Comment on "Theory of Current-Driven Domain Wall Motion: Spin Transfer versus Momentum Transfer"

Recently Tatara and Kohno (TK) [1] have proposed a theory which describes the current induced motion of a domain wall in thin ferromagnetic wires. It is suggested that there is an *intrinsic* threshold spin current $j_s^{\mathrm{cr}(1)} = (eS^2/a^3\hbar)K_\perp\lambda$ (see Ref. [1] and below for notation) for wall motion which is determined by the hard-axis (or perpendicular) magnetic anisotropy $K_\perp$. Relaxation is introduced using a Gilbert term—$(\alpha/S)\vec{S}\times(\partial\vec{S}/\partial t)$. Here I point out that this theory violates the symmetry of the problem *and* the second law of thermodynamics. I argue that this intrinsic pinning does not exist.

In Ref. [1] the authors consider a ferromagnet of spins $\vec{S}$, the orientation of which is specified locally by the Euler angles θ and ϕ. The solution for a domain wall has $\theta = \theta_0(x - X)$, $\phi_0 = 0$ where X is the coordinate of the wall center, ϕ_0 its *uniform* tilt angle, $\cos\theta_0(x) = \tanh(x/\lambda)$, and λ the wall width. The spins are coupled to the conduction electrons via an exchange interaction $H_{\mathrm{int}} = -(\Delta/S)\times\int d^3xS(x)(c^\dagger\sigma c)_x$. The model has translation invariance when the *extrinsic* pinning force, $F_{\mathrm{pin}} \equiv -(\partial V/\partial X) = 0$. The effect of the conduction electrons can be reduced to a force F_{el}, which I agree is negligible for an adiabatic wall of large width λ, and an all important torque $T_{\mathrm{el},z} = (\hbar Na^3/2\lambda e)\eta j$. This is proportional to j, the charge current density, and represents the effects of angular momentum transfer.

In the absence of a current, $T_{\mathrm{el},z} = 0$, the stationary solution $\theta = \theta_0(x - X)$, $\phi_0 = 0$ applies. Reflecting the translational invariance, the energy is independent of X. TK [1] obtain their torque transfer term, Eq. (7), in effect, by differentiating, with respect to ϕ_0, the expectation value of H_{int} for a Fermi sea which is constrained to carry a current. From their results it can be deduced that a current j adds a *potential* energy $L_\tau = -T_{\mathrm{el},z}\phi_0$ to the effective Lagrangian, L_S, their Eq. (1). When a current j is suddenly turned on, in the absence of relaxation ($\alpha = 0$), the finite $T_{\mathrm{el},z}$ solutions of their Eqs. (4) and (5) (reproduced below) have the wall moving with a velocity $v_0 = (a^3/2eS)\eta j$. This solution has $\phi_0 = 0$ and the same energy as the $j = 0$ stationary solution. However, due to the potential energy $T_{\mathrm{el},z}\phi_0$, the ground state is stationary with a tilt angle $\phi_0^j = T_{\mathrm{el},z}/K_\perp NS^2 \propto j$. When Gilbert damping is present, the velocity relaxes to zero, appropriate for this ground state, and a $j_s^{\mathrm{cr}(1)}$ exists.

However, symmetry prohibits such an energy $T_{\mathrm{el},z}\phi_0$. When $K_\perp = 0$, the system has rotational symmetry about the x axis. Since $\partial/\partial\phi_0$ is the generator of such rotations, any derivative with respect to ϕ_0 *must* be proportional to $K_\perp$. This is evidently *not* the case.

In fact [2], the current should appear in the effective spin Lagrangian *density* as a real space Berry phase term $\mathcal{L}_\tau = \hbar v_0 S(\cos\theta - 1)(\partial\phi/\partial x)$. This is a charge *kinetic* term consistent with the x-axis symmetry and the resulting equations of motion, in the absence of relaxation, are *identical* to Eqs. (4) and (5) of TK [1]. When this $\mathcal{L}_\tau$ is combined with the (time Berry phase) spin kinetic term $\hbar S(\cos\theta - 1)\dot{\phi}$ the result is a simple Galilean transformation. Specifically, a wall solution with velocity v becomes one with velocity $v + v_0$ with no change in the energy. Thus a wall with $\phi_0 = 0$ and velocity v_0 is still the ground state and cannot relax; i.e., a finite α, as it appears in their Eqs. (4) and (5), *cannot* be justified. This invalidates the solution Eq. (12) and the result Eq. (14) that there is an intrinsic critical current $j_s^{\mathrm{cr}(1)}$.

I contend that the theory of TK [1] violates the second law of thermodynamics. Consider an ideal *closed* system comprising a perfectly conducting ferromagnetic wire connected directly to a pair of charge reservoirs, of energy $U(q)$, where q is a charge per area for one reservoir, defined such that $\dot{q} = j$. The Lagrangian is $L_\tau = -(\hbar NS/\lambda)X\dot{\phi}_0 - (\hbar/e)\dot{q}\phi_0 - (1/2)K_\perp NS^2\phi^2 - U(q)$ and yields the equations of motion, including Gilbert relaxation: $\dot{\phi}_0 + \alpha(\dot{X}/\lambda) = 0$, $(\dot{X}/\lambda) - \alpha\dot{\phi}_0 = (SK_\perp/\hbar)\phi_0 + (\dot{q}/eNS)$, and $(\hbar/e)\dot{\phi}_0 = \mathcal{E}$, where $\mathcal{E} = dU/dq$ is the *effective* electromotive force of the reservoirs and where the angle ϕ_0 is assumed to be small. The first two equations are again the TK Eqs. (4) and (5) [1], while the last defines the (back) emf $(\hbar/e)\dot{\phi}_0$ and which is equal to the (direct) emf, $\mathcal{E}$, in the absence of resistance. The Hamiltonian, i.e., energy, $H = U(q) + (1/2)K_\perp NS^2\phi_0^2$. In the absence of relaxation, $\alpha = 0$, $\mathcal{E} = dU/dq = 0$ corresponding to the minimum of $U(q) \approx uq^2$, for small q, to within a constant. The sliding solution described above is the absolute ground state with $H = 0$. Putting the system in contact with the heat bath, i.e., for $\alpha \neq 0$, causes $|\phi_0| \neq 0$, $|q| \neq 0$ and $H > 0$. The two equations for $\dot{\phi}_0$ require $\alpha < 0$ consistent with $dH/dt > 0$. Energy is taken from the heat bath and given to the system, in a process which can be made periodic. This is a clear violation of the second law.

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