Observation of Strong Quantum Depletion in a Gaseous Bose-Einstein Condensate

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We studied quantum depletion in a gaseous Bose-Einstein condensate. An optical lattice enhanced the atomic interactions and modified the dispersion relation resulting in strong quantum depletion. The depleted fraction was directly observed as a diffuse background in the time-of-flight images. Bogoliubov theory provides a semiquantitative description for our observations of depleted fractions in excess of 50%.

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The advent of Bose-Einstein condensates (BECs) in 1995 extended the study of quantum fluids from liquid helium to superfluid gases with a 100×10^6 times lower density. These gaseous condensates featured relatively weak interactions and a condensate fraction close to 100%, in contrast to liquid helium where the condensate fraction is only 10% [1]. As a result, the gaseous condensates could be quantitatively described by a single macroscopic wave function shared by all atoms, which is the solution of a nonlinear Schrödinger equation. This equation provided a mean-field description of collective excitations, hydrodynamic expansion, vortices and sound propagation [2].

The fraction of the many-body wave function which cannot be represented by the macroscopic wave function is called the quantum depletion. In a homogenous BEC, it consists of admixtures of higher momentum states into the ground state of the system. The fraction of the quantum depletion η_0 can be calculated through Bogoliubov theory: $\eta_0 = 1.505\sqrt{\rho a_s^3}$ where ρ is the atomic density and a_s is the s-wave scattering length [3]. For ^{23}Na at a typical density of 10^{14} cm^{-3}, the quantum depletion is 0.2%.

For the last decade, it has been a major goal of the field to map out the transition from gaseous condensates to liquid helium. Beyond-mean-field effects of a few percent were identified in the temperature dependence of collective excitations in a condensate [4,5]. The quantum depletion increases for higher densities—however, at densities approaching 10^{15} cm^{-3} the lifetime of the gas is dramatically shortened by three-body collisions. Attempts to increase the scattering length near a Feshbach resonance were also limited by losses [6,7]. Recently, several studies of strongly interacting molecular condensates were performed [8–10]. In lower dimension systems, the effect of interactions is enhanced. Strongly correlated systems, which are no longer superfluid, were observed in 1D systems [11,12], and in optical lattices [13,14]. Quantum depletion in 1D was studied in Refs. [15–17], where condensation and quantum depletion are finite-size effects and disappear in the thermodynamic limit. The transition between a three-dimensional quantum gas and a quantum liquid has been largely unexplored.

In this Letter, we report the first quantitative study of strong quantum depletion in a superfluid gas. Exposing atoms to an optical lattice enhances quantum depletion in two ways. First, the lattice increases the local atomic density, which enhances the interactions. The increased density leads to enhanced interactions (by up to an order of magnitude in our experiment), ultimately limited by inelastic collisional losses. The second effect of the lattice is to modify the dispersion relation $T(k)$, which is simply $T(k) = \hbar^2 k^2/2m$ for free atoms. The effect of a weak lattice can be accounted for by an increased effective mass. For a deep lattice, when the width of the first band becomes comparable or smaller than the interaction energy, the full dispersion relation is required for a quantitative description.

In addition to enhancing the quantum depletion, an optical lattice also enables its direct observation in time of flight. For a harmonic trap, the quantum depletion cannot be observed during ballistic expansion in the typical Thomas-Fermi regime. Because the mean-field energy (divided by $\hbar$) is much greater than the trap frequency, the cloud remains locally adiabatic during the expansion. The condensate at high density transforms adiabatically into a condensate at low density with diminishing quantum depletion. Therefore, the true momentum distribution of the trapped condensate including quantum depletion and, for the same reason, phonon excitations can only be observed by *in situ* techniques such as Bragg spectroscopy [18,19]. In an optical lattice, the confinement frequency at each lattice site far exceeds the interaction energy, and the time-of-flight images are essentially a snapshot of the frozen-in momentum distribution at the time of the lattice switch-off, thus allowing for a direct observation of the quantum depletion.

The experiment setup is similar to that of our previous work [20]: A ^{23}Na BEC containing up to 5×10^5 atoms in the $|F = 1, m_F = -1\rangle$ state was loaded into a crossed optical dipole trap. The number of condensed atoms was controlled through three-body decay in a compressed trap, after which the trap was relaxed to allow further evaporation and rethermalization. A vertical magnetic field gradient was applied to compensate for gravity and avoid sagging in the weak trap. The final trap frequencies were $\omega_{x,y,z} = 2\pi \times 60, 60, 85$ Hz. The mean Thomas-Fermi radius was ~ 12 μm for 1.7×10^5 atoms.

The lattice beams were derived from the same single-mode infrared laser at 1064 nm used for the crossed optical dipole trap. All five beams were frequency shifted by at least 20 MHz with respect to each other via acousto-optical modulator to eliminate cross interference between different beams. The three lattice beams had a $1/e^2$ waist of ~ 90 μm at the condensate, and were retro-reflected to form standing waves. The two horizontal beams were orthogonal to each other, while the third beam was tilted $\sim 20°$ with respect to the vertical axis due to limited optical access. One- and two-dimensional lattices were formed using either one or both of the horizontal beams. The trap parameters were chosen such that during the ramping of the optical lattice potential, the overall Thomas-Fermi radii remained approximately constant in order to minimize intraband excitations. All the measurements were performed at a peak lattice site occupancy number ~ 7, as determined by a trade off between small three-body losses and good signal-to-noise ratio.

The optical lattice was linearly ramped up to a peak potential of $22 \pm 2E_R$ in time τ_{ramp}, and then linearly ramped back down at the same speed. This ramp sequence was interrupted at various times by a sudden switch-off of all lattice and trapping potentials (<1 μs). Absorption images were taken after 10 ms time of flight, reflecting the momentum distribution of the system at the instant of release (Fig. 1). Based on the number of atoms remaining in the condensate after the full ramping sequence ($\gtrsim 80\%$), we concluded that $\tau_{\text{ramp}} \gtrsim 1$ ms satisfies the intraband adiabaticity condition. In the following discussion, all measurements were performed for $\tau_{\text{ramp}} = 50$ ms.

The loss and revival of the interference contrast, as illustrated in Fig. 1, has been associated with the quantum phase transition from a superfluid to a Mott-insulator state [13,14]. The presence of sharp interference peaks indicates coherence and superfluid behavior, whereas the presence of a single broad peak indicates the insulating phase. However, as we show in this Letter, even before the lattice depth exceeds the critical value for the phase transition, a diffuse background gradually emerges as a result of quantum depletion. The interference peaks represent the population in the zero quasimomentum state, and the diffuse background represents the population in the rest of the Brillouin zone. We only account for the lowest energy band as the population in higher bands remains negligible for our parameters.

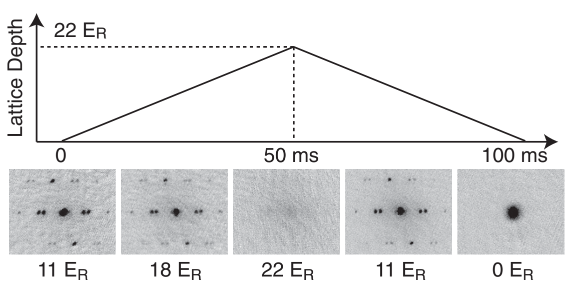

FIG. 1. Interference patterns in time of flight: The ramping sequence was interrupted as the lattice is ramped up $(11, 18, 22E_R)$ and back down $(11, 0E_R)$. The time of flight was 10 ms; the field of view is 861 μm $\times$ 1075 μm.

In the time-of-flight images, we masked off the sharp interference peaks and performed a two-dimensional Gaussian fit on the diffuse background peak. After the lattice was fully ramped down, most of the atoms remained in the condensed fraction while a small fraction (up to 20%) were heated and distributed across the first Brillouin zone likely due to the technical noise and imperfect adiabaticity of the ramp. A linear interpolation was used to subtract this small heating contribution (up to 10% at the maximum lattice depth) and obtain the quantum depletion N_{qd}/N, where N_{qd} is the number of atoms in the diffuse background peak (quantum depletion) and N the total number.

We performed this measurement for one-, two-, and three-dimensional optical lattices, and the main results are shown in Fig. 2. The quantum depletion became significant for lattice depths of $\gtrsim 10E_R$ for a three-dimensional lattice ($E_R = \hbar^2 k_{\text{latt}}^2/2m$, where $k_{\text{latt}} = 2\pi/\lambda_{\text{latt}}$ is the lattice wave number). Note that the Mott-insulator transition starts to occur only at lattice depths $\gtrsim 16E_R$ (see below). N_{qd}/N remained small for one- and two-dimensional lattices.

A theoretical description of quantum depletion can be derived from the Bogoliubov theory, which is the standard theory to describe the ground state properties of a weakly interacting system. The population in the (quasi-) momentum state k is given by [3,21,22]

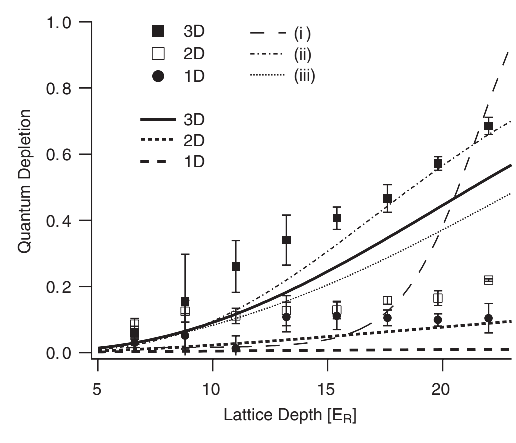

FIG. 2. Quantum depletion of a ^{23}Na BEC confined in a one-, two-, and three-dimensional optical lattice: the data points with statistical error bars are measured N_{qd}/N; the three thick curves are the theoretical calculation of N_{qd}/N using Bogoliubov theory and local density approximation. For comparison, also shown are (thin curves): (i) the (smoothed out) Mott-insulator fraction N_{MI}/N based on a mean-field theory; (ii) the calculated quantum depletion for a *homogeneous* system of per-site occupancy number $n = 1$ and (iii) $n = 7$.

$$v_k^2 = \frac{T(k) + n_0 U - \sqrt{2T(k)n_0 U + T^2(k)}}{2\sqrt{2T(k)n_0 U + T^2(k)}} \quad (1)$$

where $T(k)$ is the kinetic energy, n_0 is the occupancy number [per cubic lattice cell of $(\lambda_{\rm latt}/2)^3$] in the zero momentum state, and U is the on-site interaction energy [23–26]. Incidentally, v_k is one of the coefficients in the Bogoliubov transformation. The total occupancy number n is given by the sum of n_0 and the quantum depletion: $n = n_0 + \sum_{k\neq 0} v_k^2$. For a given density n, the quantum depletion can be obtained by using Eq. (1) and the appropriate dispersion relation $T(k)$.

A band structure calculation was performed to obtain the on-site interaction U (and also the tunneling rate J) as functions of the lattice depths, shown in Fig. 3. In calculating U, we use a Wannier density function along the dimensions with a lattice, and a uniform density in the ones without. J is independent of the lattice wavelength or atomic mass for a given lattice depth (all energies measured in E_R).

The quantum depletion for a lattice of uniform occupation is obtained by integrating over the first Brillouin zone: $n = n_0 + \int v_{\mathbf{k}}^2 d\mathbf{k}$. For a sufficiently deep lattice ($\gtrsim 5E_R$), the dispersion relation is given by

$$T(\mathbf{q}) = 4J\sum_{i=1}^{d}\sin^2(q_i\pi) + 4E_R\sum_{i=d}^{3} q_i^2, \quad (2)$$

where dimensions 1 through d are assumed to have a lattice present and $\mathbf{q} = \mathbf{k}\lambda_{\rm latt}/4\pi$.

For an inhomogeneous system such as a harmonically confined condensate, we apply the result from the homogeneous system to shells of different occupancy numbers using the local density approximation (as the dependence of the quantum depletion on the occupancy number is slowly varying). The calculated quantum depleted fractions are plotted in Fig. 2. The semiquantitative agreement between the observed quantum depletion and the simple Bogoliubov theory, even for values around 50%, is the main result of this Letter. The remaining discrepancies may be due to unaccounted heating, a systematic overestimate of the background, and the inadequacies of Bogoliubov theory for large quantum depletion.

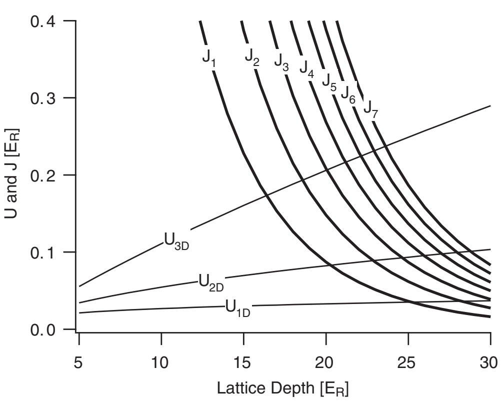

FIG. 3. On-site interaction U and tunneling rate J for a ^{23}Na BEC in an optical lattice at 1064 nm: $U_{d\rm D}$ is for a d-dimensional lattice. In a three-dimensional lattice, the superfluid to Mott-insulator transition for occupancy number n occurs when $J_n = 6(2n+1+2\sqrt{n(n+1)})J$ equals $U_{\rm 3D}$ [see Eq. (3)]. The horizontal locations of the crossing points where $J_n = U_{\rm 3D}$ are the critical lattice depths.

In free space, the dispersion curve is a continuous parabola. Both the number of populated states and the population in each state increases with the atomic density ρ, and the quantum depletion η_0 is proportional to $\rho^{1/2}$. This still holds for shallow lattices ($\lesssim 5E_R$) where the quantum depletion (η) does not saturate the lowest band: $\eta = \eta_0(\frac{\epsilon^*_{\rm MF}m^*}{\epsilon_{\rm MF}m})^{3/2}$ ($\epsilon_{\rm MF}$ and m are the free space mean-field energy and atomic mass; $\epsilon^*_{\rm MF}$ and m^* are the lattice-enhanced mean-field energy and effective mass) [27].

The situation changes for deeper lattices as the interaction energy U becomes comparable to the width of the first energy band (approximately $4J$). In this regime, the quantum depletion starts to saturate the lowest band, but the higher bands remain virtually empty due to the large band gap. While the population $N_{\rm qd}$ in the lowest band continues to increase with the atomic density $\rho = n/(\lambda_{\rm latt}/2)^3$, the quantum depleted fraction $N_{\rm qd}/N$ actually decreases with ρ [see Fig. 2 where the calculated quantum depletion is bigger for $n = 1$ than for $n = 7$ at large lattice depths ($\gtrsim 9E_R$)].

The fact that the observed quantum depletion for one- or two-dimensional lattices remained small provides further evidence for our interpretation of the diffuse background as quantum depletion. In the dimension with a lattice present, the band width is proportional to the tunneling rate J which decreases exponentially with the lattice depth. The interaction energy U increases much slower with the lattice depth. Therefore the flattening of the dispersion relation contributes more significantly to the increased quantum depletion. Since this flattening does not occur in the dimension without a lattice, the quantum depletion for one- or two-dimensional lattices is expected to increase much slower compared to three-dimensional lattice, consistent with our observation.

For a three-dimensional lattice, quantum depletion and the superfluid to Mott-insulator transitions are two consequences of admixing higher momenta into the many-body wave function. For increasing interactions, the ground state wave function is first a depleted superfluid, then develops strong correlations and suppressed density fluctuations and finally turns into the Mott insulator. In our measurements, the insulating regions appear fully in the diffuse background together with the quantum depletion of the superfluid regions. A lower bound for our measured $N_{\rm qd}/N$ is therefore provided by calculating the fraction of atoms in the Mott-insulator phase. According to the mean-field theory in a homogenous bosonic lattice system [23–26], the critical value $(U/J)_c$ at which the phase transition occurs for occupancy number n is given by

$$(U/J)_c = z[2n + 1 + 2\sqrt{n(n+1)}] \tag{3}$$

where z is the number of nearest neighbors ($z = 2d$ for a d-dimensional cubic lattice). For an inhomogeneous system such as a trapped condensate, shells of different occupancy numbers enter the Mott-insulator phase from outside as the lattice potential is increased and U/J exceeds the critical values.

In our experiment, the peak occupancy number is ~ 7. From Fig. 3, the critical lattice depths in a three-dimensional cubic lattice for $n = 1, 2, 3, 4, 5, 6, 7$ are $16.3, 18.5, 20.0, 21.2, 22.1, 22.9, 23.6E_R$ respectively. The integrated Mott-insulator fraction N_{MI}/N as a function of lattice depths is plotted in Fig. 2. Instead of a step function with jumps at each critical lattice depth, we use a smooth spline curve for N_{MI}/N, which is more realistic given the fluctuations associated with the atom numbers and lattice depths. The measured N_{qd}/N was significantly greater than N_{MI}/N.

In the case of one- and two-dimensional lattices, a Mott-insulator transition would only occur for lattice depths much larger than those in our experiment. Note that Eq. (3) is not directly applicable as the dimensions without lattice beams in our system are only loosely confined and cannot be considered frozen [25]. In addition, n in Eq. (3) is the number of atoms *per lattice site* and far exceeds 7 for the lower-dimensional lattices.

In conclusion, we conducted a quantitative study of quantum depletion in a gaseous BEC through the application of an optical lattice, and found reasonable agreement with a model based on Bogoliubov theory in the predominantly superfluid regime. A complementary study was recently reported by Gerbier *et al.* [28,29] of the nonvanishing visibility of the interference peaks in a Mott insulator as a result of the admixture of particle-hole states. The two works together give a complete description of the ground state in both the superfluid and insulating phases. More elaborate theoretical treatments for the intermediate case have been presented in Refs. [30–32].

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