Natural Electroweak Breaking from a Mirror Symmetry

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We present "twin Higgs models," simple realizations of the Higgs boson as a pseudo Goldstone boson that protect the weak scale from radiative corrections up to scales of order 5–10 TeV. In the ultraviolet these theories have a discrete symmetry which interchanges each standard model particle with a corresponding particle which transforms under a twin or a mirror standard model gauge group. In addition, the Higgs sector respects an approximate global symmetry. When this global symmetry is broken, the discrete symmetry tightly constrains the form of corrections to the pseudo Goldstone Higgs potential, allowing natural electroweak symmetry breaking. Precision electroweak constraints are satisfied by construction. These models demonstrate that, contrary to the conventional wisdom, stabilizing the weak scale does not require new light particles charged under the standard model gauge groups.

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In the standard model (SM) the weak scale is unstable under quantum corrections. This suggests the existence of new physics at or close to a TeV that protects the Higgs mass parameter of the SM against radiative corrections. While the exact form that such new physics takes is unknown, there are several interesting alternatives. One possibility, first proposed in [1,2], is that the Higgs boson is naturally light because it is the pseudo Goldstone boson of an approximate global symmetry. This idea has recently experienced a revival in the form of little Higgs theories [3,4] (for a clear review and more references, see [5]) that protect the Higgs mass from radiative corrections up to scales of order 5–10 TeV.

In this Letter we propose a class of simple alternative realizations of the Higgs boson as a pseudo Goldstone boson that also protect the weak scale from radiative corrections up to scales of order 5–10 TeV. In the ultraviolet these theories have a discrete Z_2 symmetry which interchanges each SM particle with a corresponding particle which transforms under a twin or a mirror SM gauge group. In addition, the Higgs boson sector of the theory respects an approximate global $SU(4)$ symmetry. Although the weak and electromagnetic interactions, as well as the top Yukawa coupling, violate the global symmetry, they all respect the discrete interchange symmetry. When $SU(4)$ is broken to $SU(3)$, the discrete symmetry tightly constrains the form of corrections to the pseudo Goldstone Higgs potential, allowing natural electroweak symmetry breaking.

Although the smaller Yukawa couplings need not respect the discrete symmetry, naturalness constrains the masses of the twin partners not to exceed a few hundred GeV. Precision electroweak constraints are satisfied by construction, since although these new particles may be very light, they do not transform under the SM gauge groups. This is in contrast to little Higgs theories where these constraints are often a severe problem [6].

We illustrate the basic idea by way of a simple example where the global symmetry is realized linearly. Consider a complex scalar field, H, that transforms as a fundamental under a global $SU(4)$ symmetry. The potential for this field is given by

$$V(H) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2. \tag{1}$$

Since the mass squared of H is negative, it will develop a vacuum expectation value (VEV), $\langle |H| \rangle = m/\sqrt{2\lambda} \equiv f$, that breaks $SU(4) \to SU(3)$ yielding 7 massless Nambu-Goldstone bosons. We now break the $SU(4)$ explicitly by gauging an $SU(2)_A \times SU(2)_B$ subgroup. The field H transforms as (H_A, H_B), where H_A is a doublet under $SU(2)_A$ and H_B is a doublet under $SU(2)_B$. At the end of the day, we will identify $SU(2)_A$ with $SU(2)_L$ of the SM. Since $SU(4)$ is now broken explicitly, the would-be Goldstone bosons pick up a mass that is proportional to the explicit breaking. Specifically, gauge loops contribute a quadratically divergent mass to the components of H as

$$\Delta V = \frac{9 g_A^2 \Lambda^2}{64\pi^2} H_A^\dagger H_A + \frac{9 g_B^2 \Lambda^2}{64\pi^2} H_B^\dagger H_B + \cdots, \tag{2}$$

a loop factor below the cutoff Λ of the theory. The mechanism in our model hinges on the following simple observation. Suppose we now impose an additional Z_2 symmetry, which we label "twin parity," which interchanges H_A and H_B and also interchanges the gauge bosons of $SU(2)_A$ with those of $SU(2)_B$. This symmetry forces the two gauge couplings to be equal, $g_A = g_B \equiv g$. The gauge contribution to the mass of H is now

$$\Delta V = \frac{9 g^2 \Lambda^2}{64\pi^2} (H_A^\dagger H_A + H_B^\dagger H_B) = \frac{9 g^2 \Lambda^2}{64\pi^2} H^\dagger H, \tag{3}$$

which is invariant under $SU(4)$ and therefore does not contribute a mass to the Goldstone bosons. In other words, imposing twin parity constrains the quadratically divergent mass terms to have an $SU(4)$ invariant form. The

Goldstone bosons are therefore completely insensitive to quadratic divergences from gauge loops.

Gauge loops will, however, contribute a logarithmically divergent term to the potential that is not $SU(4)$ symmetric and has the general form $\kappa(|H_A|^4 + |H_B|^4)$, where κ is of order $g^4/16\pi^2 \log(\Lambda/gf)$. Provided Λ is not very much larger than f this leads to the would-be Goldstone bosons acquiring a mass of order $g^2 f/4\pi$. This is of the order of the weak scale for f of order 1 TeV. Notice that we could have obtained exactly the same result by imposing "mirror parity"—invariance under $t \to t$, $\vec{x} \to -\vec{x}$ along with the interchange of every particle in sector A with its CP conjugate in B.

At this point we note that the Higgs potential of Eq. (1) actually possesses a larger global $O(8)$ symmetry of which $U(4)$ is merely a subgroup, and the 7 Goldstone bosons we have identified can also be thought of as emerging from the breaking of $O(8)$ to $O(7)$. In particular, this $O(8)$ symmetry includes the custodial $SU(2)$ of the Higgs potential in the standard model.

This approach to stabilizing the weak scale against quantum corrections from gauge loops can be generalized to include all the other interactions in the SM. To do this, we gauge two copies of the SM, A and B, with our SM being SM_A. We can then extend the discrete symmetry in either of the following two ways: (1) Interchange every SM_A particle with the corresponding particle in SM_B, or (2) impose $t \to t$, $\vec{x} \to -\vec{x}$ along with the interchange of every SM_A particle with its CP conjugate in SM_B. These symmetries, while similar, are distinct. Each one relates the gauge and Yukawa interactions in the A sector to those in the B sector. While the former is a simple generalization of twin parity which we label "twin symmetry," the latter extends mirror parity to the familiar mirror symmetry [7]. Either choice of the discrete symmetry ensures that any quadratically divergent contribution to the Higgs mass has a form $\propto \Lambda^2(|H_A|^2 + |H_B|^2)$, which is harmless due to its accidental $SU(4)$ symmetry. Although quantum corrections to the quartic are, in general, not $SU(4)$ invariant, once again these lead to only logarithmically divergent contributions to the mass of the pseudo Goldstone Higgs field, allowing for a natural hierarchy between f and the weak scale.

At one loop the largest contribution to the pseudo Goldstone Higgs potential arises from the top Yukawa coupling and is logarithmically sensitive to the cutoff. However, in the twin symmetric case it is straightforward to make this contribution finite. One possible approach is to enlarge the approximate global symmetry of the top Yukawa coupling to $SU(6) \times SU(4) \times U(1)$ with the $[SU(3)_c \times SU(2) \times U(1)]_{A,B}$ subgroups being gauged. We do this by introducing the following chiral fermions:

$$\begin{aligned} Q_L &= (\mathbf{6}, \bar{\mathbf{4}}) \\ &= (\mathbf{3}, \mathbf{2}; \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \mathbf{3}, \mathbf{2}) + (\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}; \mathbf{3}, \mathbf{1}) \\ &\equiv q_A + q_B + \tilde{q}_A + \tilde{q}_B, \\ T_R &= (\bar{\mathbf{6}}, \mathbf{1}) = (\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \bar{\mathbf{3}}, \mathbf{1}) \equiv t_A + t_B, \end{aligned} \quad (4)$$

which transform as shown under $SU(6) \times SU(4)$ and under $[SU(3) \times SU(2)]^2$, where we have suppressed the hypercharge quantum numbers. One can then write an $SU(4)$ invariant Yukawa coupling

$$yHQ_LT_R + \text{H.c.} \quad (5)$$

The $SU(4)$ symmetric matter content contains exotic left-handed quarks, $\tilde{q}_{A,B}$, that are charged under color of one sector and the weak group of the twin, and vice versa. We introduce additional fermions with opposite charge assignment, $\tilde{q}^c_{A,B}$ with which the exotic quarks can get a Z_2 symmetric mass $M(\tilde{q}^c_A \tilde{q}_A + \tilde{q}^c_B \tilde{q}_B)$. The mass parameter M is the only source of $SU(4)$ breaking in the top sector, and it only breaks this symmetry softly. The top contribution to the Higgs potential in this model will then be finite at one loop.

We now construct a realistic twin symmetric model that implements these symmetries nonlinearly. The linear model we have been working with should be considered as merely one possibility for a UV completion of the nonlinear one, and others may well exist. The pseudo Goldstone fields of the nonlinear model are those which survive after integrating out the radial mode of the field H in the linear model. We parametrize these degrees of freedom as

$$H = \exp\left(\frac{i}{f} h^a t^a\right) \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix} + i \begin{pmatrix} h^1 \\ h^2 \\ h^3 \\ h^0 \end{pmatrix} + \cdots, \quad (6)$$

where $h^{1,\ldots,3}$ are complex and h^0 is real. In general, the effective theory for these fields will contain all of the operators allowed by the nonlinearly realized $SU(4)$ symmetry, suppressed by the cutoff scale Λ. However, in order to suppress custodial $SU(2)$ violation we assume that the symmetry which is nonlinearly realized is in fact $O(8)$. This provides additional restrictions on the form of the interactions in the effective theory below Λ, allowing precision electroweak constraints from higher dimensional operators to be naturally satisfied. Assuming the theory is strongly coupled at the cutoff, we can estimate $\Lambda \sim 4\pi f$. However, any potential for the pseudo Goldstone fields can emerge only from those interactions which violate the global $O(8)$ symmetry, specifically their gauge and Yukawa couplings. In particular, the electroweak gauge interactions and the top Yukawa contribute the most to the pseudo Goldstone potential and must therefore be studied in detail. We will thus calculate the contributions to the one-loop Coleman-Weinberg (CW) potential [8] from these couplings. At one loop the gauge and top sectors contribute separately, simplifying the calculation.

As before, we gauge two copies of the SM, A and B. The VEV f breaks $SU(2)_B \times U(1)_B$ down to a single $U(1)$, giving W_B and Z_B masses of order gf. The $SU(2)_A$ doublet $h^T \equiv (h^1, h^2)$ is left uneaten and is identified as the SM Higgs boson. The couplings of the pseudo Goldstone fields to the $SU(2) \times U(1)$ gauge fields and their mirror partners are given by expanding out $H = (H_A, H_B)$ in terms of the pseudo Goldstone bosons as given by Eq. (6) in the interaction

$$\left|\left(\partial_\mu + igW_{\mu,A} + \frac{i}{2}g'B_{\mu,A}\right)H_A\right|^2 + (A \rightarrow B). \quad (7)$$

A simple way of calculating the effective potential is to calculate the vacuum energy as a function of the field dependent masses of all of the fields in the theory. In the absence of quadratic divergences this leads to the formula

$$V_{\rm CW} = \pm\frac{1}{64\pi^2}\sum_i M_i^4\left(\log\frac{\Lambda^2}{M_i^2} + \frac{3}{2}\right), \quad (8)$$

where the sum is over all degrees of freedom, the sign being negative for bosons and positive for fermions. Writing the Higgs potential in the form

$$V(h) = m_h^2 h^\dagger h + \lambda_h (h^\dagger h)^2 + \cdots, \quad (9)$$

we find that the contribution to the Higgs mass term from the gauge sector is

$$m_h^2 = \frac{6g^2 M_{W_B}^2}{64\pi^2}\left(\log\frac{\Lambda^2}{M_{W_B}^2} + 1\right) + \frac{3(g^2 + g'^2)M_{Z_B}^2}{64\pi^2}\left(\log\frac{\Lambda^2}{M_{Z_B}^2} + 1\right), \quad (10)$$

where $M_{W_B}^2 = g^2 f^2/2$ and $M_{Z_B}^2 = (g^2 + g'^2)f^2/2$. Equation (10) holds if electromagnetism in the twin sector is an unbroken gauge symmetry as in the SM. However, it is also possible that QED in the twin sector is a broken symmetry and that the twin photon has a mass. This could arise if, for example, the hypercharge gauge boson in the twin sector has a mass M_B which softly breaks the twin symmetry. We do not specify a dynamical origin for this mass since it is technically natural for the dynamics which generate it to lie at scales above the cutoff Λ. In the limit that $M_B^2 \gg g'^2 f^2$ the second term in Eq. (10) becomes approximately

$$\frac{3g^2 M_{W_B}^2}{64\pi^2}\left(\log\frac{\Lambda^2}{M_{W_B}^2} + 1\right) + \frac{3g'^2 M_B^2}{64\pi^2}\left(\log\frac{\Lambda^2}{M_B^2} + 1\right). \quad (11)$$

The contribution to the Higgs quartic from this sector is small and can be neglected.

We now turn to the top sector. The couplings of the pseudo Goldstone fields to the top quark are obtained by expanding out H as in Eq. (6) in the $SU(4) \times SU(6)$ invariant interaction $(yHQ_L T_R + \text{H.c.})$ of Eq. (5). The h dependent masses of the fields in the top sector are determined from this and from the $SU(4)$ breaking mass term, and can be expressed as

$$m_{t_A}^2 = \frac{y^2 M^2}{M^2 + y^2 f^2} h^\dagger h, \qquad m_{T_A}^2 = M^2 + y^2 f^2,$$
$$m_{t_B}^2 = y^2 f^2, \qquad m_{T_B}^2 = M^2, \quad (12)$$

to leading order in $|h|^2$, where we have assumed for simplicity that y is real. This leads to the following contributions to the Higgs potential of Eq. (9):

$$m_h^2 = \frac{3}{8\pi^2}\frac{y^2 M^2}{M^2 - y^2 f^2}\left(M^2 \log\frac{m_{T_A}^2}{m_{T_B}^2} - y^2 f^2 \log\frac{m_{T_A}^2}{m_{t_B}^2}\right),$$

$$\lambda_h = -\frac{m_h^2}{3f^2} + \frac{3}{16\pi^2}\frac{y^4 M^4}{(M^2 + y^2 f^2)^2}\log\frac{m_{T_A}^2}{m_{t_A}^2} + \frac{3}{16\pi^2}\frac{y^4 M^4 (M^2 + y^2 f^2)}{(M^2 - y^2 f^2)^3}\log\frac{m_{T_B}^2}{m_{t_B}^2} - \frac{3}{32\pi^2}\left[\frac{4y^4 M^4}{(M^2 - y^2 f^2)^2} + \frac{y^4 M^4}{(M^2 + y^2 f^2)^2}\right]. \quad (13)$$

In order to generate a mild hierarchy $\langle h\rangle < f$ so that in the strong coupling limit the cutoff $\Lambda \sim 4\pi f$ is of order 5 TeV, we add to the theory a "μ term" that softly breaks the discrete Z_2 twin symmetry. This term takes the form $\mu^2 H_A^\dagger H_A$ and contributes to m_h^2 and λ_h. In addition, since the smaller Yukawa couplings do not contribute significantly to the Higgs potential, we do not require them to respect the discrete symmetry. In this nonlinear model, the absence of quadratically divergent contributions to the Higgs mass can be understood as a consequence of cancellations between the familiar SM loop corrections and new loop corrections that arise from the (mostly nonrenormalizable) couplings of the Higgs boson to the twin sector.

For phenomenological purposes we divide twin symmetric models into two classes—those where the top sector is extended as in Eq. (5), and those where it is not. As we now explain, the experimental constraints in these two cases are different. In the first case the exotic quarks $\tilde{q}_{A,B}$ and $\tilde{q}^c_{A,B}$, which are charged under both $U(1)_A$ and $U(1)_B$, lead to kinetic mixing between the photon and its twin partner at one loop [9]. Since the experimental constraints on such mixing are very severe, the twin photon must be heavy. In the second case, however, there are no particles charged under both sets of gauge groups, and a preliminary analysis does not reveal any nonzero contribution to the kinetic mixing term up to three-loop order. In this scenario it may therefore be phenomenologically allowed for the twin photon to be massless, provided a kinetic mixing term is not present at the cutoff. The mirror symmetric model shares the same phenomenology as the twin symmetric model without the extended top sector.

We now study each of these two scenarios in more detail, starting with theories with the extended top sector. In this case the strongest bound arises from the requirement that the twin neutrinos (and the twin photon itself) do not contribute significantly to the energy density of the universe at the time of big bang nucleosynthesis [10,11]. This constraint can be satisfied if the following two conditions are met: There is large entropy production during the QCD phase transition, significantly more than during the corresponding transition in the twin sector, and the two sectors are not in thermal equilibrium at any time after the QCD phase transition. Since the dynamics of the QCD phase transition is expected to be sensitive to the number of light quarks and their masses, which are not constrained to be the same in the two sectors, it is certainly plausible that the first condition is satisfied. What about the second? If the mixing term is zero at the cutoff and is generated only at the one-loop level through the exchange of the exotic quarks, the two sectors will not be in equilibrium below a few hundred MeV provided the twin photon mass M_B is larger than a few hundred GeV. In such a scenario the twin electron cannot go out of the bath by annihilating into photons once the temperature falls below its mass, as in the SM. Instead the twin electron must be extremely light so as not to contribute too much to the energy density of the universe at late times. We expect that twin baryons will constitute some or all of the dark matter in the universe, depending on the baryon asymmetry in the mirror sector.

Although this model predicts the existence of new light twin states, the fact that these particles are not charged under the SM gauge group implies that it may not be easy to test. In particular, precision electroweak constraints are easily satisfied. However, one possibility is to look for invisible decays of the SM Higgs boson into twin fermions [12]. The relevant vertex arises from substituting the expansion Eq. (6) into the Yukawa coupling of H_B to twin fermions. The branching ratio for invisible Higgs decays is of order $|\langle h \rangle / f|^2$.

We now estimate the fine-tuning in this class of models for two sets of parameters. For $f = 800\,\text{GeV}$, $\Lambda \sim 4\pi f \approx 10\,\text{TeV}$, $M = 6.0$ TeV, and $M_B = 1$ TeV, we find that in order to obtain the SM values of M_W and M_Z we need the soft Z_2 breaking parameter $\mu \approx 240\,\text{GeV}$. The Higgs mass is then about 120 GeV. Estimating the fine-tuning as $\partial \log M_Z^2 / \partial \log \mu^2$ we find that it is of order 13% (1 in 8). For $f = 500\,\text{GeV}$, $\Lambda \sim 4\pi f \approx 6$ TeV, $M = 5.5$ TeV, and $M_B = 1$ TeV, we find that the soft Z_2 breaking parameter μ needs to be around 150 GeV. The Higgs mass is again around 120 GeV and the fine-tuning 38% (1 in 3). This shows that these models stabilize the weak scale up to 5–10 TeV.

Let us now turn to mirror symmetric models (and twin symmetric models without the extended top sector). We are specifically interested in the scenario where the mirror photon is massless, since it appears current experimental bounds cannot exclude this possibility. This class of models also predict new light mirror fermions. These may now have tiny fractional electric charges if the kinetic mixing term between the photon and its mirror partner is very small but nonzero. Apart from this, the phenomenological implications are expected to be similar to the case with the extended top sector. We now estimate the fine-tuning for one specific parameter choice. Note that the formulas of the previous section generalize to the case without the extended top sector when the limit $M \to \Lambda$ is taken, up to finite terms. For $f = 800$ GeV, $\Lambda \sim 4\pi f \approx 10$ TeV, we find that the Higgs mass is 166 GeV and the fine-tuning is $\sim 11\%$ (1 in 9). For $f = 500$ GeV, $\Lambda \sim 4\pi f \approx 6$ TeV, we get a Higgs mass of 153 GeV with a fine-tuning of 31% (1 in 3). This shows that this class of models also stabilizes the weak scale up to 5–10 TeV.

In summary, we have constructed a new class of models where the Higgs boson emerges as a pseudo Goldstone boson whose mass is protected against radiative corrections up to scales of order 5–10 TeV. These theories demonstrate that, contrary to the conventional wisdom, stabilizing the weak scale does not require new light particles transforming under the SM gauge groups.

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