

Clean Prediction of CP -Violating Processes ψ , ϕ , and $Y(1S)$ Decay to $K_S K_S$ and $K_L K_L$

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The ratio of $K_S K_S$ ($K_L K_L$) and $K_S K_L$ production rates is calculated by considering $K^0 - \bar{K}^0$ oscillation in $J/\psi \rightarrow K^0 \bar{K}^0$ decay. The theoretical uncertainty due to strong interaction in J/ψ decay is completely canceled in the ratio; therefore, the absolute branching fractions of the CP -violating processes of $J/\psi \rightarrow K_S K_S$ and $K_L K_L$ can be cleanly and model-independently determined in the case that $J/\psi \rightarrow K_S K_L$ decay is precisely measured. In the future τ -charm factory, the expected CP violating process of $J/\psi \rightarrow K_S K_S$ should be reached. It is important to measure J/ψ to $K_S K_S$ and $K_S K_L$ decays simultaneously, so that many systematic errors will be canceled. More precise measurements are suggested to examine the predicted isospin relation in $J/\psi \rightarrow K \bar{K}$ decays. All results can be extended to decays of other vector quarkonia, ϕ , $\psi(2S)$, and $Y(1S)$, etc.

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In the standard model (SM), CP violation arises from an irreducible weak phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1]. CP violation has been established in both K and B systems. Currently, all experimental measurements are consistent with the CKM picture of CP violation, and the CKM is, very likely, the dominant source of CP violation in low energy flavor-changing processes [2]. However, the surprising point is that the CKM mechanism for CP violation fails to account for the baryogenesis [3]. It is crucial to probe CP violation in various reactions, to see the correlations between different processes and probe the source of CP violation.

In this Letter, we consider the possible CP asymmetric observation in $J/\psi \rightarrow K^0 \bar{K}^0$ decay, in which $K_S K_S$ and $K_L K_L$ pairs can be formed in addition to $K_S K_L$. Within the SM, the possible CP -violating decay processes of $J/\psi \rightarrow K_S K_S$ and $K_L K_L$ are due to $K^0 - \bar{K}^0$ oscillation, here we assume that possible strong multi-quark effects that involve sea-quarks play no role in $J/\psi \rightarrow K_S K_S$, $K_L K_L$, and $K_S K_L$ decays [4]. The J/ψ decays will provide another opportunity to understand the source of CP violation. The amplitude for J/ψ decaying to $K^0 \bar{K}^0$ is $\langle K^0 \bar{K}^0 | H | J/\psi \rangle$, and the $K^0 \bar{K}^0$ pair system is in a state with charge parity $C = -1$, which can be defined as

$$|K^0 \bar{K}^0\rangle^{C=-1} = \frac{1}{\sqrt{2}} [|K^0\rangle | \bar{K}^0\rangle - | \bar{K}^0\rangle | K^0\rangle]. \quad (1)$$

Although there is weak current contribution in $J/\psi \rightarrow K^0 \bar{K}^0$ decay, which may not conserve charge parity, the $K^0 \bar{K}^0$ pair cannot be in a state with $C = +1$. The reason is that the relative orbital angular momentum of the $K^0 \bar{K}^0$ pair must be $l = 1$ because of angular momentum conservation. A boson-pair with $l = 1$ must be in an antisymmetric state, the antisymmetric state of particle-anti-particle pair must be in a state with $C = -1$. This conclusion can also be illustrated by a direct calculation.

For explicitness, let us denote the particle state $|K^0\rangle$ with momentum p_1 as $|K^0(p_1)\rangle$, and $| \bar{K}^0\rangle$ with momentum p_2 as $| \bar{K}^0(p_2)\rangle$. The general structure of the effective Hamiltonian for $J/\psi \rightarrow K^0 \bar{K}^0$ decay is

$$H = C_W \sum_{i=d,s} \bar{q}_i \gamma^\mu (a - b \gamma_5) q_i \bar{c} \gamma_\mu (a' - b' \gamma_5) c, \quad (2)$$

where q_i ($i = d, s$) and c denote the quarks d, s and c , C_W is the Wilson coefficient, and a, b, a' , and b' are the relevant coefficients for the vector and axial-vector currents. Then the amplitude for J/ψ decaying into a state $|K^0(p_1) \bar{K}^0(p_2)\rangle$ is

$$\langle K^0(p_1) \bar{K}^0(p_2) | H | J/\psi \rangle = C_W a' \langle K^0(p_1) \bar{K}^0(p_2) | \sum_{i=d,s} \bar{q}_i \gamma^\mu (a - b \gamma_5) q_i | 0 \rangle \cdot \langle 0 | \bar{c} \gamma_\mu c | J/\psi \rangle, \quad (3)$$

where $\langle 0 | \bar{c} \gamma^\mu \gamma_5 c | J/\psi \rangle = 0$ has been used. From the Lorentz structure of the above matrix element, we have the following decomposition

$$\langle K^0(p_1) \bar{K}^0(p_2) | \sum_{i=d,s} \bar{q}_i \gamma^\mu (a - b \gamma_5) q_i | 0 \rangle = F^+(p_1 + p_2)^\mu + F^-(p_1 - p_2)^\mu, \quad (4)$$

where F^+ and F^- are Lorentz invariant form factors. For the vector current induced vacuum- J/ψ matrix element, there is the common decomposition

$$\langle 0 | \bar{c} \gamma_\mu c | J/\psi \rangle = f_{J/\psi} m_{J/\psi} \epsilon_\mu, \quad (5)$$

where $f_{J/\psi}$ is the decay constant, and ϵ_μ the polarization vector of J/ψ .

Substituting Eqs. (4) and (5) into Eq. (3), and using $\epsilon \cdot (p_1 + p_2) = \epsilon \cdot p = 0$, where p is the momentum of J/ψ , we can obtain

$$\langle K^0(p_1)\bar{K}^0(p_2)|H|J/\psi\rangle = C_W a' f_{J/\psi} m_{J/\psi} F^- \epsilon \cdot (p_1 - p_2). \quad (6)$$

Exchanging p_1 and p_2 , we can get another amplitude

$$\langle \bar{K}^0(p_1)K^0(p_2)|H|J/\psi\rangle = C_W a' f_{J/\psi} m_{J/\psi} F^- \epsilon \cdot (p_2 - p_1). \quad (7)$$

The above two equations directly give

$$\langle K^0(p_1)\bar{K}^0(p_2) + \bar{K}^0(p_1)K^0(p_2)|H|J/\psi\rangle = 0, \quad (8)$$

which shows that the amplitude for J/ψ decaying into the $K^0\bar{K}^0$ pair with charge parity $C = +1$ is zero. Therefore, only a state with $C = -1$ can be produced in $J/\psi \rightarrow K^0\bar{K}^0$ decay.

Note that Eq. (3) is based on the factorization approximation. However, the conclusion that the $K^0\bar{K}^0$ pair produced from a vector meson decay must be in the $C = -1$ state does not depend on the factorization approximation, it can be drawn directly from the momentum conservation. The above computation should be only viewed as a complementary illustration rather than demonstration.

Next we shall analyze the time evolution of the $K^0\bar{K}^0$ system produced in J/ψ decay.

The weak eigenstates of the $K^0 - \bar{K}^0$ system are $|K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$ and $|K_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle$ with eigenvalues $\mu_S = m_S - \frac{i}{2}\Gamma_S$ and $\mu_L = m_L - \frac{i}{2}\Gamma_L$, respectively, where the m_S and Γ_S (m_L and Γ_L) are the mass and width of K_S (K_L) meson. Following the $J/\psi \rightarrow K^0\bar{K}^0$ decay, the K^0 and \bar{K}^0 will go separately and the time evolution of the particle states $|K^0(t)\rangle$ and $|\bar{K}^0(t)\rangle$ are given by $|K^0(t)\rangle = \frac{1}{2p}(e^{-i\mu_S t}|K_S\rangle + e^{-i\mu_L t}|K_L\rangle)$ and $|\bar{K}^0(t)\rangle = \frac{1}{2q} \times (e^{-i\mu_S t}|K_S\rangle - e^{-i\mu_L t}|K_L\rangle)$, respectively. Then the time evolution of the $K^0 - \bar{K}^0$ system with $C = -1$ is

$$\begin{aligned} |K^0\bar{K}^0(t_1, t_2)\rangle^{C=-1} &= \frac{1}{\sqrt{2}}[|K^0(t_1)\rangle|\bar{K}^0(t_2)\rangle \\ &\quad - |\bar{K}^0(t_1)\rangle|K^0(t_2)\rangle] \\ &= \frac{1}{2\sqrt{2}pq}[g_{LS}|K_L\rangle|K_S\rangle \\ &\quad - g_{SL}|K_S\rangle|K_L\rangle], \end{aligned} \quad (9)$$

where $g_{LS} = e^{-i\mu_L t_1 - i\mu_S t_2}$ and $g_{SL} = e^{-i\mu_S t_1 - i\mu_L t_2}$. Since the states $|K_S\rangle$ and $|K_L\rangle$ are unorthogonal, we have $\langle K_S|K_L\rangle = \langle K_L|K_S\rangle = |p|^2 - |q|^2$ and $\langle K_S|K_S\rangle = \langle K_L|K_L\rangle = 1$. Then the amplitudes to find the $K_S K_S$, $K_S K_L$, $K_L K_S$, and $K_L K_L$ pairs are

$$\begin{aligned} A_1(t_1, t_2) &\equiv \langle K_S K_S|K^0\bar{K}^0(t_1, t_2)\rangle^{C=-1} \\ &= \frac{1}{2\sqrt{2}pq}[(|p|^2 - |q|^2)(g_{LS} - g_{SL})], \end{aligned} \quad (10)$$

$$\begin{aligned} A_2(t_1, t_2) &\equiv \langle K_S K_L|K^0\bar{K}^0(t_1, t_2)\rangle^{C=-1} \\ &= \frac{1}{2\sqrt{2}pq}[g_{LS} - (|p|^2 - |q|^2)g_{SL}], \end{aligned} \quad (11)$$

$$\begin{aligned} A_3(t_1, t_2) &\equiv \langle K_L K_S|K^0\bar{K}^0(t_1, t_2)\rangle^{C=-1} \\ &= \frac{1}{2\sqrt{2}pq}[(|p|^2 - |q|^2)g_{LS} - g_{SL}], \end{aligned} \quad (12)$$

$$\begin{aligned} A_4(t_1, t_2) &\equiv \langle K_L K_L|K^0\bar{K}^0(t_1, t_2)\rangle^{C=-1} \\ &= \frac{1}{2\sqrt{2}pq}[(|p|^2 - |q|^2)(g_{LS} - g_{SL})]. \end{aligned} \quad (13)$$

Therefore, one can find $A_1(t_1, t_2) = A_4(t_1, t_2)$ and $A_3(t_1, t_2) = -A_2(t_2, t_1)$. If CP is conserved, i.e., $|q/p| = 1$, or the two particles are observed at the same time, namely, $t_1 = t_2$, then $A_1(t_1, t_2) = A_4(t_1, t_2) = 0$ for $K_S K_S$ and $K_L K_L$ cases because Bose-Einstein statistics prevents two identical bosons from being in an antisymmetric state. In other words, only K_S and K_L can be seen at the same time in $J/\psi \rightarrow K^0\bar{K}^0$ decay.

Squaring the amplitudes $A_i(t_1, t_2)$'s, one can get the time-dependent possibilities to find $K_S K_S$, $K_S K_L$, and $K_L K_L$ pairs

$$\begin{aligned} \frac{d^2 \mathcal{P}[K_S(t_1), K_S(t_2)]}{dt_1 dt_2} &\equiv \mathcal{N}_f |A_1(t_1, t_2)|^2 \\ &= \mathcal{N}_f \frac{(|p|^2 - |q|^2)^2}{4|pq|^2} \\ &\quad \times e^{-\Gamma(t_1+t_2)}[\cosh(y\Gamma(t_2 - t_1)) \\ &\quad - \cos(x\Gamma(t_2 - t_1))], \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{d^2 \mathcal{P}[K_L(t_1), K_S(t_2)]}{dt_1 dt_2} &\equiv \mathcal{N}_f |A_2(t_1, t_2)|^2 \\ &= \mathcal{N}_f \frac{1}{8|pq|^2} e^{-\Gamma(t_1+t_2)} [e^{y\Gamma(t_2-t_1)} \\ &\quad - 2(|p|^2 - |q|^2)^2 \cos(x\Gamma(t_2 - t_1)) \\ &\quad + (|p|^2 - |q|^2)^4 e^{-y\Gamma(t_2-t_1)}], \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{d^2 \mathcal{P}[K_S(t_1), K_L(t_2)]}{dt_1 dt_2} &\equiv \mathcal{N}_f |A_3(t_1, t_2)|^2 \\ &= \mathcal{N}_f |A_2(t_2, t_1)|^2, \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{d^2 \mathcal{P}[K_L(t_1), K_L(t_2)]}{dt_1 dt_2} &\equiv \mathcal{N}_f |A_4(t_1, t_2)|^2 \\ &= \mathcal{N}_f |A_1(t_1, t_2)|^2, \end{aligned} \quad (17)$$

where \mathcal{N}_f is a common normalization factor, $\Gamma = \frac{\Gamma_S + \Gamma_L}{2}$, $x = \frac{\Delta m}{\Gamma}$, and $y = \frac{\Delta\Gamma}{2\Gamma}$ (Δm is the mass difference of K_L and K_S , i.e., $\Delta m = m_L - m_S$, while $\Delta\Gamma = \Gamma_L - \Gamma_S$ is the width difference).

The time-integrated possibilities to observe $K_S K_S$, $K_S K_L$, and $K_L K_L$ pairs, which are normalized by the widths of K_S and K_L , i.e., $\mathcal{N}_f = \Gamma_S \Gamma_L$, are

$$\begin{aligned} \mathcal{P}(K_S, K_S) &\equiv \Gamma_S \Gamma_L \int_0^\infty dt_2 \int_0^\infty dt_1 |A_1(t_1, t_2)|^2 \\ &= \frac{\Gamma_S \Gamma_L}{\Gamma^2} \frac{(|p|^2 - |q|^2)^2}{4|pq|^2} \left(\frac{1}{1-y^2} - \frac{1}{1+x^2} \right), \end{aligned} \quad (18)$$

$$\begin{aligned} \mathcal{P}(K_S, K_L) &\equiv \Gamma_S \Gamma_L \int_0^\infty dt_2 \int_0^\infty dt_1 [|A_2(t_1, t_2)|^2 + |A_3(t_1, t_2)|^2] \\ &= \frac{\Gamma_S \Gamma_L}{\Gamma^2} \frac{1}{4|pq|^2} \left(\frac{1}{1-y^2} - \frac{2(|p|^2 - |q|^2)^2}{1+x^2} \right. \\ &\quad \left. + \frac{(|p|^2 - |q|^2)^4}{1-y^2} \right), \end{aligned} \quad (19)$$

$$\begin{aligned} \mathcal{P}(K_L, K_L) &\equiv \Gamma_S \Gamma_L \int_0^\infty dt_2 \int_0^\infty dt_1 |A_4(t_1, t_2)|^2 \\ &= \mathcal{P}(K_S, K_S). \end{aligned} \quad (20)$$

The final amplitude squared for the decay process $J/\psi \rightarrow K_1 K_2$, where $K_1 K_2$ can be $K_S K_S$, $K_S K_L$, or $K_L K_L$, which is from the time evolution of the $K^0 - \bar{K}^0$ system, is

$$|A(J/\psi \rightarrow K_1 K_2)|^2 = \mathcal{P}(K_1, K_2) |\langle K^0 \bar{K}^0 | H | J/\psi \rangle|^2, \quad (21)$$

where $\mathcal{P}(K_1, K_2)$ can be obtained from Eqs. (18)–(20). Finally, the partial widths of the $J/\psi \rightarrow K_S K_S$, $K_S K_L$, and $K_L K_L$ are

$$\Gamma(\psi \rightarrow K_S K_S) = \mathcal{P}(K_S, K_S) C |\langle K^0 \bar{K}^0 | H | \psi \rangle|^2, \quad (22)$$

$$\Gamma(\psi \rightarrow K_S K_L) = \mathcal{P}(K_S, K_L) C |\langle K^0 \bar{K}^0 | H | \psi \rangle|^2, \quad (23)$$

$$\Gamma(\psi \rightarrow K_L K_L) = \mathcal{P}(K_L, K_L) C |\langle K^0 \bar{K}^0 | H | \psi \rangle|^2, \quad (24)$$

where $C = \frac{1}{3} \frac{1}{8\pi} \frac{|\mathbf{P}|}{m_{J/\psi}^2}$ is the phase space factor, $m_{J/\psi}$ is the J/ψ mass and \mathbf{P} is the three-momentum of the final particles in the rest frame of J/ψ . Combining Eqs. (20), (22), and (24), one can get $\Gamma(J/\psi \rightarrow K_S K_S) = \Gamma(J/\psi \rightarrow K_L K_L)$. The ratio R_{SS} (R_{LL}) of $K_S K_S$ ($K_L K_L$), and $K_S K_L$ production rates is obtained

$$R_{SS} \equiv \frac{\Gamma(J/\psi \rightarrow K_S K_S)}{\Gamma(J/\psi \rightarrow K_S K_L)} = (|p|^2 - |q|^2)^2 \frac{x^2 + y^2}{1 + x^2}, \quad (25)$$

and

$$R_{LL} \equiv \frac{\Gamma(J/\psi \rightarrow K_L K_L)}{\Gamma(J/\psi \rightarrow K_S K_L)} = R_{SS}. \quad (26)$$

In the ratios, the phase space factor C and the strong matrix element squared $|\langle K^0 \bar{K}^0 | H | J/\psi \rangle|^2$ are completely canceled, which ensures that the ratios are completely free from uncertainty caused by strong interaction in $J/\psi \rightarrow K^0 \bar{K}^0$ decay. The expected value of the ratio, R_{SS} (R_{LL}), can be determined by using experimental measured values of x , y and $|p|^2 - |q|^2$, where $|p|^2 - |q|^2$ is related to the CP asymmetry parameter δ_L in semileptonic K_L decay

$$\begin{aligned} \delta_L &\equiv \frac{\Gamma(K_L \rightarrow l^+ \nu \pi^-) - \Gamma(K_L \rightarrow l^- \nu \pi^+)}{\Gamma(K_L \rightarrow l^+ \nu \pi^-) + \Gamma(K_L \rightarrow l^- \nu \pi^+)} \\ &= |p|^2 - |q|^2. \end{aligned} \quad (27)$$

One can further express R_{SS} (R_{LL}) completely in terms of mixing and CP asymmetry parameters in kaon decays, by combining Eqs. (25)–(27)

$$R_{SS} = R_{LL} = (\delta_L)^2 \frac{x^2 + y^2}{1 + x^2}, \quad (28)$$

where the experimental result of δ_L is $(3.27 \pm 0.12) \times 10^{-3}$ [5]. From the measured values of mass difference and widths of K_L and K_S [5], one can get $x = 0.946$ and $y = -0.997$. The uncertainties on x and y are negligible. Therefore, $\frac{x^2 + y^2}{1 + x^2} \approx 1$ and $R_{SS} = R_{LL} \approx (\delta_L)^2$. The dominated uncertainty of R_{SS} (R_{LL}) is from the error of the measured value of δ_L . The values of the ratios R_{SS} and R_{LL} can be obtained

$$R_{SS} = R_{LL} = (10.66 \pm 0.78) \times 10^{-6}, \quad (29)$$

which are model independent and theoretically clean. The total uncertainty of R_{SS} (R_{LL}) is about 7%, and can be improved in the future if more precise measurement of the CP violation parameter in $K_L \rightarrow l^\pm \nu \pi^\mp$ decays is obtained. In general, one can measure J/ψ to $K_S K_S$ and $K_S K_L$ decays simultaneously in the future τ -charm factory, so that some of the systematic errors can be canceled in the ratio. It is very interesting that the ratio can be extended to other quarkonia which can decay into $K\bar{K}$ final states, for example, in ϕ , $\psi(2S)$, or $\Upsilon(1S) \rightarrow K^0 \bar{K}^0$ decays, we have

$$\begin{aligned} R_{SS} &\equiv \frac{\Gamma(J/\psi \rightarrow K_S K_S)}{\Gamma(J/\psi \rightarrow K_S K_L)} = \frac{\Gamma(\psi(2S) \rightarrow K_S K_S)}{\Gamma(\psi(2S) \rightarrow K_S K_L)} \\ &= \frac{\Gamma(\phi \rightarrow K_S K_S)}{\Gamma(\phi \rightarrow K_S K_L)} = \frac{\Gamma(\Upsilon(1S) \rightarrow K_S K_S)}{\Gamma(\Upsilon(1S) \rightarrow K_S K_L)}. \end{aligned} \quad (30)$$

With current available experimental data, such as, $\mathcal{B}(J/\psi \rightarrow K_S K_L) = (1.82 \pm 0.04 \pm 0.13) \times 10^{-4}$ [6], $\mathcal{B}(\psi(2S) \rightarrow K_S K_L) = (5.24 \pm 0.47 \pm 0.48) \times 10^{-5}$ [7], and $\mathcal{B}(\phi \rightarrow K_S K_L) = (33.7 \pm 0.5)\%$ [5], the branching fractions of CP -violating decay processes of J/ψ , $\psi(2S)$, and $\phi \rightarrow K_S K_S$ ($K_L K_L$) can be extracted as the following:

$$\begin{aligned} \mathcal{B}(J/\psi \rightarrow K_S K_S) &= (1.94 \pm 0.20) \times 10^{-9}, \\ \mathcal{B}(\psi(2S) \rightarrow K_S K_S) &= (0.56 \pm 0.08) \times 10^{-9}, \\ \mathcal{B}(\phi \rightarrow K_S K_S) &= (3.59 \pm 0.27) \times 10^{-6}, \end{aligned} \quad (31)$$

which are the first model-independent predictions in rare quarkonium decays. With more precise measurements of $K_S K_L$ decays, the above errors can be reduced further. It is interesting that the $\phi \rightarrow K_S K_S$ decay can be even reached at the current KLOE experiment at the DAΦNE accelerator.

The CP -violating decay processes of $J/\psi \rightarrow K_S K_S$ and $\psi(2S) \rightarrow K_S K_S$ had been searched for by the BESII Collaboration. The upper limits on the branching fractions

at 95% C.L. are set: $\mathcal{B}(J/\psi \rightarrow K_S K_S) < 1.0 \times 10^{-6}$ and $\mathcal{B}(\psi(2S) \rightarrow K_S K_S) < 4.6 \times 10^{-6}$, respectively [8]. The current bounds of the production rates are beyond the sensitivity for testing R_{SS} . However, the BESIII experiment will start to take data in the middle of 2007. About $10 \times 10^9 J/\psi$ and $3 \times 10^9 \psi(2S)$ data samples can be collected per year's running according to the designed luminosity of BEPCII in Beijing [9]. Thus, both $J/\psi \rightarrow K_S K_S$ and $\psi(2S) \rightarrow K_S K_S$ will be reached with data taking in a few years at BEPCII. The $\phi \rightarrow K_S K_S$ will also be easily accessible at the future DAΦNEII Frascati ϕ factory at which the designed luminosity is about $1.0 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ [10], and the signal is clean and free from backgrounds because it is just near the $K\bar{K}$ threshold. $Y(1S) \rightarrow K_S K_S$ and $K_S K_L$ was studied at the B factory [10], for example, with the current luminosity at KEK-B, about $4.0 \times 10^9 Y(1S)$ events can be collected with 1 yr running. It will be very interesting to collect more data at the future Super- B factory to test the R_{SS} .

Because of isospin symmetry, we have $\langle K^0 \bar{K}^0 | H | J/\psi \rangle = \langle K^+ K^- | H | J/\psi \rangle$, then, it is straightforward to obtain the following relation by neglecting the phase space difference

$$\begin{aligned} \mathcal{B}(J/\psi \rightarrow K_S K_L) &= \mathcal{A} \mathcal{B}(J/\psi \rightarrow K^+ K^-) \\ &\cong \mathcal{B}(J/\psi \rightarrow K^+ K^-), \end{aligned} \quad (32)$$

where \mathcal{A} is the correction factor due to $K^0 - \bar{K}^0$ mixing, which can be derived from Eqs. (19) and (23)

$$\begin{aligned} \mathcal{A} &= \frac{\Gamma_S \Gamma_L}{\Gamma^2} \frac{1}{4|pq|^2} \left\{ \frac{1}{1-y^2} - \frac{2(|p|^2 - |q|^2)^2}{1+x^2} + \frac{(|p|^2 - |q|^2)^4}{1-y^2} \right\} \\ &= 0.972, \end{aligned} \quad (33)$$

where we have used the relation $\frac{\Gamma_S \Gamma_L}{\Gamma^2} \frac{1}{1-y^2} = 1$. The uncertainty on \mathcal{A} is tiny, at the order of 10^{-5} . The correction factor \mathcal{A} is slightly smaller than 1, which is caused by $K^0 - \bar{K}^0$ mixing.

Equations (32) and (33) show that the ratio $\mathcal{A} = \frac{\mathcal{B}(J/\psi \rightarrow K_S K_L)}{\mathcal{B}(J/\psi \rightarrow K^+ K^-)} \neq 1$ even if isospin symmetry is an exact symmetry. However isospin symmetry is only an approximate symmetry, it is usually violated by a few percent level. Electromagnetic effects may be even particularly important for the $K^+ K^-$ final state. Therefore, the true value of the factor \mathcal{A} may be even largely different from unity, which should be the total effects of $K^0 - \bar{K}^0$ mixing and isospin violation. The isospin violating effect must be taken into account when comparing the experimental value of the ratio $\mathcal{A} = \frac{\mathcal{B}(J/\psi \rightarrow K_S K_L)}{\mathcal{B}(J/\psi \rightarrow K^+ K^-)}$ with the theoretical predic-

tion. However further study of the isospin violating effect is beyond the scope of this Letter.

One more remark is in the following. Soft photons can be emitted from the initial and final states in vector quarkonia to $K\bar{K}$ decays. The radiation of the soft photons in the decays allows the $K^0 \bar{K}^0$ in a $C = +1$ state. Such a process with a soft photon in the $K_S K_S$ or $K_L K_L$ final state is not CP violating. The detection of the soft photons depends on the sensitivity of the detectors. Therefore the soft-photon-radiation process is an experimental background for the test of the R_{SS} and R_{LL} predictions. This background should be subtracted in experiment.

In conclusion, we have studied the CP -violating decay processes of J/ψ , $\psi(2S)$, ϕ , and $Y(1S)$ quarkonia to $K_S K_S$ and $K_L K_L$. The ratio R_{SS} (R_{LL}) of $K_S K_S$ ($K_L K_L$) and $K_S K_L$ production rates has been constructed in a model-independent and theoretically clean way. Simultaneous measurements of vector quarkonium decays to both $K_S K_S$ and $K_S K_L$ pairs are suggested at higher luminosity $e^+ e^-$ machines, so that many systematic errors can be canceled. With the current experimental information, the absolute branching fractions of the CP -violating processes are first predicted. The isospin relation is obtained by considering $K^0 - \bar{K}^0$ mixing effect.

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