

Positronium Collapse and the Maximum Magnetic Field in Pure QED

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A maximum value for the magnetic field is determined, which provides the full compensation of the positronium rest mass by the binding energy in the maximum symmetry state and disappearance of the energy gap separating the electron-positron system from the vacuum. The compensation becomes possible owing to the falling to the center phenomenon. The maximum magnetic field may be related to the vacuum and describe its structure.

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A common feature of compact astronomical objects: white dwarfs, neutron stars, and accretion disks around black holes, is a very strong magnetic field. It is believed that neutron stars possess the strongest observed magnetic fields. The field strength is in the range from $\sim 10^8$ G to $\sim 10^{14}$ G for radio pulsars identified with rotation-powered neutron stars [1], and may be as high as $\sim 10^{15}$ G for soft gamma-ray repeaters [2], or even higher ($\sim 10^{16}$ – 10^{17} G) for the sources of cosmological gamma-ray bursts [3]. Much more intense magnetic fields have been conjectured to be involved in several astrophysical phenomena. For instance, superconductive cosmic strings, if they exist, may have magnetic fields up to $\sim 10^{47}$ – 10^{48} G in their vicinities [4]. Magnetic fields of $\sim 10^{47}$ G may be also produced in our Universe at the beginning of the inflation [5]. The fundamental physical problems are how large the field strength can be in nature, and how the properties of the vacuum change when magnetic fields approach the extremity. It is accepted that magnetic fields are stable in pure quantum electrodynamics (QED), and another interaction (weak or strong) or magnetic monopoles have to be involved to make the magnetized vacuum unstable [6].

In this Letter, working solely in QED, we find that there exists a maximum value of the magnetic field that delimits the range of its values admitted without revising QED. Its value is many orders of magnitude less than $B = B_0 \exp(3\pi/\alpha)$ (here $\alpha = e^2/4\pi \simeq 1/137$, $B_0 = m^2/e = 4.4 \times 10^{13}$ G, m is the electron mass, $\hbar = c = 1$ throughout), the value that restricts the range of validity of QED due to the lack of asymptotic freedom [7]. The maximum magnetic field causes the shrinking of the energy gap between an electron and positron.

We exploit the unboundedness from below of the energy spectrum for sufficiently singular attractive potentials, known as the “falling to the center” [8]. For e^+e^- system in a magnetic field B this phenomenon is caused by the ultraviolet singularity of the photon propagator and occurs in the limit $B \rightarrow \infty$. It takes place independently of whether nonrelativistic or relativistic description is used, and is associated with dimensional reduction due to the charged particles being restricted to the lowest Landau levels [see the pioneering work of Loudon [9] and other works out of which most important for our present theme are [9–11]]. Using the Bethe-Salpeter (BS) equation for the electron-positron system, with the relative motion of the two particles treated in a strictly relativistic way, we show that at a maximum value of the magnetic field, which may be of an astrophysical significance, the rest energy of the system is compensated for by the mass defect; i.e., the system is not separated from the vacuum by an energy gap. We refer to this situation as a collapse of positronium.

In processing the formalism and interpreting the results, especially while discussing the vacuum structure, we use the theory of the falling to the center developed in [12] that implies deviations from the standard quantum theory manifesting themselves when extremely large electric fields near the singularity become important [13].

We proceed from the $(3+1)$ -dimensional BS equation in an approximation, which is the ladder approximation once the photon propagator (in the coordinate space) is taken in the Feynman gauge: $D_{ij}(x) = g_{ij}D(x^2)$, $x^2 \equiv x_0^2 - \mathbf{x}^2$, g_{ij} is the metrics, $g_{ii} = (1, -1 - 1 - 1)$. In an asymptotically strong magnetic field this equation may be written in the following $(1+1)$ -dimensional form [see our detailed paper [14] for the derivation], covariant under the Lorentz transformations along the axis 3:

$$\left(i\vec{\delta}_{\parallel} - \frac{\hat{P}_{\parallel}}{2} - m\right)\Theta(t, z)\left(-i\vec{\delta}_{\parallel} - \frac{\hat{P}_{\parallel}}{2} - m\right) = i8\pi\alpha \sum_{i=0,3} D\left(t^2 - z^2 - \frac{P_{\perp}^2}{(eB)^2}\right)g_{ii}\gamma_i\Theta(t, z)\gamma_i \quad (1)$$

where $\Theta(t, z)$ is the 4×4 (Ritus transform of) BS amplitude, $t = x_0^e - x_0^p$ and $z = x_3^e - x_3^p$ are the differences of the coordinates of the electron (e) and positron (p) along the time x_0 and along the magnetic field $\mathbf{B} = (0, 0, B_3 = B)$, respectively. P_{\parallel} and P_{\perp} are projections of the total (generalized) momentum of the positronium onto the $(0, 3)$ subspace

and the (1, 2) subspace, respectively. Only two Dirac gamma matrices, $\gamma_{0,3}$, are involved, $\hat{\partial}_{\parallel} = \gamma_0 \partial / \partial t + \gamma_3 \partial / \partial z$, $\hat{P}_{\parallel} = P_0 \gamma_0 - P_3 \gamma_3$.

Equation (1) is valid in the coordinate domain, where the argument of the function D is greater than the electron Larmour radius squared $(L_B)^2 = (eB)^{-1}$. When $B = \infty$, the domain of validity covers the whole exterior of the light cone $z^2 - t^2 \geq 0$. The Lorentz-invariant expansion of Θ over matrix basis contains four independent scalar components, whose number diminishes to three if $P_{\parallel} = 0$. The argument of the original photon propagator $(x^e - x^p)^2$ has proved to be replaced in (1) by $t^2 - z^2 - (\tilde{x}_{\perp}^e - \tilde{x}_{\perp}^p)^2 = t^2 - z^2 - P_{\perp}^2 / (eB)^2$, where $\tilde{x}_{\perp}^{e,p}$ are the center of orbit coordinates of the two particles in the transverse plane. Now that after the dimensional reduction this subspace no longer exists these substitutes for the transverse particle coordinates themselves: $\tilde{x}_{\perp}^{e,p}$ are not coordinates, but quantum numbers of the transverse momenta. The mechanism of replacement of a coordinate by a quantum number is the same as in [10].

In deriving Eq. (1) the expansion over the complete set of Ritus matrix eigenfunctions [15] was used in [14] that accumulate the dependence on the transverse spatial and spinorial degrees of freedom. This expansion yields an infinite set of equations, where different pairs of Landau quantum numbers n^e, n^p are entangled, Eq. (1) being the equation for the $n^e = n^p = 0$ component that decouples from this set in the limit $B = \infty$.

In the ultrarelativistic limit $P_0 = P_3 = 0$ Eq. (1) is solved by the most symmetric Ansatz $\Theta = I\Phi$, where I is the unit matrix, and becomes

$$(-\square_2 + m^2)\Phi(t, z) = i16\pi\alpha D\left(t^2 - z^2 - \frac{P_{\perp}^2}{(eB)^2}\right)\Phi(t, z). \quad (2)$$

Here $\square_2 = -\partial^2 / \partial t^2 + \partial^2 / \partial z^2$ is the Laplace operator. Equations for the rest two invariant coefficients, other than the singlet Φ , are considered in [14].

The ultraviolet singularity on the light cone ($x^2 = 0$) of the free photon propagator, $D(x^2) = -(i/4\pi^2)1/x^2$, after one substitutes this expression into Eq. (2) taken for the lowest energy state $P_{\perp}^2 = 0$, leads to falling to the center in the Schrödinger-like differential equation

$$-\frac{d^2\Psi(s)}{ds^2} + \left(m^2 - \frac{1}{4s^2}\right)\Psi(s) = \frac{4\alpha}{\pi} \frac{1}{s^2}\Psi(s), \quad (3)$$

to which the radial part of Eq. (2) is reduced in the most symmetrical case, when the wave function $\Phi(x) = s^{-1/2}\Psi(s)$ does not depend on the hyperbolic angle ϕ in the spacelike region of the two-dimensional Minkowsky space, $t = s \sinh\phi$, $z = s \cosh\phi$, $s = \sqrt{z^2 - t^2}$.

The solution that decreases at $s \rightarrow \infty$ is given by the McDonald function with imaginary index:

$$\Psi(s) = \sqrt{s}K_{\nu}(ms), \quad \nu = i2\sqrt{\alpha/\pi} \approx 0.096i. \quad (4)$$

It behaves near the singular point $s = 0$ as

$$\left(\frac{s}{2}\right)^{1+\nu} \frac{1}{\Gamma(1+\nu)} - \left(\frac{s}{2}\right)^{1-\nu} \frac{1}{\Gamma(1-\nu)}. \quad (5)$$

Here the Euler Γ functions appear. The falling to the center manifests itself [8] in the complexity of the exponents in (5) that makes the both asymptotic terms oscillating and equal in rights. The falling to the center holds for any positive α , the genuine value $\alpha = 1/137$ included, unlike the case without the magnetic field, where nonphysically large values of the fine structure constant, $\alpha > \pi/8$, are required to provide it [14,16]. Equation (3) is valid in the interval

$$s_0 \leq s \leq \infty, \quad s_0 \gg (eB)^{-1/2}. \quad (6)$$

Thus, the Larmour radius serves as a regularizing length. According to [12], the singular Eq. (3) should be considered as the generalized eigenvalue problem with respect to α . The operator in the left-hand side is self-adjoint provided the boundary condition is imposed,

$$\Psi(s_0) = 0, \quad (7)$$

that treats the Larmour radius as the lower edge of the normalization box. The discrete eigenvalues $\alpha_n(s_0)$ condense in the limit $B = \infty$ ($s_0 = 0$) to become a continuum of states that make the Hilbert space of vectors orthogonal with the singular measure $s^{-2}ds$. The latter fact allows us to normalize them to δ functions and interpret as free particles emitted and absorbed by the singular center. In order that the Larmour radius might be treated as the edge of the box it is necessary that s_0 be much smaller than the electron Compton length, $s_0 \ll m^{-1} \approx 3.9 \times 10^{-11}$ cm. Then the small-distance asymptotic regime is reached, and the behavior of the system “behind the horizon,” $s < s_0$ —where the two-dimensional Eqs. (1)–(3) are not valid—is not important. In this way the existence of the limit $s_0 \rightarrow 0$, impossible in the standard theory, is achieved [17].

Beginning with a certain small value of the argument ms , the function $K_{\nu}(ms)$ oscillates, as $s \rightarrow 0$, and takes the zero value infinitely many times. To find the largest value of s_0 , for which the boundary problem (3) and (7) can be solved, one can use (5). Then Eq. (7) reads

$$\nu \ln \frac{ms_0}{2} = i \arg \Gamma(\nu + 1) - i\pi n, \quad n = 0, \pm 1, \pm 2, \dots \quad (8)$$

Since $|\nu|$ is small we may exploit the approximation $\Gamma(1 + \nu) \approx 1 - \nu C_E$, where $C_E = 0.577$ to get

$$\ln \left(\frac{ms_0}{2}\right) = -\frac{n}{2} \sqrt{\frac{\pi^3}{\alpha}} - C_E, \quad n = 1, 2, \dots \quad (9)$$

We have expelled the nonpositive integers n from here, since they would lead to the roots for ms_0 of the order of or larger than unity in contradiction to the adopted condition $s_0 \ll m^{-1}$. For such values Eq. (5) is not valid. It may be

checked that there are no other zeros of McDonald function, apart from (9). The maximum value for s_0 is provided by $n = 1$:

$$s_0^{\max} = \frac{2}{m} \exp\left\{-\frac{1}{2}\sqrt{\frac{\pi^3}{\alpha}} - C_E\right\} \approx 10^{-14} \text{ m}^{-1}, \quad (10)$$

i.e., s_0^{\max} is about 14 orders of magnitude smaller than m^{-1} and makes $\sim 10^{-25}$ cm. By demanding, in accord with (6), that the value of s_0^{\max} should exceed the Larmour radius

$$s_0^{\max} \gg L_B = (eB)^{-1/2} \quad \text{or} \quad B \gg \frac{1}{e(s_0^{\max})^2}, \quad (11)$$

one establishes how large the magnetic field should be in order that the boundary problem might have a solution, in other words, that the point $P_0 = \mathbf{P} = 0$ might belong to the spectrum. Therefore, if the magnetic field exceeds the maximum value of

$$B_{\max} \approx \frac{1}{e(s_0^{\max})^2} \approx \frac{m^2}{4e} \exp\left\{\frac{\pi^{3/2}}{\sqrt{\alpha}} + 2C_E\right\}, \quad (12)$$

the positronium ground state with the center-of-mass 4-momentum equal to zero exists [18]. The value of B_{\max} is $1.6 \times 10^{28} B_0 \sim 10^{42}$ G. This is a few orders of magnitude smaller than the magnetic field that may be in the vicinity of superconductive cosmic strings [4]. Excited positronium states may also reach the spectral point $P_\mu = 0$, but this occurs for magnetic fields, tens orders of magnitude larger than (12)—to be found in the same way from (9) with $n = 2, 3, \dots$.

The ultrarelativistic state $P_\mu = 0$ has the internal structure of what was called a “confined state,” belonging to kinematical domain called “sector III” in [12], i.e., the one whose wave function behaves as a standing wave combination of free particles near the lower edge of the normalization box and decreases as $\exp(-ms)$ at large distances. The effective “Bohr radius,” i.e., the value of s that provides the maximum to the wave function (4) makes $s_{\max} = 0.17 \text{ m}^{-1}$. This is much less than the standard Bohr radius $(e^2 m)^{-1}$. The wave function is concentrated within the limits $0.006 \text{ m}^{-1} < s < 1.1 \text{ m}^{-1}$. But the effective region occupied by the confined state is still much closer to $s = 0$, since the probability density of the confined state is the wave function squared *weighted with the measure* $s^{-2} ds$ *singular in the origin* [12] and is hence concentrated near the edge of the normalization box $s_0 \approx 10^{-25}$ cm. The electric fields in the e^+e^- system at such distances are about 10^{43} V/cm. There is no evidence that the standard quantum theory (SQT) should be valid under such conditions. This fact encourages the use of the theory of Ref. [12] above that differs from SQT in that it excludes the short distances beyond the normalization box. [Note, nevertheless, the reserves [13,17].]

Compare the value (12) with the analogous value, obtained earlier [10] by extrapolating the semirelativistic result concerning the positronium binding energy in a

magnetic field to extreme relativistic region:

$$B_{\max}|_{\text{nonrel}} \approx \frac{\alpha^2 m^2}{e} \exp\left\{\frac{2\sqrt{2}}{\alpha}\right\} \approx 10^{164} B_0. \quad (13)$$

Such is the magnetic field that makes the binding energy of the lowest energy state equal to $-2m$. We see that the relativistically enhanced attraction has resulted in a drastically lower value of the maximum magnetic field.

Let us estimate possible effects of radiative corrections [see Ref. [19] for details].

May the vacuum polarization screen the attraction force between the electron and positron in such a way as to prevent the positronium collapse? The three photon polarization eigenmodes give the contributions into the photon propagator [20,21], to be used in the BS equation,

$$D_a(x) = -\frac{1}{(2\pi)^4} \int \frac{\exp(ikx) d^4k}{k^2 + \kappa_a(k)}, \quad a = 1, 2, 3, \quad (14)$$

which contain the polarization tensor eigenvalues $\kappa_a(k)$ in the denominator. When calculated [20] in the one-loop approximation, $\kappa_{1,3}$ grow with the field as $(\alpha/3\pi) \times \ln(B/B_0)$, but this remains yet small (~ 0.04) for the fields of the order of (12). The interaction between charged particles carried by these two photon modes remains practically the same as it was without the vacuum polarization and thus continues to support the collapse. However, the eigenvalue κ_2 , besides the logarithmic growth, also contains a term that increases with B linearly [21,22]:

$$\begin{aligned} \kappa_2(k) &= \frac{\alpha B m^2 (k_0^2 - k_3^2)}{\pi B_0} \exp\left(-\frac{k_\perp^2}{2m^2} \frac{B_0}{B}\right) \\ &\times \int_{-1}^1 \frac{(1 - \eta^2) d\eta}{4m^2 - (k_0^2 - k_3^2)(1 - \eta^2)} \end{aligned} \quad (15)$$

and might seemingly screen the interaction carried by the mode 3 photon. Here k_0 is the photon energy, k_3 and \mathbf{k}_\perp are its momentum components along and transverse the magnetic field, $k^2 = k_0^2 - \mathbf{k}^2$. However, once the electron-positron separation in the plane perpendicular to the magnetic field $\mathbf{x}_\perp = \mathbf{x}_\perp^e - \mathbf{x}_\perp^p$ is restricted to the domain inside the Larmour radius $|\mathbf{x}_\perp| \ll (eB)^{-1/2}$, the integration in (14) gets essential contribution from large $|k_\perp| \gg (eB)^{1/2}$. In this domain the exponential in (15) suppresses the linearly growing term in the denominator of D_2 . This is how the exponentially strong spatial dispersion opposes the screening [19].

Among the mass radiative corrections, the so-called \ln^2 terms are potentially dangerous for the present gap-shrinking effect, since they yield a competing growth of the corrected electron mass, essential at the scale of (12). Substituting the one-loop corrected [23] electron mass \tilde{m} ,

$$\tilde{m} \approx m \left(1 + \frac{\alpha}{4\pi} \ln^2 \frac{B}{B_0}\right), \quad (16)$$

for m , and $L_B = (eB)^{-1/2}$ for s_0^{\max} into Eq. (10), we

estimate that the maximum field increases only by a factor of ~ 10 . The most recent results concerning the mass correction that take into account the vacuum polarization diagrams inside the mass operator [24] read that the \ln^2 terms do not, as a matter of fact, appear at all, thanks to the presence of the growing term (15).

At $B = B_{\max}$ the total energy and momentum of a positronium in the ground state are zero. This state is not separated from the vacuum by an energy gap, and it is the one of maximum symmetry in the coordinate and spin space. Hence, it may be related to the vacuum and describe its structure.

What happens when the magnetic field exceeds the maximum value (12)? To answer this question one would have to solve (a more complicated) BS equation with P_{\parallel} nonzero and spacelike. In that case we transit from the “sector III of confined states” to the “deconfinement sector IV” [12] where solutions are free waves both near $s = 0$ and $s = \infty$ and correspond to delocalized states of mutually free electron and positron—each on its Larmor orbit—moving along the magnetic field. These are capable of screening the magnetic field and put a limit to its further growing. A more detailed discussion of this hypothesis that traces an analogy with the known problem of a Dirac electron in the Coulomb field of a supercharged nucleus [25] and also dwells on the structure of the corresponding translationally-non-invariant vacuum state is presented in [14]. For the present, we state that the hypervalue (12) is such a value of the magnetic field, the exceeding of which would cause restructuring of the vacuum and demand a revision of QED.

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