W Boson Production Cross Section at the Large Hadron Collider with $\mathcal{O}(\alpha_s^2)$ Corrections

Kirill Melnikov
Department of Physics and Astronomy, University of Hawaii, 2505 Correa Road, Honolulu, Hawaii 96822, USA

Frank Petriello
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706, USA
and Fermi National Laboratory, P.O. Box 500, MS 106, Batavia, Illinois 60510, USA
(Received 31 March 2006; published 16 June 2006)

We compute the $\mathcal{O}(\alpha_s^2)$ QCD corrections to the fully differential cross section $pp \to WX \to l\nu X$, retaining all effects from spin correlations. The knowledge of these corrections makes it possible to calculate with high precision the W boson production rate and acceptance at the CERN Large Hadron Collider (LHC), subject to realistic cuts on the lepton and missing energy distributions. For certain choices of cuts we find large corrections when going from next-to-leading order (NLO) to next-to-next-to-leading order in perturbation theory. These corrections are significantly larger than those obtained by parton-shower event generators merged with NLO calculations. Our result may be used to assess and significantly reduce the QCD uncertainties in the many studies of W boson production planned at the LHC.

DOI: 10.1103/PhysRevLett.96.231803 PACS numbers: 13.85.Qk, 12.38.Lg, 14.70.Fm

Production of electroweak gauge bosons is a vital component of the hadron collider physics program. The large production rates for this channel at the CERN Large Hadron Collider (LHC) will facilitate several important precision measurements. LHC experiments plan to determine the W boson mass and width with errors of $\delta M_W \sim 15$ MeV and $\delta\Gamma_W \sim 50$ MeV, respectively [1]. The Weinberg angle $\sin\theta_W$ can be extracted from the forward-backward asymmetry of the lepton pair in $pp \to Z \to e^+e^-$ with a precision of 1×10^{-4}. The precision possible in these channels at high luminosities makes these measurements competitive with LEP results. Searching for deviations from predictions in dilepton events with large invariant mass, missing energy, or transverse momentum probes extensions of the standard model which contain new gauge bosons.

In addition to its high rate, electroweak gauge boson production has a simple, distinct experimental signature. This also makes it a useful process for calibrating and monitoring machine and detector performance. Z and W production can be used to determine and monitor the hadronic and partonic luminosities at the LHC [2]. This requires a theoretical prediction for the cross section to the highest possible precision, since this error propagates into all other measurements through the luminosity uncertainty. Determination of the LHC luminosity to 1% accuracy is the ultimate goal of this procedure [2]. This sets the precision required for theoretical predictions.

When the desired precision on the production cross section is at the few percent level, many subtle effects must be included. Both $\mathcal{O}(\alpha)$ electroweak effects and $\mathcal{O}(\alpha_s^2)$ QCD effects must be calculated. The electroweak corrections to $pp \to W \to l\nu$ were computed in [3], where the importance of final state photon radiation in the W decay was observed. Next-to-leading order (NLO) computations of the QCD corrections to electroweak gauge boson production were first obtained in the late seventies [4]. The W boson momentum distribution was investigated via resummation techniques in [5]. Currently, the next-to-next-to-leading order (NNLO) QCD corrections are known for both the inclusive production cross section [6] and for the gauge boson rapidity distributions [7]. The NNLO corrections are typically in the few percent range at the LHC, and must be included in both precision electroweak studies and the luminosity determination.

Existing calculations of the NNLO QCD corrections to this process do not include all effects needed for a percent-level theoretical prediction. Phenomenological applications of Z and W production require significant cuts on the phase space of the final state leptons. For example, all LHC experiments will impose constraints on the transverse momenta and rapidities of the final state charged leptons. Cuts on the missing energy will also be employed to identify the neutrino from the W decay. Calculations that treat the leptons inclusively are therefore not fully realistic. They can be used to make estimates, but they are not sufficient for precision measurements.

The calculation of the full NNLO QCD corrections is complicated by the spin-one nature of the gauge bosons. If they were spin-zero bosons, fully differential results could be obtained from Ref. [7], where the rapidity distributions for Z and W bosons were computed through NNLO. That result could be combined with the known double differential distribution in transverse momentum and rapidity [8] to fully determine the gauge boson kinematics. The decay of spin-zero bosons in their rest frame is isotropic, and the final state distribution of interest could be obtained by assuming a flat decay distribution in the gauge boson rest frame. There would be no correlation between the production and decay of the boson. However, since the Z and W

are spin-one bosons, the $Z \to l^+ l^-$ and $W \to l\nu$ decays are not isotropic, and there are "spin correlations" between the production and decay channels. The importance of spin correlations for the production of gauge bosons at the LHC was recently emphasized in [9].

The computation of the differential cross section for $pp \to WX \to l\nu X$ through NNLO in QCD is a difficult theoretical challenge. While techniques for performing differential NLO calculations have been known for many years [10], the corresponding technology for obtaining NNLO results is still in its infancy. In a recent series of papers [11], we have developed a method for performing these calculations. This technique features an automated extraction of infrared singularities from the real radiation matrix elements and a numerical cancellation of these divergences with the virtual corrections. We describe below the application of this method to the computation of the W production cross section with all spin correlations included.

We compute the partonic cross sections $ij \to l\nu X$ as perturbative expansions in the strong coupling constant α_s. We specialize here to W^- production. At LO, the W^- boson is produced in the collision of an up-type antiquark and a down-type quark. At NLO, gluon-quark and gluon-antiquark scatterings also contribute. A variety of partonic processes contributes at NNLO. These have been enumerated in great detail in [6]. In our discussion below we use $\bar{u}d \to W^- X \to e\bar{\nu}X$ as an example, since it contains all the complexities present in the full calculation. All partonic channels have been included in our result.

There are three distinct contributions contained in the $\bar{u}d$ initiated process: the two-loop virtual corrections, the one-loop virtual corrections to single gluon emission, and tree-level double-real radiation processes with two additional partons in the final state. These must be combined in the presence of an infrared-safe measurement function to produce a finite result. We use dimensional regularization to regulate all ultraviolet, soft, and collinear divergences.

The two-loop virtual corrections to the $\bar{u}d$ process are straightforward to compute. They are very similar to the $\mathcal{O}(\alpha_s^2)$ corrections to the quark form factor studied in [12]; however, the W production calculation must include the two-loop corrections to nonsinglet axial current. Care must be taken to define this correctly in $d = 4 - 2\epsilon$ dimensions; we discuss this issue further below. We use the implementation of the Laporta algorithm [13] described in Ref. [14] to reduce all required two-loop integrals to a minimal set of master integrals. The master integrals needed for this computation are well known [12].

We obtain the one-loop correction to the single gluon emission process $\bar{u}d \to e\bar{\nu}_e g$ using a combination of two methods. We first use the Laporta algorithm to express all one-loop Feynman integrals relevant for this process through master integrals. The master integrals must be integrated over the final state phase space subject to the kinematic constraints under consideration. It is not possible to perform this integration analytically, since we want an expression valid for arbitrary cuts. Numerical integration is also not straightforward because of soft and collinear singularities. We employ the method developed in [11] to extract the singularities in a constraint-independent way as poles in ϵ before integrating over the phase space numerically. This technique maps the final state phase space onto the unit hypercube and uses iterated sector decomposition [15] to extract all soft and collinear singularities.

We use essentially the same algorithm to compute the double-real radiation corrections. A detailed description of this method, which studies both the one and two parton emission corrections, can be found in [11].

We now discuss a few new features of this calculation, first explaining how we treat the axial current in d dimensions. This issue arises from Dirac structures of the form $\mathrm{Tr}_{\mathrm{H}}[\Gamma^{(1)}\gamma_5]\mathrm{Tr}_{\mathrm{L}}[\Gamma^{(2)}\gamma_5]$, where $\Gamma^{(1,2)}$ denote generic products of Dirac matrices and $\mathrm{Tr}_{\mathrm{H,L}}$ refer to traces over hadronic and leptonic degrees of freedom, respectively. These traces do not vanish when the final state phase space is sufficiently constrained. A consistent extension of the axial current to d dimensions is given in [16]. It utilizes an anti-commuting γ_5 and contains additional renormalizations relative to the vector current in order to maintain the Ward identities. We use this prescription in our calculation.

Even after all three components of the hard-scattering cross section are combined, collinear counterterms are needed to remove initial state collinear singularities. In [11] these collinear counterterms were treated analytically. Such an approach is not sufficiently flexible to handle cuts on the W decay products. However, it is straightforward to extend the numerical approach used for the other NNLO components in [11] to obtain the desired results.

We have essentially two checks on our calculation. First, considering different cuts on the electron transverse momentum and rapidity as well as on the missing energy, we verify cancellation of the divergences in the W^- production cross section. Because the divergences start at $1/\epsilon^4$ at NNLO, the cancellation of all divergences through $1/\epsilon$ provides a stringent check on the calculation. We also check that the vector and axial contributions are separately finite, as required. A second check is obtained by integrating fully over the final state phase space and comparing against known results for the inclusive cross section. We find excellent agreement with the results of [6] for all partonic channels.

We now discuss the results of our calculation. We first present the input parameters. We use the parton distribution functions of [17] at the appropriate order in α_s. We use $m_W = 80.451$ GeV and work in the narrow width approximation, although this restriction can be easily removed. We set $|V_{\mathrm{ud}}| = 0.974$, $|V_{\mathrm{us}}| = |V_{\mathrm{cd}}| = 0.219$, and $|V_{\mathrm{cs}}| = 0.996$, and obtain $|V_{\mathrm{ub}}|$ and $|V_{\mathrm{cb}}|$ from unitarity of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix. We neglect contributions from the top quark; these have been shown to be small in the inclusive cross section [6]. For electroweak input parameters, we use $\sin^2\theta_W = 0.2216$, $\alpha_{\rm QED}(m_Z) = 1/128$, and $\mathrm{Br}(W \to e\nu) = 0.1068$. We set the factorization and renormalization scales to a common value, $\mu_r = \mu_f = \mu$, and employ various choices of μ in our numerical study.

We find that NNLO corrections depend on the cuts and can change rapidly from very small to fairly substantial. We consider cuts of the form

$$p_\perp^e > p_\perp^{e,\min}, \qquad |\eta^e| < 2.5, \qquad E_\perp^{\rm miss} > 20 \text{ GeV}, \quad (1)$$

and use the values $p_\perp^{e,\min} = 20$, 30, 40, 50 GeV. The choices $p_\perp^{e,\min} = 20$ and 40 GeV were considered in the study of [9]. $p_\perp^{e,\min} = 20$ GeV is similar to cuts that will be employed by the ATLAS and CMS collaborations, while $p_\perp^{e,\min} = 40$ GeV was chosen in [9] to illustrate the potential sensitivity of the QCD radiative corrections to experimental cuts. We first present results for the lepton invariant mass distribution for on-shell W^- production in Table I, to give a feeling for the magnitude of the cross section for each set of cuts. The numerical precision for all NNLO numbers is 1% or better. We note that the row labeled ''Inc'' denotes the fully inclusive cross section.

There are a few things to notice about these numbers. First, at LO, there is no additional hadronic radiation in the final state for the lepton and neutrino to recoil against, so the transverse momentum is restricted to $p_\perp^e < m_W/2$. The cross section is therefore very small for $p_\perp^{e,\min} = 40$ GeV, and vanishes for $p_\perp^{e,\min} = 50$ GeV. This restriction is lifted at NLO when there is an additional parton for the W to recoil against. Very near this boundary, the width of the W can be an important effect. It will induce a (tiny) cross section for $p_\perp^{e,\min} = 50$ GeV at LO, and it will shift the result for $p_\perp^{e,\min} = 40$ GeV since this value is close to $m_W/2$. We have not included the W width in our results. However, we have checked using the results in [9] that finite width effects change the acceptance by only 7% at LO, and by less at NLO. We are therefore confident that our discussion and conclusions are not affected by this omission.

Another feature to notice is that the corrections are large for higher choices of $p_\perp^{e,\min}$ and that the dependence on $p_\perp^{e,\min}$ is strong. For example, for the scale choice $\mu = m_W$, we observe a 22% increase when going from a LO to NLO inclusive cross section, followed by a decrease of 2.5% when NNLO corrections are included. We obtain similar results for $p_\perp^{e,\min} = 20$, 30 GeV. However, the pattern of corrections is much different for $p_\perp^{e,\min} = 40$, 50 GeV; we find corrections of 18–27% for $p_\perp^{e,\min} = 40$ GeV and 33–37% for 50 GeV when going from NLO to NNLO, depending on the choice of scale.

The remaining scale dependences can be seen from Table I. We define the scale dependences of the cross section σ_X as $\Delta\sigma_X = 2(\max[\sigma_X(\mu)] - \max[\sigma_X(\mu)])/(\max[\sigma_X(\mu)] + \max[\sigma_X(\mu)])$, where we take the maximum and minimum values from among the three studied scale choices. $\Delta\sigma_X$ therefore gives the scale variation uncertainty band for the observable σ_X. The scale dependence is reduced to the percent level or less for the inclusive case, and for the cuts $p_\perp^{e,\min} = 20$, 30 GeV. Moreover, the NNLO results lie within the NLO uncertainty bands. This is not the case for the other two choices of $p_\perp^{e,\min}$; here, the scale dependence actually increases to the 5%–8% level at NNLO, and the NLO scale dependence completely underestimates the higher-order radiative corrections. This is not completely unexpected, since for these values additional partons to recoil against only appear at NLO. The NNLO results therefore serve as the first radiative corrections for these $p_\perp^{e,\min}$ choices. However, it indicates the care that must be taken when using the scale variation as a measure of the theoretical error.

Another important quantity to study is the experimental acceptance, defined as the ratio of the cross section after cuts over the inclusive cross section. We present the acceptances at NLO and NNLO in Table II. We again note that for the choices $p_\perp^{e,\min} = 40$, 50 GeV, the NLO scale dependences completely underestimate the NNLO corrections. The NNLO shifts in acceptances are very large for $p_\perp^{e,\min} = 40$, 50 GeV, reaching 25% for 40 GeV and 40% for 50 GeV. For the other choices of cuts, the NNLO acceptances are identical to the NLO ones within numerical errors, indicating stabilization of the perturbative expansion.

The cross sections and acceptances for the transverse momentum cuts $p_\perp^{e,\min} = 20, 40$ GeV were recently studied in [9]. The primary tool used in that analysis was the Monte Carlo event generator MC@NLO, which consistently combines NLO corrections with the HERWIG parton shower [18]. In [9], MC@NLO is used to estimate the importance of

TABLE I. The lepton invariant mass distribution $d\sigma/dM^2$, $M = m_W$, for on-shell W production in the reaction $pp \to W^- X \to e^- \bar{\nu} W$, in pb/GeV^2, for various choices of $p_\perp^{e,\min}$, GeV and $\mu = m_W/2$, m_W, $2m_W$.

$p_\perp^{e,\min}$	LO	NLO	NNLO
Inc	11.70, 13.74, 15.65	16.31, 16.82, 17.30	16.31, 16.40, 16.50
20	5.85, 6.96, 8.01	7.94, 8.21, 8.46	8.10, 8.07, 8.10
30	4.305, 5.12,5.89	6.18, 6.36, 6.54	6.18, 6.17, 6.22
40	0.628, 0.746, 0.859	2.07, 2.10, 2.11	2.62, 2.54, 2.50
50	0, 0, 0	0.509, 0.497, 0.480	0.697, 0.651, 0.639

TABLE II. Acceptances at NLO and NNLO for various choices of $p_{\perp}^{e,\min}$ and $\mu = m_W/2,\ m_W,\ 2m_W$.

$p_{\perp}^{e,\min}$ (GeV)	A(NLO)	A(NNLO)
20	0.487, 0.488, 0.489	0.497, 0.492, 0.491
30	0.379, 0.378, 0.378	0.379, 0.376, 0.377
40	0.127, 0.125, 0.122	0.161, 0.155, 0.152
50	0.0312, 0.0295, 0.0277	0.0427, 0.0397, 0.0387

QCD effects beyond those included in NLO calculations. Differences between the cross sections and acceptances of a few percent are found when comparing NLO and MC@NLO for both $p_{\perp}^{e,\min}$ choices. The authors of [9] then conclude that higher-order corrections beyond those in MC@NLO are unlikely to change significantly the results they find.

We believe that the few percent differences between MC@NLO and NLO cannot be used as an estimate of higher-order corrections. This is because few percent shifts coming from hard emissions generically occur in process-dependent radiative corrections that cannot be described by parton showers. We can support this assertion with the following observations. It follows from Table II in [9] that adding the parton shower to the LO cross section fails to properly predict the NLO cross section. For $p_{\perp}^{e,\min} = 20$ GeV, adding the parton shower to the LO result *decreases* the cross section by 8%, while the NLO correction increases it by 4%. While both HERWIG and NLO corrections increase the LO result for $p_{\perp}^{e,\min} = 40$ GeV, the magnitude of the shifts differ by 45% relative to the LO cross section. In addition, the NNLO corrections to the cross sections presented in Table I differ significantly from the estimate of these corrections in [9]. We conclude that the results for the cross sections and acceptances obtained in [9] cannot be used if few percent precision is required. This is particularly true for cuts where hard gluon emissions are expected to be large, such as for $p_{\perp}^{e,\min} = 40$, 50 GeV. We note that since the corrections are large for $p_{\perp}^{e,\min} = 50$ GeV, which is well above the LO kinematic boundary for the electron transverse momentum at $m_W/2$, our results are not caused by large logarithms which spoil the perturbative expansion.

In this Letter we report on the computation of the NNLO QCD corrections to the fully differential cross section $pp \to WX \to l\nu X$ at the LHC, with all spin correlations included. For inclusive enough cuts, this calculation provides the percent-level theoretical accuracy needed for the W cross section when realistic experimental cuts are imposed on the final state leptons. We find that the QCD corrections exhibit significant dependence on the lepton minimum transverse momentum. For high values of this cut, the corrections may be very different than the inclusive NNLO results. Our calculation can be easily extended to include Z production, finite width effects, and $p\bar{p}$ collisions. The last case is particularly interesting because of its importance for the Tevatron Run II physics program. These extensions will be discussed in detail in a forthcoming publication.

K. M. is supported in part by the DOE Grant No. DE-FG03-94ER-40833, Outstanding Junior Investigator program, and by the Alfred P. Sloan Foundation. F. P. is supported in part by the University of Wisconsin Research Committee with funds provided by the Wisconsin Alumni Foundation.

[1] For a review, see P. M. Nadolsky, AIP Conf. Proc. **753**, 158 (2005); U. Baur, hep-ph/0511064.
[2] M. Dittmar, F. Pauss, and D. Zürcher, Phys. Rev. D **56**, 7284 (1997); V. A. Khoze *et al.*, Eur. Phys. J. C **19**, 313 (2001); W. T. Giele, *et al.*, hep-ph/0104053.
[3] S. Dittmaier and M. Kramer, Phys. Rev. D **65**, 073007 (2002); U. Baur *et al.*, Phys. Rev. D **59**, 013002 (1999).
[4] G. Altarelli *et al.*, Nucl. Phys. B **157**, 461 (1979); J. Kubar-Andre *et al.*, Phys. Rev. D **19**, 221 (1979); K. Harada *et al.*, Nucl. Phys. B **155**, 169 (1979); Nucl. Phys. B**165**, 545(E) (1980); P. Aurenche and J. Lindfors, Nucl. Phys. B **185**, 274 (1981).
[5] C. Balazs and C.-P. Yuan, Phys. Rev. D **56**, 5558 (1997); F. Landry *et al.*, Phys. Rev. D **67**, 073016 (2003).
[6] R. Hamberg, W. L. van Neerven, and T. Matsuura, Nucl. Phys. B **359**, 343 (1991); **644**, 403(E) (2002); R. V. Harlander and W. B. Kilgore, Phys. Rev. Lett. **88**, 201801 (2002).
[7] C. Anastasiou *et al.*, Phys. Rev. D **69**, 094008 (2004).
[8] R. K. Ellis *et al.*, Nucl. Phys. B **211**, 106 (1983); R. Gonsalves *et al.*, Phys. Rev. D **40**, 2245 (1989); P. B. Arnold and M. H. Reno, Nucl. Phys. B **319**, 37 (1989); **330**, 284(E) (1990).
[9] S. Frixione and M. L. Mangano, J. High Energy Phys. 05 (2004) 056.
[10] R. K. Eliis *et al.*, Nucl. Phys. B **178**, 421 (1981); W. T. Giele and E. W. N. Glover, Phys. Rev. D **46**, 1980 (1992); Z. Kunszt and D. E. Soper, Phys. Rev. D **46**, 192 (1992); S. Frixione *et al.*, Nucl. Phys. B **467**, 399 (1996); S. Catani and M. H. Seymour, Nucl. Phys. B **485**, 291 (1997); **510**, 503(E) (1997).
[11] C. Anastasiou, K. Melnikov, and F. Petriello, Phys. Rev. D **69**, 076010 (2004); Phys. Rev. Lett. **93**, 032002 (2004); **93**, 262002 (2004); Nucl. Phys. B **724**, 197 (2005).
[12] R. Gonsalves, Phys. Rev. D **28**, 1542 (1983); G. Kramer *et al.*, Z. Phys. C **34**, 497 (1987); **42**, 504(E) (1989).
[13] S. Laporta, Int. J. Mod. Phys. A **15**, 5087 (2000).
[14] C. Anastasiou *et al.*, J. High Energy Phys. 07 (2004) 046.
[15] T. Binoth and G. Heinrich, Nucl. Phys. B **585**, 741 (2000); see also K. Hepp, Commun. Math. Phys. **2**, 301 (1966); A. Denner *et al.*, Nucl. Phys. B **479**, 495 (1996).
[16] S. Larin, Phys. Lett. B **303**, 113 (1993).
[17] A. D. Martin *et al.*, Phys. Lett. B **531**, 216 (2002).
[18] S. Frixione and B. R. Webber, J. High Energy Phys. 06 (2002) 029;S. Frixione *et al.*, J. High Energy Phys. 08 (2003) 007.