region), or one double zero and two T regions around. It can certainly have a single simple zero or no zeros at all.

So the choice of possible types of global causal structure is precisely the same as for the general Schwarzschild–de Sitter solution with arbitrary mass and cosmological constant.

Equation (6) does not contain ε, hence this result [*the Global Structure Theorem* [21]] equally applies to normal and phantom fields. It holds for any sign and shape of $V(\phi)$ and under any assumptions on the asymptotics. BHs with scalar hair (respecting the no-hair theorems) are not excluded. Examples of (singular) BHs with both normal [e.g., [28–30]] and phantom [31] scalar hair are known. However, BHs with a regular center are ruled out since their existence requires a minimum of $B(\rho)$.

The Hawking temperature of a horizon $\rho = h$ is determined [32] as $T_H = \kappa/(2\pi)$, where κ is the surface gravity at $\rho = h$. In our system, by (7),

$$\kappa = |A'(h)|/2 \quad \text{and} \quad A'(h) = 2(\rho_0 - h)/r^2(h). \quad (8)$$

Let us determine the possible kinds of nonsingular solutions without restricting the shape of $V(\phi)$. Assuming no pathology at intermediate ρ, regularity is provided by the system behavior at the ends of the ρ range. The latter may be classified as a regular infinity ($r \to \infty$), which may be flat, de Sitter, or AdS (other possible variants, like $r^2 \sim \rho$, are of lesser interest), a regular center, and the intermediate case $r \to r_0 > 0$. Any oscillatory behavior of $r(\rho)$ is ruled out by the constant sign of r''.

Suppose we have a regular infinity as $\rho \to \infty$, so that $V \to V_+ =$ const while the metric becomes Minkowski (M), de Sitter (dS), or AdS according to the sign of V_+. In all cases $r \approx \rho$ at large ρ.

For $\varepsilon = +1$, due to $r'' \leq 0$, $r \to 0$ at some $\rho = \rho_c$, which means a center, and the only possible regular solutions have a regular center and an AdS, flat, or dS asymptotic; in the latter case, the causal structure coincides with that of de Sitter space time.

For $\varepsilon = -1$, there exist similar solutions with $\rho_c < \rho < \infty$ and a regular center. However, due to $r'' \geq 0$, there are also others, in which $\rho \in \mathbb{R}$ and either $r \to r_0 =$ const > 0 or $r \to \infty$ as $\rho \to -\infty$. All kinds of regular behavior are thus possible at the "negative" end. In particular, if $r \to r_0$, we get $A \approx -\rho^2/r_0^2$, i.e., a T region comprising a highly anisotropic KS cosmology with one scale factor (r) tending to a constant while the other (A) inflates. The scalar field tends to a constant, while $V(\phi) \to 1/r_0^2$.

Thus there are three kinds of regular asymptotics at one end, $\rho \to \infty$ (M, dS, AdS), and four at the other, $\rho \to -\infty$:

the same three plus $r \to r_0$, simply r_0 for short. (The asymmetry has appeared since we did not allow $r \to$ const as $\rho \to \infty$. The inequality $r'' > 0$ forbids nontrivial solutions with two such r_0 asymptotics.) This makes nine combinations shown in Table I. Moreover, each of the two cases labeled KS* actually comprises three types of solutions according to the properties of $A(\rho)$: there can be two simple horizons, one double horizon, or no horizons between two dS asymptotics. Recalling 3 kinds of solutions with a regular center, we obtain as many as 16 qualitatively different classes of globally regular configurations of phantom scalar fields.

Examples of each behavior may be found in an algorithmic manner by properly choosing the function $r(\rho)$ and invoking the inverse problem method: $B(\rho)$ and $A(\rho)$ are then obtained from Eq. (7) [and $B(\rho)$ always behaves as described above]; after that $\phi(\rho)$ is yielded by Eq. (5) and $V(\rho)$ by Eq. (4). A critical requirement is that $r(\rho)$ must satisfy the inequality $r'' \le 0$ for $\varepsilon = 1$ and $r'' \ge 0$ for $\varepsilon = -1$. The function $V(\phi)$ is restored from known $V(\rho)$ and $\phi(\rho)$ provided the latter is monotonic, which is the case if everywhere $r'' \neq 0$.

The potential V tends to a constant and, moreover, $dV/d\phi \to 0$ at each end of the ρ range. Therefore, any model from the above classes requires a potential with at least two zero-slope points (not necessarily extrema) at different values of ϕ. Suitable potentials are, e.g., $V = V_0 \cos^2(\phi/\phi_0)$ and the Mexican hat potential $V = (\lambda/4) \times (\phi^2 - \eta^2)^2$ where V_0, ϕ_0, λ, η are constants. A flat infinity certainly requires $V_+ = 0$, while a de Sitter asymptotic can correspond to a maximum of V since phantom fields tend to climbing up the slope of the potential rather than rolling down, as is evident from Eq. (3). Accordingly, Faraoni [9], considering spatially flat isotropic phantom cosmologies, has shown that if $V(\phi)$ is bounded above by $V_0 = \text{const} > 0$, the de Sitter solution is a global attractor. Very probably this conclusion extends to KS cosmologies after isotropization.

We will now give a transparent analytic example, leaving for the future more elaborated models with better motivated potentials. So we put $\varepsilon = -1$,

$$r = (\rho^2 + b^2)^{1/2}, \qquad b = \text{const} > 0, \tag{9}$$

and use the inverse problem scheme. Equation (7) gives

TABLE I. Regular solutions with $\rho \in \mathbb{R}$ for $\varepsilon = -1$. Each row corresponds to a certain asymptotic behavior as $\rho \to +\infty$, each column—to $\rho \to -\infty$. The mark "sym" refers to combinations obtained from others by symmetry $\rho \leftrightarrow -\rho$.

	AdS	M	dS	r_0
AdS	wormhole	wormhole	black hole	black hole
M	sym	wormhole	black hole	black hole
dS	sym	sym	KS*	KS*

$$\begin{aligned} B(\rho) &= A(\rho)/r^2(\rho) \\ &= \frac{c}{b^2} + \frac{1}{b^2+\rho^2} + \frac{\rho_0}{b^3}\left(\frac{b\rho}{b^2+\rho^2} + \tan^{-1}\frac{\rho}{b}\right), \end{aligned} \tag{10}$$

where $c = \text{const}$. Equations (4) and (5) then lead to expressions for $\phi(\rho)$ and $V(\rho)$:

$$\phi = \pm\sqrt{2}\tan^{-1}(\rho/b) + \phi_0, \tag{11}$$

$$V = -\frac{c}{b^2}\frac{r^2+2\rho^2}{r^2} - \frac{\rho_0}{b^3}\left(\frac{3b\rho}{r^2} + \frac{r^2+2\rho^2}{r^2}\tan^{-1}\frac{\rho}{b}\right) \tag{12}$$

with $r = r(\rho)$ given by (9). In particular,

$$B(\pm\infty) = -\frac{1}{3}V(\pm\infty) = \frac{2bc \pm \pi\rho_0}{2b^3}. \tag{13}$$

Choosing in (11), without loss of generality, the plus sign and $\phi_0 = 0$, we obtain for $V(\phi)$ ($\psi := \phi/\sqrt{2}$):

$$\begin{aligned} V(\phi) = &-\frac{c}{b^2}(3 - 2\cos^2\psi) \\ &- \frac{\rho_0}{b^3}[3\sin\psi\cos\psi + \psi(3 - 2\cos^2\psi)]. \end{aligned} \tag{14}$$

The solution behavior is controlled by two integration constants: c that moves $B(\rho)$ up and down, and ρ_0 showing the maximum of $B(\rho)$. Both $r(\rho)$ and $B(\rho)$ are even functions if $\rho_0 = 0$; otherwise $B(\rho)$ loses this symmetry.

In the simplest case $\rho_0 = c = 0$ we obtain the so-called Ellis wormhole [22]: $V \equiv 0$ and $A \equiv 1$.

Solutions with $\rho_0 = 0$ but $c \neq 0$ describe symmetric structures: wormholes with two AdS asymptotics if $c > 0$ and solutions with two dS asymptotics if $c < 0$. If $0 > c > -1$, there is an R region in the middle, bounded by two simple horizons, at $c = -1$ they merge into a double horizon, and $c < -1$ leads to a pure KS cosmology.

If $\rho_0 \neq 0$, the two asymptotics are different. In solutions flat at $\rho = \infty$, it holds $2bc = -\pi\rho_0$ while the Schwarzschild mass, defined in the usual way, is $m = \rho_0/3$. According to (13), for $\rho_0 < 0$ we obtain a wormhole with $m < 0$ and an AdS metric at the far end, corresponding to the cosmological constant $V_- < 0$. For $\rho_0 > 0$, when $V_- > 0$, there is a regular BH with $m > 0$ and a dS asymptotic far beyond the horizon. As any asymptotically flat BH with a simple horizon, it has a Schwarzschild-like causal structure, but the singularity $r = 0$ in the Carter-Penrose diagram is replaced by $r = \infty$.

The horizon radius depends on both parameters m and $b = \min r(\rho)$ and cannot be smaller than b, which also plays the role of a scalar charge: $\psi \approx \pi/2 - b/\rho$ at large ρ. Since $A(0) = 1 + c$, the throat $\rho = 0$ is located in the R region if $c > -1$, i.e., if $3\pi m < 2b$, at the horizon if $3\pi m = 2b$ and in the T region beyond it if $3\pi m > 2b$.

Such regular BHs combine the properties of BHs, whose main feature is a horizon, and wormholes, whose main feature is a throat, $r = r_{\min} > 0$. The above relations between m and b show (and it is probably generically true) that if the BH mass dominates over the scalar charge, the throat is invisible to a distant observer, and the BH looks almost as usual in general relativity. However, a possible BH explorer now gets a chance to survive for a new life in an expanding KS universe.

One may also speculate that our Universe could appear from collapse to a phantom BH in another, "mother" universe and undergo isotropization (e.g., due to particle creation) soon after crossing the horizon. The KS nature of our Universe is not excluded observationally [33] if its isotropization had happened early enough, before the last scattering epoch (at redshifts $z \gtrsim 1000$). The same idea of a Null Bang instead of a Big Bang (cosmological expansion starting from a horizon rather than a singularity) was discussed in [20] for a system with a de Sitter vacuum core and a regular center in the R region.

Let us note in conclusion that the present analysis, which has revealed a wealth of regular solutions including BHs, is easily extended to more sophisticated phantom models, e.g., to those of k-essence type. Indeed, for the scalar field Lagrangian $L = P(X) - 2V(\phi)$ where $X = g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ and P is an arbitrary function, Eqs. (6) and (7) remain unchanged while the crucial inequality $r'' \geq 0$ holds if the theory satisfies the "phantom condition" $dP/dX < 0$. k-essence-type theories, among other merits, are known to avoid inadmissible sound velocities and the stabilization problem [34,35].

Other obvious generalizations are scalar-tensor theories of gravity and, as their subclass, nonminimally coupled scalar fields with Lagrangians including const $\times R\phi^2$. Such theories are conformally related to (1), and the conformal factors, if well behaved, do not change the causal and asymptotic properties of the solutions.

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