Signatures for Multi-α-Condensed States

Tz. Kokalova,[1,*] N. Itagaki,[1,2] W. von Oertzen,[1,3] and C. Wheldon[1]

[1]SF7, Hahn-Meitner-Institut Berlin, Glienicker Straße 100, D-14109 Berlin, Germany
[2]Department of Physics, University of Tokyo, Hongo, 113-0033 Tokyo, Japan
[3]Freie Universität Berlin, Fachbereich Physik, Arnimallee 14, D-14195 Berlin, Germany

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An experimental way of testing Bose-Einstein condensation of α clusters in the atomic nucleus is reported. The enhancement of cluster emission and the multiplicity partition of possible emitted clusters could be direct signatures for the condensed states. The barrier for the emission of clusters, such as ^{8}Be and ^{12}C$^{*}(0_2^+)$, is calculated and compared with the barrier for the sequential emission of 2 or 3α particles from the compound nucleus. For the calculations, a simple approach using a folded Woods-Saxon potential is used.

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Introduction.—The Bose-Einstein condensation of atoms is a recent and exciting research topic due to ongoing pioneering experiments. However, the study of related effects in atomic nuclei is much less advanced. Over the past couple of years, α condensation in the atomic nucleus has attracted increasing theoretical interest [1–6] centered around 0^+ states in light, α-conjugate nuclei. For these states, a description in which all the α particles occupy the same 0s orbital works well. The radial extension of these states is considerably increased compared to normal bound states, and, thus, the Pauli principle among α particles does not play an important role, and the state can be described as a condensate of α clusters. This ''condensed'' 0s orbital is completely different from the ''normal'' s orbit for each nucleon that arises from the single-particle potential. However, for the existence of such special levels, a potential for α clusters is needed that is very shallow and spreads widely in space.

The strongest candidates for condensed states in light nuclei appear around the threshold energy of $N\alpha$ systems. The ground state of ^{8}Be, which is just above the decay threshold energy for two α clusters, is the simplest example [1]. The second 0^+ state of ^{12}C [^{12}C$^{*}(0_2^+)$, $E_x =$ 7.65 MeV], which lies approximately 300 keV above the three-α-threshold energy and is well known as the Hoyle state [7], is also a strong candidate for a condensate. The wave function of this condensate and that of a microscopic three-α-model state from Ref. [8] have a squared overlap equal to more than 70% [3,4]. Furthermore, experimental efforts to confirm the theoretically proposed condensed state in ^{16}O, with $E_x \approx 14$ MeV, are ongoing [9].

It is challenging to extend this idea to heavier systems and clarify specific physical quantities that could be the unambiguous signature for an α-condensed state. The threshold states in heavier nuclei are considered to form states which have properties of a Bose gas (or liquid), because of their low excitation energy for decay into many α particles and the small relative kinetic energy between the α particles. In these states, the de Broglie wavelength of relative motion of the α particles is much larger than the radius of the nucleus, leading to an enhanced emission of coherent α particles [e.g., ^{8}Be and ^{12}C$^{*}(0_2^+)$]. Therefore, the emission of ^{8}Be and ^{12}C$^{*}(0_2^+)$ from moderately excited compound nuclei with $N \approx Z$ should, in various ways, give access to the special properties of these states, because of the coherent behavior of the α particles and properties of Bose-Einstein states in compound nuclei [10]. In this Letter, signatures of a multi-α-condensed state are investigated.

In a recent experimental study of ^{8}Be and ^{12}C$^{*}(0_2^+)$ emission using the germanium-detector array GASP and the silicon-detector sphere ISIS [11,12], a dramatic change in the population of the states of the residual nucleus has been observed. The ^{8}Be and ^{12}C$^{*}(0_2^+)$ have been detected as a pileup of signals in individual silicon detectors (i.e., correlated α particles). Using the reaction ^{28}Si + ^{24}Mg $\rightarrow$ ^{52}Fe* and recording particle-γ coincidences, the statistical (i.e., sequential) three-α-particle emission and ^{12}C$^{*}(0_2^+)$ emission have been compared. In the latter, a fourth α particle is emitted from the residual nucleus with a large probability, whereas the statistical α-chain decay practically ends with the emission of three-α particles. It is the nature of such observations that will be addressed here.

In previous work [5], the limiting number of α particles in a condensate is reported to be ten, due to the increase in the Coulomb energy. However, the case with an additional core (for example, ^{40}Ca) coupled to condensed α clusters is considered here. This stabilizes the state due to the core-condensate interaction.

Signatures for a multi-α-condensed state.—In the following two subsections, we propose two experimental signatures for the identification of a condensed state. We take, as an example, a condensate of α particles around a core of ^{40}Ca.

Enhancement of cluster emission.—One of the signatures for multi-α-condensed states is the enhancement of cluster emission due to the decrease in the barrier for such decays. Such an enhanced cluster emission [^{8}Be and ^{12}C$^{*}(0_2^+)$], using the reaction ^{28}Si + ^{24}Mg $\rightarrow$ ^{52}Fe* at a beam energy of 130 MeV [12,13], has already been detected. The big differences observed in the γ-ray spectra between the sequential α-particle emission and cluster emission show that the clusters are preformed in the compound nucleus and subsequently emitted. Furthermore, the kinetic energy released by $^{12}C^*(0_2^+)$ emission is much smaller than that during the competing, sequential three-α emission. These features are considered to be characteristic of α-condensed states. Here we explain the decrease in the Coulomb barrier as a mechanism for these phenomena.

In general, α-condensed states can be described by the following wave function:

$$\Psi = \int d\vec{R}_1 d\vec{R}_2 \cdots d\vec{R}_n \mathcal{A} G_1(\vec{R}_1) G_2(\vec{R}_2) G_3(\vec{R}_3) \cdots G_n(\vec{R}_n) \times \exp[-(\vec{R}_1^2 + \vec{R}_2^2 + \vec{R}_3^2 \cdots + \vec{R}_n^2)/b^2] = \mathcal{A} \prod_{i=1}^{n} \int d\vec{R}_i G_i(\vec{R}_i) \exp[-\vec{R}_i^2/b^2]. \quad (1)$$

All the nucleons are antisymmetrized by the operator $\mathcal{A}$, and four nucleons in each α cluster are described by a Gaussian-type wave function $[G_i(\vec{R}_i)]$ centered at $\vec{R}_i$. Here the wave function is integrated over the parameters $\{\vec{R}_i\}$. Thus, all α clusters occupy the same s orbit with the oscillator parameter b, and this parameter b is extremely large (~ 10 fm) for a condensed state [1]. When the wave function has such a spatial extension, the folded potential has a much lower Coulomb barrier compared to the case with a normal wave function. This difference can be estimated in a simplified way. Suppose that N α clusters in a condensed state are emitted, for example, from a compound nucleus with a ^{40}Ca core. We introduce a parameter R, which describes the center of mass of these N α particles forming a condensed state. The folded potential $V(R)$, calculated as a function of R, is

$$V(R) = F \int dr r^2 v(r) \exp[-2(r-R)^2/b^2], \quad (2)$$

where F is a normalization coefficient. Here $\exp[-2(r-R)^2/b^2]$ is the density distribution of the emitted system and $v(r)$ is an interaction between the ^{40}Ca core (20 protons and 20 neutrons) and the emitted system ($2N$ protons and $2N$ neutrons) and comprises Woods-Saxon and Coulomb potentials:

$$v(r) = 4N \times \frac{V_0}{1 + \exp(r - R_0/d)} + 20 \times 2N \times \frac{e^2}{r}, \quad (3)$$

where V_0, R_0, and d are standard values of -50 MeV, 5.8 fm, and 0.65 fm, respectively.

In Fig. 1, the folded potential V for $^{12}C^*(0_2^+)$ emission ($N = 3$) is shown as a function of the distance R between the $^{12}C^*(0_2^+)$ and the ^{40}Ca core. The solid, dashed, and dotted lines correspond to the cases of α clusters in a condensed state ($1/b^2 = 0.01$ fm^{-2}), cluster state ($1/b^2 = 0.081$ fm^{-2}), and normal (ground) state ($1/b^2 = 0.128$ fm^{-2}), respectively. As can be seen from the figure, the barrier is significantly lower for a condensate. For example, the difference in the barrier heights between the ground (cluster) state and the condensate is 15.6 (13.0) MeV at $R = 10$ fm. Therefore, for example, if the system has an excitation energy of 10 MeV, the condensate can decay via $^{12}C^*(0_2^+)$ emission, whereas for the statistical 3α state the probability to penetrate the barrier will be negligible.

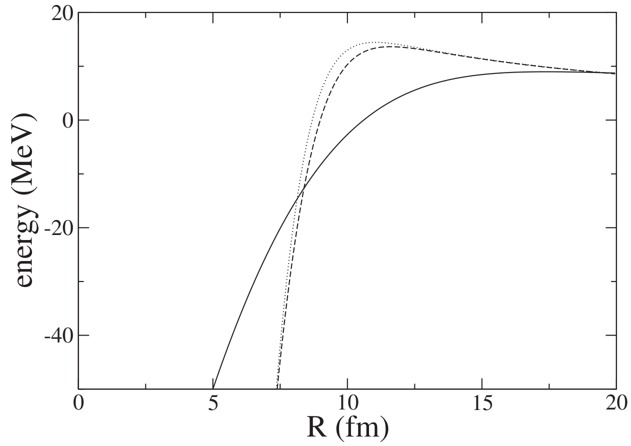

FIG. 1. The folded potential (V in the text) for ^{12}C emission as a function of the distance (R in the text) between the ^{12}C and the ^{40}Ca core. The solid, dashed, and dotted lines correspond to the condensed, cluster, and ground states of ^{12}C, respectively.

In such a case, if one can populate the condensate, a higher α multiplicity compared to statistical emission can be expected, which agrees with our recent experimental results [12]. Also, this is considered to be the main reason the kinetic energy released during $^{12}C^*(0_2^+)$ emission from the $^{52}Fe^*$ compound nucleus is drastically smaller than in the sequential emission process of three-α particles.

To enable a comparison between the statistical process and condensate decay, the folded potential V for sequentially emitted α particles as a function of the distance R is shown in Fig. 2.

After the calculations of the barrier in the case of cluster emission, we can compare the difference in the barriers at 10 fm for the condensate, cluster, and ground states in ^{12}C (see Fig. 1). The calculated difference between the ground or cluster state and the condensate is 10–15 MeV. For a good comparison of the calculated barriers to the experiment, observations of the minimum energy (i.e., the beginning of the energy distribution curves) are needed. In the experiment in Ref. [12], this was not possible because of the Si detectors and associated electronics ($E_\alpha^{\min} = 14.9$ MeV). However, it is thought that the difference between the maxima of the energy distributions is propor-

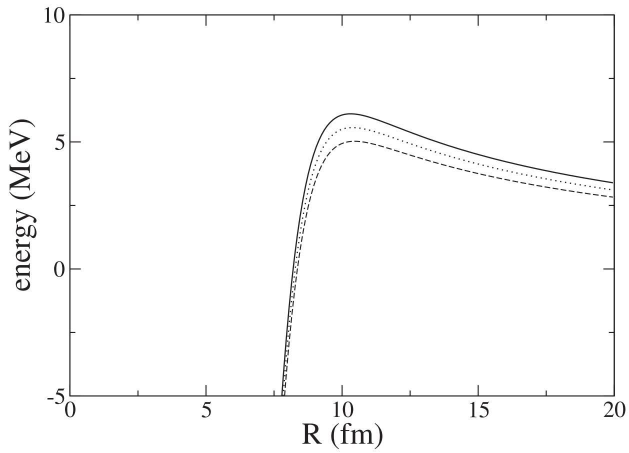

FIG. 2. The folded potential (V in the text) for the sequential emission of α particles as a function of the distance (R in the text) between an α particle and the core nucleus. The solid, dotted, and dashed lines correspond to ^{48}Cr (compound 1α), ^{44}Ti (compound $2\alpha s$), and ^{40}Ca (compound $3\alpha s$) residual nuclei, respectively.

tional to the difference in the starting energy, which corresponds approximately to the difference between the barriers. (Note that the difference between the statistical-α and coherent-α emission maxima in Ref. [12] is $\approx$ 14 MeV.) A new experiment at Grand Accelerateur National d'Ions Lourdes, France, is planned, which will provide the opportunity to measure the full energy distributions, enabling an extensive comparison between the theoretical calculations and the experimental data.

Other results on the γ-ray spectroscopy in coincidence with cluster emission [e.g.,^{12}C$^*(0_2^+)$] have been reported by Torilov *et al.* in Ref. [11], for which this special reaction mechanism provided the opportunity to populate levels selectively in the residual nuclei (in this case, ^{40}Ca). Furthermore, because of the lower energy removed from the compound nucleus in the case of ^{12}C$^*(0_2^+)$ emission compared to the statistical emission, following cluster emission an additional α channel is populated, namely, ^{36}Ar. These results are in good agreement with extended Hauser-Feshbach calculations [14], which include the statistical and light-cluster emission processes but not condensation. For the ratio between the 3α channel and ^{12}C$^*(0_2^+)$, the calculations give 4.8 and what we experimentally observe is 3.1. Note that these values are without taking into account the efficiency for the detection of different particles and that in the experimental setup used there was a gap at forward angles, where the loss of particles will be bigger for the ^{12}C$^*(0_2^+)$ channel.

Extending the barrier calculations to a system with ten-α particles ($N = 10$) outside of the ^{40}Ca core, the difference is even more dramatic than for ^{12}C emission, as shown in Fig. 3. The solid and dotted lines correspond to α clusters in a condensed state ($1/b^2 = 0.01$ fm^{-2}) and a normal (ground) state ($1/b^2 = 0.045$ fm^{-2}), respectively. The difference in the barrier heights between these states at $R = 10$ fm is 27 MeV.

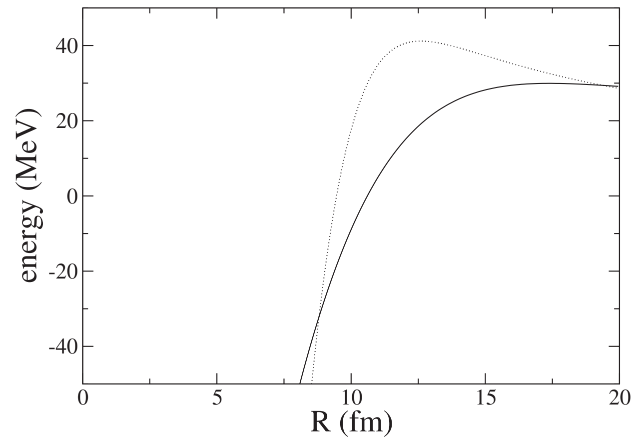

FIG. 3. The folded potential (V in the text) for the emission of ^{40}Ca as a function of the distance (R in the text) between two ^{40}Ca nuclei. The solid line and the dotted line correspond to the condensed and ground states of ^{40}Ca, respectively.

Multiplicity and partitions of emitted clusters.—Now the α multiplicity of the decay process is examined. For each residual nucleus, the ^{8}Be multiplicity, ^{12}C$^*(0_2^+)$ multiplicity, etc., can be experimentally determined from the particle-γ coincidences. The decay of the compound nucleus will consist of a large variety of partitions (decay modes) D represented by the widths Γ_i (the total width being Γ):

$$\Gamma = \sum_{i=1}^{D} \Gamma_i, \tag{4}$$

where

$$i = n_j\alpha + n_k{}^8\mathrm{Be} + n_l{}^{12}\mathrm{C}^*(0_2^+) + n_m{}^{16}\mathrm{O}^* + \cdots, \tag{5}$$

n_j, n_k, n_l, and n_m being different numbers of emitted particles j, k, l, and m, respectively, the sum $(1n_j + 2n_k + 3n_l + \cdots)$ being N. The partitions $(n_j, n_k, n_l, \ldots)$ are determined experimentally where decays with large n_k, n_l, etc., represent a signature of the decay of condensed states in the compound nucleus. Specifically, the total charge removed from the compound nucleus by these decays will considerably exceed the expected total charge removed by *statistical* decay via α particles and/or clusters.

Considering the role of the above outlined partitions in more detail, since in condensed states all of the α clusters occupy the same $0s$ orbit, any kind of partition into subsystems, which are also condensed states, is possible. This means that the initial compound state has equal probabilities for all of the possible partitions involving α-condensed subsystems. Experimentally, one could register a compound state consisting of, for example, ten condensed α particles $[\Psi(10\alpha)]$ and measure decay probabilities from the same level via different decay patterns, e.g., five ^{8}Be cluster systems ($|^{8}\mathrm{Be}\rangle$), or two ^{8}Be and two $^{12}\mathrm{C}^{*}(0_2^+)$ ($|^{12}\mathrm{C}\rangle$), or one ^{8}Be and two ^{16}O condensed states ($|^{16}\mathrm{O}\rangle$). This can be written as:

$$|\Psi(10\alpha)\rangle = C_1|^{8}\mathrm{Be}\rangle|^{8}\mathrm{Be}\rangle|^{8}\mathrm{Be}\rangle|^{8}\mathrm{Be}\rangle|^{8}\mathrm{Be}\rangle = C_2|^{8}\mathrm{Be}\rangle|^{8}\mathrm{Be}\rangle|^{12}\mathrm{C}\rangle|^{12}\mathrm{C}\rangle = C_3|^{8}\mathrm{Be}\rangle|^{16}\mathrm{O}\rangle|^{16}\mathrm{O}\rangle\ldots, \quad (6)$$

and the coefficients $(C_1, C_2, C_3, \ldots)$ can be deduced by analyzing the decay probabilities and showing that they are equal. More specifically, the decay widths (Γ_i for the ith channel) and decay energies (E) are obtained directly from experiment and are dependent on the square of the overlap between the initial state and the observed channel ($|Ob_i\rangle$). The amplitudes for each channel (C_i) just below the barrier are such that:

$$\Gamma_i(E) \propto |\langle\Psi(10\alpha)|Ob_i\rangle|^2 = |C_i|^2 \prod_{j=1}^{N} P_j(E), \quad (7)$$

and here, penetration factors $[P_j(E)]$ for the Coulomb barrier can be calculated theoretically. Knowing the measured decay widths and substituting in the calculated penetration factors yields the penetration-factor coefficients $(C_1, C_2, C_3, \ldots)$, which, for a condensate, should all be the same. For the statistical decay of α particles, even if a cluster state is populated as the initial, compound nucleus state, these coefficients would be very different and this process can be separated from the decay of an α-condensed state. Specifically, condensed states emitting clusters and normal cluster emission can be distinguished.

Summary.—Two experimental signatures, valid for both light and heavy nuclei, for the decay of multi-α-condensed states have been discussed: observed decay widths and the difference in the kinetic energies of the emitted clusters, due to the large Coulomb barrier differences. It has been shown using a simple folded Woods-Saxon potential that such states can be distinguished experimentally from competing processes in which states decay via, for example, sequential α-particle emission. However, for the precise calculations of the barrier with the appropriate interaction, more theoretical input is needed.

Furthermore, using the measured decay widths to obtain the coefficients of the penetration factors, normal cluster emission and condensate decay via cluster emission are distinguishable.

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*Corresponding author.
Electronic address: kokalova@hmi.de

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