Detectability of Dissipative Motion in Quantum Vacuum via Superradiance

Woo-Joong Kim,[1] James Hayden Brownell,[1] and Roberto Onofrio[1,2]

[1]*Department of Physics and Astronomy, Dartmouth College, 6127 Wilder Laboratory, Hanover, New Hampshire 03755, USA*
[2]*Dipartimento di Fisica "G. Galilei," Università di Padova, Via Marzolo 8, Padova 35131, Italy*

We propose an experiment for generating and detecting vacuum-induced dissipative motion. A high frequency mechanical resonator driven in resonance is expected to dissipate mechanical energy in quantum vacuum via photon emission. The photons are stored in a high quality electromagnetic cavity and detected through their interaction with ultracold alkali-metal atoms prepared in an inverted population of hyperfine states. Superradiant amplification of the generated photons results in a detectable radio-frequency signal temporally distinguishable from the expected background.

PACS numbers: 12.20.Fv, 42.50.Lc, 42.50.Pq, 85.85.+j

Introduction.—Macroscopic quantum effects are suitable to bridge the gap between quantum theory and general relativity. In this context, observable effects due to the change in the boundary conditions of quantum fields, like the creation of particles in an expanding universe [1] or the Casimir force [2], may provide crucial information. So far the attention has been mainly focused on *conservative* Casimir forces, with measurements performed in a variety of geometries ranging from the original parallel plane [3,4] to the sphere plane [5–9] and crossed cylinders [10]. Meanwhile, there have been several theoretical attempts at studying the *dissipative* contribution of vacuum fluctuations to understand their interplay with the relativity of motion [11–13]. In principle, dissipative Casimir forces should be evidenced as a further damping source in the nonuniformly accelerated motion of micromechanical resonators already implemented for measuring the conservative component of the force. However, given the level of dissipations coming from more technical sources, a direct detection of the dissipative Casimir force seems out of experimental reach. Instead of focusing the attention on deviations from conservative motion, the dissipation induced in vacuum could be more easily detected by looking at the radiated photons that are less contaminated from other sources of noise [14]. This phenomenon, also known as dynamical Casimir effect [see [15] for an updated review], can be understood both as the creation of particles under nonadiabatic changes in the boundary conditions of quantum fields, or as classical parametric amplification where the zero point energy of a vacuum field mode is exponentially amplified in time. Theoretical analysis indicates that under parametric amplification in an electromagnetic cavity an initial state of N_0 photons with frequency within the resonance bandwidth of the fundamental mode of the cavity ω is transformed into a squeezed state with an average number of photons growing in time as [16–18]

$$N_{\text{Cas}}(t) = N_0 \sinh^2(\Omega \epsilon t), \quad (1)$$

assuming that the parametric resonance condition with a mechanical driving at a frequency $\Omega = 2\omega$ is fulfilled. The product $\Omega \epsilon$ represents the squeezing parameter, with the modulation depth $\epsilon = v/c$, where v is the velocity of the resonator and c the speed of light. This exponential growth is eventually limited by the photon leakage of the cavity expressed through the optical quality factor Q_{opt}, which saturates at the hold time $\tau = Q_{\text{opt}}/\omega$, reaching a maximum photon population

$$N_{\text{Cas}}^{\max} = N_{\text{Cas}}(\tau) = N_0 \sinh^2(2Q_{\text{opt}}\epsilon). \quad (2)$$

In this Letter, we discuss a generation mechanism for Casimir photons and a nearly quantum-limited photodetection scheme in the radio-frequency range based on the interaction of the generated photons with an excited population of atoms. This proposed experiment, initially sketched in [19], exploits in addition the high gain of superradiant emission to boost the expected signal to detectable levels, and a schematic outline of its components is shown in Fig. 1. An extremely weak signal of Casimir photons will trigger the emission of an intense, time-compressed, superradiant pulse whose characteristic delay time will provide the signature of mechanically induced vacuum radiation.

Generation of Casimir photons.—Current thin film technology makes feasible mechanical motions in the GHz range, with the highest frequency reported to date $\Omega/2\pi = 3.0$ GHz and a modulation depth of $\epsilon = 10^{-8}$ [20]. This has been obtained through a film bulk acoustic resonator (FBAR) [21–23], consisting of a vibrating aluminum nitride (AlN) film of thickness corresponding to one half of the acoustic wavelength, sandwiched between two electrodes. The average number of photons at saturation, Eq. (2), depends on the product of two parameters, Q_{opt} and ϵ, which can be on the order of 10^8 and 10^{-8} respectively. The average number of photons in the cavity is very sensitive to this product, with $N_{\text{Cas}}^{\max} = 1.4, 13, 740$ for values of $Q_{\text{opt}}\epsilon = 0.5, 1, 2$ for a vacuum state with $N_0 = 1$, respectively. The expected saturated power initiated by

200402-1

(a) Inversion of atomic population

(b) Generation and amplification of Casimir photons

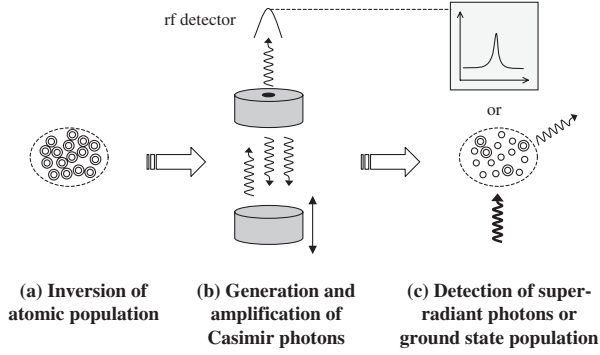**(c) Detection of super-radiant photons or ground state population**

FIG. 1.   Generation and detection of photons irradiated through vacuum-induced damping of motion. The two-level atoms are optically pumped to the maximum angular momentum state of the spin-orbit manifold which only allows for a single, down-ward, circularly polarized, magnetic dipole transition. The atoms are then sent through the cavity (a). The Casimir photons are generated through mechanical modulation of one FBAR resonator (b). An amplified superradiant pulse is triggered by a weak Casimir signal and detected by radio-frequency electronics or atomic fluorescence (c).

Casimir emission is

$$P_{\text{Cas}} = N_{\text{Cas}}^{\max} \frac{\hbar\omega}{\tau}. \tag{3}$$

For a 3.0 GHz FBAR resonator and a benchmark value of $Q_{\text{opt}}\epsilon \simeq 1$ at the edge of current technology, the saturated power 3×10^{-22} W is too low to be directly detectable. This demands the use of an efficient, nearly quantum-limited, photon detector in the radio-frequency range.

Detection of Casimir photons.—Ultrasensitive atomic detection schemes can be exploited for detecting Casimir photons by preparing an ensemble of population-inverted atoms in a hyperfine state whose transition frequency corresponds to the cavity resonance. An additional amplification process is available in which the weak Casimir signal triggers the stimulated emission of the ensemble of atoms. This effect is a form of superradiance [24,25]. One favorable feature to be exploited for the proposed scheme is that the hyperfine splitting of the ground states for alkali-metal atoms ranges from 0.2 GHz for Li to 9 GHz for Cs, conveniently matching the operating frequencies of FBAR resonators achieved or achievable in the near future [26]. The hyperfine transition in the ground state occurs through a magnetic dipole interaction, and its natural lifetime in free space is approximately

$$T_1 \approx \frac{3\pi\epsilon_0\hbar c^5}{\mu_B^2\omega^3}, \tag{4}$$

where μ_B is the Bohr magneton and ϵ_0 the electric permittivity in vacuum. This natural lifetime in free space is favorably reduced inside a resonant cavity due to the modification of density of states [28,29]

$$T_1^{\text{cav}} = \frac{4\pi^2}{3Q_{\text{opt}}} \frac{V}{\lambda^3} T_1, \tag{5}$$

where V is the cavity volume. For a few GHz cavity with 1 cm^2 cross-sectional area and $Q_{\text{opt}} = 10^8$, the natural lifetime is reduced by a factor of 10^{10}. In spite of this cavity-enhanced spontaneous rate, the typical hyperfine transition lifetime for the alkali-metal atoms is still impractically long, on the order of 10^3–10^5 s. The super-radiant lifetime—the characteristic time scale for superradiant evolution when N_{at} atoms are enclosed within the cavity—is $T_{\text{SR}} = T_1^{\text{cav}}/N_{\text{at}}$. Hence the emission time scale for the experiment is further reduced in the milli-second range for $N_{\text{at}} \approx 10^8$ or less. The peak power of the superradiant pulse is

$$P_{\text{SR}} = N_{\text{at}} \frac{\hbar\omega}{T_{\text{SR}}}, \tag{6}$$

increasing quadratically with the number of atoms. Considering as before a few GHz resonator with 10^8 atoms and $T_{\text{SR}} = 10^{-3}$ s, yields $P_{\text{SR}} = 10^{-13}$ W, a billionfold improvement over the initial power estimated in Eq. (3).

The superradiant emission can be detected in either of two ways. First, a power or field detector can be coupled to the cavity. The detector should be fast enough to resolve one superradiant lifetime. Such direct measurement would be preferred although the coupling mechanism itself is likely to reduce the quality factor of the cavity significantly in order to attain sufficient coupling efficiency. Micro-bolometers mounted on etched "spiderwebs" have an ultimate sensitivity of 10^{-16} W/$\sqrt{\text{Hz}}$ in the GHz range [30]. Spectrum analyzers are sensitive to sub-fW rf power of kHz bandwidth [31], and the temporal profile of the burst can be reconstructed through vector analysis. Second, the exiting atoms can be interrogated resonantly with the lower hyperfine state to ascertain the lower state population and therefore the energy released into the cavity. Either D-line fluorescence or ionization current can be monitored at the thousand atom level sensitivity [32]. The average delay time can be inferred by varying the time the atoms stay within the cavity. Coherent D-line excitation may also generate free-induction decay, due to the coherent magnetic moment developed on the hyperfine transition as a consequence of the amplification process, that would have a clearer signature. The two detection techniques are complementary to each other, and could be used in coincidence to further reject spurious signals.

Background rejection.—Casimir-generated photons are not the only seed to trigger the stimulated amplification process. In particular, any atom decaying spontaneously will also trigger a superradiant burst. In this case, the process is more commonly known as superfluorescence. The temporal envelope of the photon burst allows for discrimination among the triggering sources. Indeed, the average delay between the initial stimulation of the atomic population depends on the number of atoms and resonant photons N_{ph} initially present [27]:

$$T_{\text{D}} = T_{\text{SR}} \ln\left[\frac{N_{\text{at}}}{1 + N_{\text{ph}}}\right]. \qquad (7)$$

The delay is typically around ten superradiant lifetimes but with an inherent uncertainty due to quantum fluctuations. Both the delay and its uncertainty shrink as the number of initial photons increases. Measuring the delay can then indicate the number of initial Casimir photons. Tailoring the atomic number can further distinguish the Casimir signal from superfluorescent pulses. In order for the superradiant pulse to develop fully, the growth rate must exceed any decay process, which is primarily Doppler dephasing in the atomic cloud, and the atoms must remain in the interaction region for a time longer than the delay time. Then superfluorescence will be suppressed relative to Casimir superradiance provided that the atoms will be removed from the cavity after the expected Casimir delay time but prior to the superfluorescence delay $T_{\text{D}}^{(0)}$ [obtained with $N_{\text{ph}} = 0$ in Eq. (7)].

Experimental approach.—The Casimir photon population is allowed to reach saturation before introducing the prepared atoms. The atoms can be trapped and cooled with standard magneto-optical techniques, optically pumped and then transported into the cavity via optical tweezers. The existing photons then trigger a coherent pulse so long as the superradiant delay time is less than the cavity hold time. The direct use of an atomic beam is prevented by the short interaction time available in this configuration. While, based on Eq. (6), it looks advantageous to increase the number of atoms, an upper bound is imposed by the necessity to resolve the delay time as in Eq. (7). A major advantage of the proposed scheme is that outside the cavity the atoms are effectively inert due to the long hyperfine lifetime. Furthermore, the atoms are not resonant with the direct emission of photons at frequency Ω originating from antenna dipole irradiation due to the mechanical oscillation.

The atom number, N_{at}, and the interaction time are the primary adjustable parameters. The maximum sensitivity is obtained when the former is adjusted so that the superradiant lifetime is comparable to the detector speed (or the transfer speed for the interrogation technique) and the latter is slightly less than the superfluorescent delay time. In Table I, we summarize the various time scales and photon production rates involved in our proposed scheme for hyperfine transitions of different alkali-metal species. Lithium is not a practical candidate due to the large cavity size, whereas cesium and rubidium require mechanical frequencies not presently available. In this regard, sodium looks promising instead, with many individual steps of our proposed experiment already demonstrated. Sodium atoms in the maximum amount of 10^8 have been trapped at a temperature of $T = 100 \, \mu$k in a Bose condensed state [33], radio-frequency transitions between hyperfine states have been intentionally driven [34], and superradiance phenomena have been observed [35]. Concerning the de-

TABLE I. Summary of relevant parameters and time scales involved in the superradiance dynamics for different alkali-metal isotopes already cooled in the μK range. The maximum number of atoms $N_{\text{at}}^{\text{max}}$ is chosen so that T_{SR} equals the assumed detector response time of 0.5 ms. The length of the electromagnetic cavity is chosen to match the hyperfine transition $L = 2c/\nu$ at frequency $\nu = \omega/2\pi$. Optical quality factor $Q_{\text{opt}} = 10^8$ of the cavity and modulation depth $\epsilon = 10^{-8}$ are assumed, yielding $N_{\text{Cas}}^{\text{max}} = 13$.

	^{6}Li	^{23}Na	^{87}Rb	^{133}Cs
ν (GHz)	0.228	1.77	6.83	9.19
L (mm)	657	84.6	21.9	16.3
T_1 (s)	8.4×10^{16}	1.8×10^{14}	3.1×10^{12}	1.3×10^{12}
T_1^{cav} (s)	3.2×10^5	4.1×10^4	1.1×10^4	8.0×10^3
$N_{\text{at}}^{\text{max}}$	6.4×10^8	8.2×10^7	2.2×10^7	1.6×10^7
$T_{\text{D}}^{(0)}$ (ms)	10.1	9.1	8.5	8.3
T_{D} (ms)	8.8	7.8	7.1	7.0
P_{Cas} (W)	2.8×10^{-23}	1.7×10^{-21}	2.5×10^{-20}	4.6×10^{-20}
P_{SR} (W)	1.9×10^{-13}	1.9×10^{-13}	2.0×10^{-13}	1.9×10^{-13}

tection speed, both microbolometers and heterodyne receivers are fast enough to resolve the shortest achievable pulse. The difference in delay times is a few T_{SR} and so to suppress the superfluorescence, the atom transfer time out of the cavity should be at most T_{SR}. Given that the transfer rate with optical tweezers is limited by the mechanical drive moving the focusing lens to roughly 10 cm/s, corresponding to a transfer time of 100 ms in and out of the cavity [36], the number of atoms required is $N_{\text{at}} = 6 \times 10^5$, corresponding to a peak power of 2×10^{-15} W.

The key parameter in our scheme is the optical quality factor. Assuming a relative error in the determination of the delay time of 10%, a situation with $Q_{\text{opt}}\epsilon = 1$ determines the borderline for the temporal discrimination between superradiance induced by Casimir photons and superfluorescence, with a significantly improved signal for $Q_{\text{opt}}\epsilon$ larger than unity. Quality factors of 10^8 [37] and 10^{10} [38] have been reported for open, Gaussian, superconducting cavities. While, as already mentioned, the transient decrease of the cavity transmission occurs on a time scale longer than the superradiant pulse, care must be taken to minimize the losses introduced by the FBAR resonator as well as ports to admit the atoms and monitor the rf power.

Scaling the size of existing resonators and dissipation of heat are other issues to be carefully addressed. Given a typical size of current FBARs of $\simeq 500 \, \mu$m^2, increasing the FBAR to 1 cm^2 could adversely introduce additional acoustic modes as well as enhance the risk of a pinhole breakdown through the AlN film. The power required to drive the FBAR is obtained by considering the kinetic energy of a vibrating material whose energy is dissipated in the time scale of Q_{m}/Ω, where Q_{m} is the mechanical quality factor, which gives $P = \rho V \Omega^3 \delta x^2 / 4Q_{\text{m}}$. Expressing the volume of the vibrating body V in terms of the cross-sectional area A and the thickness of the material,

one half of the acoustic wavelength $2\pi v_a/\Omega$, we obtain $P_{\text{FBAR}} = \rho A v_a \pi^3 \epsilon^2 c^2/Q_{\text{m}}$, independent of frequency. For a cross-sectional area of 1 cm^2, $\epsilon = 10^{-8}$, $\rho = 3.3 \times 10^3$ kg/m^3, and $v_a = 10\,400$ m/s for aluminum nitride with a typical mechanical quality factor $Q_{\text{m}} = 10^3$, the dissipated power is about 3 W [39], smaller than the maximum threshold power of $\simeq 10$ W applicable to a FBAR resonator without damaging it [40]. In principle, the mechanical quality factor can be increased up to 4000 at room temperature, even larger at cryogenic temperatures, by a careful design of the multiple reflection layer and the refinement of an annealing process [41]. To minimize the moving boundary area to reduce heat load and fabrication difficulty, the cavity could be a hollow, coaxial waveguide terminated with length equal to half the resonant wavelength. Finally, the thermal contribution to the initial photon population N_0 at 10 mK is $N_{\text{therm}} = 6 \times 10^{-4}$, negligible with respect to the Casimir contribution [42].

Conclusions.—We have proposed an experiment involving superradiant amplification to detect the dynamical Casimir photons generated by a vibrating wall in an electromagnetic cavity. Although the observation of radiated photons is limited by the current technology, the use of superradiant atoms should overcome the technical limits and make their unambiguous detection possible. The technology currently available for mechanical resonators, the analysis of various alkali-metal atoms, and the interplay of the time scales indicate that a detection scheme based upon use of sodium atoms should have realistic chances to detect photons radiated by nonuniformly accelerating bodies in quantum vacuum.

[1] E. Schrödinger, Physica (Amsterdam) **6**, 899 (1939).
[2] H. G. B. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).
[3] M. J. Sparnaay, Physica (Amsterdam) **24**, 751 (1958).
[4] G. Bressi, G. Carugno, R. Onofrio, and G. Ruoso, Phys. Rev. Lett. **88**, 041804 (2002).
[5] P. H. G. M. van Blokland and J. T. G. Overbeck, J. Chem. Soc., Faraday Trans. 1 **74**, 2637 (1978).
[6] S. K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997).
[7] U. Mohideen and A. Roy, Phys. Rev. Lett. **81**, 4549 (1998).
[8] H. B. Chan et al., Science **291**, 1941 (2001).
[9] R. S. Decca, D. López, E. Fischbach, and D. E. Krause, Phys. Rev. Lett. **91**, 050402 (2003).
[10] T. Ederth, Phys. Rev. A **62**, 062104 (2000).
[11] G. T. Moore, J. Math. Phys. (N.Y.) **11**, 2679 (1970).
[12] S. A. Fulling and P. C. W. Davies, Proc. R. Soc. A **348**, 393 (1976).
[13] M.-T. Jaekel and S. Reynaud, Rep. Prog. Phys. **60**, 863 (1997).
[14] A. Lambrecht, M.-T. Jaekel, and S. Reynaud, Phys. Rev. Lett. **77**, 615 (1996).
[15] J. Opt. B, edited by G. Barton, V. V. Dodonov, and V. I. Man'ko (Institute of Physics Publishing, Bristol, 2005), Vol. 7.
[16] V. V. Dodonov and A. B. Klimov, Phys. Rev. A **53**, 2664 (1996).
[17] G. Plunien, R. Schützhold, and G. Soff, Phys. Rev. Lett. **84**, 1882 (2000).
[18] M. Crocce, D. A. R. Dalvit, and F. D. Mazzitelli, Phys. Rev. A **64**, 013808 (2001); **66**, 033811 (2002).
[19] R. Onofrio, in *Proceedings of the Sixth International Workshop on Quantum Fields under External Conditions*, edited by K. A. Milton (Rinton, Princeton, 2004).
[20] T. Zhang et al., Chin. Phys. Lett. **22**, 694 (2005).
[21] R. Ruby, P. Bradley, J. D. Larson III, and Y. Oshmyansky, Electron. Lett. **35**, 794 (1999).
[22] A. N. Cleland and M. R. Geller, Phys. Rev. Lett. **93**, 070501 (2004).
[23] R. Gabl et al., Biosens. Bioelectron. **19**, 615 (2004).
[24] M. G. Benedict, A. M. Ermolaev, V. A. Malyshev, I. V. Sokolov, and E. D. Trifonov, *Superradiance: Multiatomic Coherent Emission* (Institute of Physics Publishing, Philadelphia, 1996).
[25] M. Gross and S. Haroche, Phys. Rep. **93**, 301 (1982).
[26] An alternative to the use of hyperfine states as atomic probes in the radio-frequency range is provided by circular Rydberg atoms as demonstrated in the superradiant regime in [27]. The use of Rydberg atoms has also been proposed for the detection of Casimir photons [14,16]. However, the strong coupling of electric dipole interaction makes Rydberg states far too sensitive, and therefore short lived, to be practically employed to detect the expected photon rate.
[27] P. Goy et al., Phys. Rev. A **27**, 2065 (1983).
[28] E. M. Purcell, Phys. Rev. **69**, 681 (1946).
[29] P. Goy, J. M. Raimond, M. Gross, and S. Haroche, Phys. Rev. Lett. **50**, 1903 (1983).
[30] A. D. Turner et al., Appl. Opt. **40**, 4921 (2001).
[31] Agilent Technologies, Inc., Palo Alto, CA.
[32] C. Y. Chen et al., Science **286**, 1139 (1999).
[33] R. Onofrio et al., Phys. Rev. Lett. **84**, 810 (2000).
[34] A. Görlitz et al., Phys. Rev. Lett. **90**, 090401 (2003).
[35] S. Inouye et al., Science **285**, 571 (1999); Phys. Rev. Lett. **85**, 4225 (2000); Nature (London) **402**, 641 (1999).
[36] T. L. Gustavson et al., Phys. Rev. Lett. **88**, 020401 (2002).
[37] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73**, 565 (2001).
[38] G. Rempe, F. Schmidt-Kaler, and H. Walther, Phys. Rev. Lett. **64**, 2783 (1990).
[39] In the attempt to overcome various issues related to mechanical resonators an alternative scheme for the photon generation has been proposed in: C. Braggio et al., Europhys. Lett. **70**, 754 (2005). In this Letter, an unrealistically large dissipated power of 3×10^8 W has been incorrectly estimated based on the damping of the entire vibrational energy in one cycle of oscillation, regardless of the mechanical quality factor of the resonator.
[40] C. Ballou (private communication).
[41] G. Yoon and J. D. Park, Electron. Lett. **36**, 1435 (2000); M. Yim, D. H. Kim, D. Chai, and G. Yoon, Electron. Lett. **39**, 1638 (2003).
[42] A. Lambrecht, M. T. Jaekel, and S. Reynaud, Europhys. Lett. **43**, 147 (1998).