Lavagna *et al.* Reply: In a recent Letter [1] we proposed an interpretation of the experimental measurement of the transmission phase shift through a quantum dot (QD) in the Kondo regime. Our starting point is the 1D single level Anderson model (SLAM) with 2 reservoirs for which we develop a scattering theory. We distinguished between the phase shift δ of the S matrix responsible for the shift $\delta_{\rm ABI}$ in the AB oscillations ($\delta_{\rm ABI} = \delta$), and the one controlling the conductance $G \sim \sin^2\delta_G$ (with $\delta_G = \delta_\sigma$), and claimed the following relation holds: $\delta_G = \delta_{\rm ABI}/2$ (or equivalently $\delta_\sigma = \delta/2$). The results obtained this way are in remarkably good agreement with experimental measurements.

In their preceding Comment, Aharony, Entin-Wohlman, Oreg, and von Delft (AE-WOvD) [2] question the validity of our main assertion, claiming that it fails in some exactly known limits as the noninteracting ($U = 0$) Anderson model. The main point of our Letter, however, was that the SLAM provides an incomplete description of the experimental device. Rather the quantum dot needs to be viewed as an artificial atom and electrons scattering off it must satisfy the generalized Levinson theorem that incorporates the Pauli principle in the many electron system. Adding this physics takes us out of the strict SLAM description. We now provide some details.

(i) First we derive Eq. (3) of JVL. The first step consists in evaluating the retarded Green's function of one electron on the site 0 for the SLAM. Using exact results for the self-energy in an interacting Fermi liquid at $T = 0$, one can show that, at any U, $\mathcal{G}_\sigma(\mu + i\eta) = \sin\delta_\sigma e^{i\delta_\sigma}/\mathrm{Im}\,\Sigma_\sigma(\mu + i\eta)$, where $\delta_\sigma = \pi n_{0\sigma}$ and $\mathrm{Im}\,\Sigma_\sigma(\mu + i\eta) = -\pi(V_L^2 + V_R^2)\rho_\sigma(\mu)$. At $U = 0$, one can check that the exact Green's function of the SLAM (cf. expression given by AE-WOvD) satisfies the latter expression. In a second step, one derives the S matrix at $T = 0$ in the absence of magnetic moment from $\hat{S}'_{k\sigma} = (\hat{I} - i\hat{T}'_{k\sigma})$ [3]. The elements of $\hat{T}'_{k\sigma}$ are given by the right-hand side of Eq. (2) of JVL. Incorporating the result above for $\mathcal{G}_\sigma(\varepsilon_k + i\eta)$, one can derive Eq. (3) of JVL leading, in the case of a symmetric QD ($V_L = V_R$), to

$$\hat{S}'_{k_F\sigma} = e^{i\delta_\sigma}\begin{pmatrix} \cos\delta_\sigma & i\sin\delta_\sigma \\ i\sin\delta_\sigma & \cos\delta_\sigma \end{pmatrix}. \quad (1)$$

Note that the latter equation for $\hat{S}'_{k_F\sigma}$ completely agrees with Eq. (2) for $\hat{S}_{k_F\sigma}$ of AE-WOvD when θ is taken equal to $\pi/4$ as it should be for a symmetric QD.

(ii) The expression we have just derived violates the generalized Levinson theorem. The Levinson theorem [see Refs. [16] and [17] of [1] and references within] in its generalized version relates the phase shift at zero energy $\delta(0)$ to the number of composite bound states N_B, formed by the incident particle and the scatterer (as that is usual in the standard Levinson), *plus* an additional number denoted by $N_{\rm Pauli}$, equal to the number of states excluded by the Pauli principle, i.e., $\delta(0) = \pi(N_B + N_{\rm Pauli})$. In the case of the electron scattering by an hydrogen atom for instance, the scattering can occur in the singlet or triplet channel. In both cases, the phase shift is found to be π which either comes from the existence of a bound state for singlet scattering, or from a number of excluded states equal to 1 for triplet case. Applying this theorem to the problem of the scattering of a spin σ electron off a QD containing a total electron number n_0, one can show that: $\delta = \pi(n_{0-\sigma} + n_{0\sigma}) = \pi n_0$. As announced, the expression (1) for $\hat{S}'_{k_F\sigma}$ violates the generalized Levinson theorem since $(1/2i)\ln\det\hat{S}'_{k_F\sigma} = \delta_\sigma = \pi n_{0\sigma}$, missing the other part related to $n_{0-\sigma}$.

(iii) Our claim and we agree with the comment of AE-WOvD, is that the 1D SLAM with 2 reservoirs is not sufficient to capture the whole physics contained in the experimental device. While it captures most of the physics, it fails to account for the many electron nature of the experimental setup. One may try to start with a many level Anderson model (MLAM) description of the system. We have chosen another route and introduced minimally the missing ingredients through an additional multiplicative phase factor C_σ in front of the S matrix of the SLAM: $\hat{S}_{k\sigma} = C_\sigma\hat{S}'_{k\sigma}$. The value of C_σ is determined in order to guarantee the generalized Levinson theorem. It is easy to check that $C_\sigma = e^{i\delta_{-\sigma}}$ which eventually leads to Eq. (4) of JVL for $\hat{S}_{k_F\sigma}$. By doing so, the total occupancy of the QD as evaluated in the 1D-SLAM is directly related to the phase shift at $T = 0$. We believe that this is precisely the quantity measured in the quantum interferometry.

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[1] A. Jerez, P. Vitushinsky, and M. Lavagna, Phys. Rev. Lett. **95**, 127203 (2005).
[2] A. Aharony, O. Entin-Wohlman, Y. Oreg, and J. von Delft, preceding Comment, Phys. Rev. Lett. **96**, 189705 (2006).
[3] The opposite sign convention for the definition of $\hat{S}$ is taken in Eq. (2) of AE-WOvD explaining the sign difference which is found.

