Thermal Background Can Solve the Cosmological Moduli Problem

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It is shown that the coherent field oscillation of moduli fields with weak or TeV scale masses can dissipate its energy efficiently if they have a derivative coupling to standard bosonic fields in a thermal state. This mechanism provides a new solution to the cosmological moduli problem without creating too much entropy at late time.

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Modern theories of high energy physics contain a number of scalar fields which have a flat potential and interact with ordinary particles only with the gravitational strength [1]. In the context of low-energy supersymmetry, these moduli fields typically have a flat potential intrinsically and acquire a mass of the order of weak or TeV scale when supersymmetry is broken [2] although other mechanisms of moduli stabilization have also been extensively discussed recently [3]. During inflation in the early universe [4], supersymmetry is spontaneously broken in a different manner than it is today due to the large vacuum energy density. Then moduli fields, which we denote by ϕ, typically acquire a mass of the order of the Hubble parameter and they settle down to a potential minimum at this stage, which is deviated from today's value at the current potential minimum by up to the gravitational scale, $\Delta\phi \lesssim M_G = 2.4 \times 10^{18}$ GeV. After inflation their mass is turned off to a much smaller value due to the disappearance of vacuum energy density and they keep their position until the Hubble parameter decreases to their eventual mass scale which is presumably of order of the weak scale or TeV scale as stated above. The scalar fields then start coherent oscillation with the initial amplitude up to M_G, which will dominate the energy density of the universe eventually. According to the conventional estimates, their lifetime is given by

$$\tau_\phi \approx \frac{M_G^2}{m_\phi^3} \approx 10^8 \left(\frac{m_\phi}{10^2 \text{ GeV}}\right)^{-3} \text{ sec}, \qquad (1)$$

that is, they decay after the primordial nucleosynthesis creating a huge amount of entropy to demolish the successful primordial nucleosynthesis [1].

In this Letter we present a new class of solution to the cosmological moduli problem by arguing that the previous estimate of the decay rate (1), which has been used in all the other proposed solutions to the problem [5] does not apply in a finite-temperature and finite-density state of the early universe and that it is much more enhanced than in the case of decay in a vacuum. As a result we show that the coherent moduli oscillation can efficiently dissipate its energy density well before the big bang nucleosynthesis.

The crucial point is to take moduli decay through derivative coupling correctly into account, such as a coupling with a kinetic term of other fields. Indeed, we expect the moduli field is coupled with gauge fields through $\frac{\phi}{M_G} F_{\mu\nu}F^{\mu\nu}$. It may also be coupled with scalar fields as $\lambda \frac{\phi}{M_G}(\partial\chi)^2$ or $\lambda \frac{\phi}{M_G}\chi\Box\chi$ where χ is a generic scalar field and λ is a dimensionless coupling constant of order unity [6]. Previously these couplings were expected to give a decay rate no different from (1) at most, because using the equation of motion, $\Box\chi = m_\chi^2\chi$, it was concluded that the derivative coupling would give a decay rate similar to the coupling $\frac{\phi}{M_G} m_\chi^2 \chi^2$, which yields (1) for $m_\phi \gtrsim 2m_\chi$.

Such a naive analysis, however, could only be valid in decay in the vacuum and would not apply in the high-temperature environment in the early universe. If χ is a standard field in the visible sector, it is strongly coupled with other degrees of freedom in the early universe and rapidly reaches thermal equilibrium. Then they acquire a thermal mass of order of $\sim gT$ in general, where g is a typical gauge coupling and T is the cosmic temperature, so that the right-hand side of the above equation of motion could be significantly enhanced. Alternatively, one may regard the derivative ∂ acting on χ as not yielding its rest mass energy m_χ but energy-momentum arising from finite-temperature environment, $\partial \sim T$ modulo some coupling. Then, the strength of interaction $\lambda \frac{\phi}{M_G}(\partial\chi)^2$ is estimated as

$$L_{\text{int}} \approx \frac{\lambda (gT)^2}{M_G} \phi\chi^2. \qquad (2)$$

The decay rate of ϕ through the above interaction should read

$$\Gamma \approx \frac{\lambda^2 g^4 T^4}{8\pi M_G^2 m_\phi}\left[1 + 2n_B\left(\frac{m_\phi}{2}\right)\right] C \approx \frac{\lambda^2 g^4 T^5}{2\pi M_G^2 m_\phi^2} C, \qquad (3)$$

where C is a suppression factor due to a large thermal mass of the decay product χ which has been given in Ref. [7] for a specific model. Here $n_B(\omega) = 1/(e^{\omega/T} - 1)$ is the thermal number density of a boson and the factor in the bracket represents the effect of induced emission [8,9]. Taking $\lambda \sim g \sim 1$, $m_\phi \sim 10^2$ GeV, $T \sim 10^{10}$ GeV, which is a typical radiation temperature at the onset of moduli oscillation $H \sim m_\phi$, we find $\Gamma \sim 3 \times 10^8 C$ GeV. Thus if C takes an appropriate value, ϕ can dissipate its energy right after it starts oscillation.

In order to examine that the above estimate is correct, we employ nonequilibrium field theory to calculate the dissipation rate of a modulus ϕ in the presence of a derivative interaction. For simplicity we consider the following model consisting of two scalar fields, ϕ and χ.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m_\phi^2 \phi^2 + \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}m_\chi^2 \chi^2 + \lambda \frac{\phi}{M_G}(\partial \chi)^2 - \frac{1}{4}g^2 \chi^4, \quad (4)$$

which also mimics interaction $\frac{\phi}{M_G}F_{\mu\nu}F^{\mu\nu}$ if we identify g with the gauge coupling strength.

We calculate the dissipation rate of ϕ under the following setup appropriate to the specific problem we are working on. First we neglect cosmic expansion since we are interested only in the case in which dissipation time is shorter than the cosmic expansion time. Second we assume χ is in a thermal state with a specific temperature $T = \beta^{-1}$ owing to the rapid thermalizing interaction due to the self-coupling. Finally, we consider the situation in which the parametric resonance [10] is ineffective, which is the case for the modulus mass range of our interest [11].

We calculate an effective action for ϕ and derive an equation of motion for its expectation value using the closed time-path formalism [12,13]. Although this method has been applied to various cosmological problems by a number of authors [14,15], to our knowledge, derivative coupling at finite temperature has not been investigated in this context yet. The one-loop effective action relevant to dissipation due to the derivative coupling is given by

$$\Gamma[\phi_c, \phi_\Delta] = -\int d^4x \phi_\Delta(x)(\Box + m_\phi^2)\phi_c(x) - \int d^4x d^4x' C(x-x')\theta(t-t')\phi_\Delta(x)\phi_c(x') + \frac{i}{2}\int d^4x d^4x' D(x-x')\phi_\Delta(x)\phi_\Delta(x') + \cdots. \quad (5)$$

Here ϕ_c and ϕ_Δ are the mean and difference of the field variable in the forward time branch ($t = -\infty$ to $+\infty$), ϕ_+, and those in the backward time branch ($t = +\infty$ to $-\infty$), ϕ_-, namely, $\phi_c \equiv (\phi_+ + \phi_-)/2$ and $\phi_\Delta \equiv \phi_+ - \phi_-$, respectively. ϕ_+ and ϕ_- should be identified with each other in the end. The kernels in (5) are defined by

$$C(x - x') \equiv \frac{\lambda^2}{M_G^2}\eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}\,\mathrm{Im}[\mathcal{G}^F_{\mu_1\mu_2}(x-x') \times \mathcal{G}^F_{\nu_1\nu_2}(x-x')], \quad \text{for } t - t' > 0, \quad (6)$$

$$D(x - x') \equiv \frac{\lambda^2}{2M_G^2}\eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2} \times \mathrm{Re}[\mathcal{G}^F_{\mu_1\mu_2}(x-x')\mathcal{G}^F_{\nu_1\nu_2}(x-x')]. \quad (7)$$

Here $\mathcal{G}^F_{\mu\nu}(x, x')$ is a finite-temperature Feynman propagator of field derivatives defined by

$$\mathcal{G}^F_{\mu\nu}(x, x') \equiv \langle\beta|T\partial_\mu\chi(x)\partial_\nu\chi(x')|\beta\rangle = \partial_{x^\mu}\partial_{x'^\nu}G^F_\chi(x-x') + i\delta_{\mu 0}\delta_{\nu 0}\delta^4(x-x'), \quad (8)$$

with $G^F_\chi(x) \equiv \langle\beta|T\chi(x)\chi(x')|\beta\rangle$.

The effective action (5) is complex-valued as a manifestation of the dissipative nature of the system. We cannot obtain any sensible equation of motion by simply differentiating with respect to a field variable because we are dealing with a real scalar field and its equation of motion should be real valued. As shown in Ref. [14], one can obtain a real-valued effective action by introducing a random Gaussian variable $\xi(x)$, which represents fluctuation related to dissipation, with the dispersion $\langle\xi(x)\xi(x')\rangle = D(x-x')$. As a result of the equation of motion for the expectation value $\phi_c(x)$ is given by

$$(\Box + m_\phi^2)\phi_c(x) + \int_{-\infty}^{t} dt' \int d^3x' C(x-x')\phi_c(x') = \xi(x). \quad (9)$$

Hereafter we omit the suffix c.

As described in Refs. [7,9], this equation can easily be solved using Fourier transform. As a result, we find the dissipation rate of zero-mode modulus oscillation is related to the imaginary part of ϕ's self-energy and given by

$$\Gamma_\phi = i\frac{\tilde{C}_0(m_\phi)}{2m_\phi}, \quad (10)$$

with $\tilde{C}_0(m_\phi)$ being the $\boldsymbol{k} = 0$ mode of the Fourier transform, $\tilde{C}_{\boldsymbol{k}}(\omega)$, of the memory kernel $C(x)$.

In order to take thermal effects of χ correctly into account, we should use the full dressed propagator, $\mathcal{G}^{F(\mathrm{drs})}_{\mu\nu}(x, x')$ to calculate $\tilde{C}_{k=0}(\omega = m_\phi)$ [7]. It is obtained by calculating

$$\mathcal{G}^{F(\mathrm{drs})}_{\mu\nu}(x, x') = \langle\beta|T\partial_\mu\chi(x)\partial_\nu\chi(x') \times \exp(-i\int \mathcal{L}_{\mathrm{int}}d^4x)|\beta\rangle, \quad (11)$$

using the Matsubara representation [16] and resummation, where

$$\mathcal{L}_{\mathrm{int}} = -\frac{1}{4}g^2\chi^4 + \lambda\frac{\phi}{M_G}(\partial\chi)^2. \quad (12)$$

Since loops generated by the second term of (12) have an extremely small effective coupling,

$$\lambda\frac{\partial_\mu}{M_G} \sim \lambda\frac{p_\mu}{M_G} \sim \lambda\frac{T}{M_G} \sim 10^{-8}\lambda, \quad \text{for } T \sim 10^{10}\ \mathrm{GeV},$$

they are safely negligible. As a result, the finite-temperature dressed propagator is given in terms of the spectral representation as

$$\mathcal{G}^{F(\text{drs})}_{\mu\nu}(\boldsymbol{p},t) = i\int\frac{d\omega}{2\pi}\left(\{[1+n_B(\omega)]\theta(t)+n_B(\omega)\theta(-t)\}\left[\frac{1}{(\omega+i\Gamma_p)^2-\omega_p'^2}-\frac{1}{(\omega-i\Gamma_p)^2-\omega_p'^2}\right]p_\mu p_\nu + i\delta_{\mu 0}\delta_{\nu 0}\right)e^{-i\omega t}, \tag{13}$$

with $p_\mu = (\omega, \boldsymbol{p})$, $\omega_p'^2 = \boldsymbol{p}^2 + m_\chi^2 + \Sigma_R(p) + \Gamma_p^2$ and $\Gamma_p = -\Sigma_I(p)/(2\omega)$, where $\Sigma_R(p)$ and $\Sigma_I(p)$ are real and imaginary parts of χ's self-energy. To the lowest nonvanishing order, we find $\Sigma_R(p) = g^2T^2/4$ and $\Gamma_p = 3g^4T^2/(128\pi\omega_p')$ [17].

Inserting (13) into (10), the dissipation rate of the coherent field oscillation is given by

$$\begin{aligned}\Gamma_\phi = {}& \frac{\lambda^2}{2m_\phi M_G^2}\int\frac{d^3p}{(2\pi)^3}\frac{1}{4\omega_p'^2}\bigg\{(2\boldsymbol{p}^2+m_T^2)^2[2n_B(\omega_p')+1]\left[\frac{2\Gamma_p}{(m_\phi-2\omega_p')^2+(2\Gamma_p)^2}-\frac{2\Gamma_p}{(m_\phi+2\omega_p')^2+(2\Gamma_p)^2}\right]\\ &+[-2(2\boldsymbol{p}^2+m_T^2)^2n_B^2(\omega_p')e^{\beta\omega_p'}\beta\Gamma_p + 4(2\boldsymbol{p}^2+m_T^2)\omega_p'\Gamma_p[2n_B(\omega_p')+1]]\\ &\times\left[\frac{m_\phi-2\omega_p'}{(m_\phi-2\omega_p')^2+(2\Gamma_p)^2}+\frac{m_\phi+2\omega_p'}{(m_\phi+2\omega_p')^2+(2\Gamma_p)^2}\right]+2m_T^4n_B^2(\omega_p')e^{\beta\omega_p'}\beta\Gamma_p\frac{2m_\phi}{m_\phi^2+(2\Gamma_p)^2}\bigg\},\end{aligned} \tag{14}$$

to the first order in Γ_p. Here $m_T^2 = m_\chi^2 + \Sigma_R = m_\chi^2 + g^2T^2/4$ is the finite-temperature mass. We find the last term is dominant in (14), to yield

$$\Gamma_\phi \approx \frac{\lambda^2 g^4 m_T^2 T^3}{512\pi^3 M_G^2 m_\phi^2} = \frac{\lambda^2 g^6 T^5}{2048\pi^3 M_G^2 m_\phi^2} = 1.6\times 10^{-5}\frac{\lambda^2 g^6 T^5}{M_G^2 m_\phi^2}. \tag{15}$$

The above result has the same form as (3) with the suppression factor $C = g^2/(1024\pi^2)$ depending only on the coupling constant as expected [7].

We now discuss cosmological implications of the above result (15). Taking $g \sim \lambda \sim 1$, $m_\phi \sim 10^2$ GeV, and $T \sim 10^{10}$ GeV as before, we find a value as large as $\Gamma_\phi \sim 3\times 10^4$ GeV, which is much larger than the cosmic expansion rate in the interested regime. This does not guarantee, however, that the moduli fields dissipate its energy immediately, because the above expression has been obtained under several assumptions. So we investigate its significance more carefully here.

Our result (15) has been obtained using perturbative analysis, which requires $m_\phi > \Gamma_\phi$, and under the assumption that the scalar field is rapidly oscillating in the expansion time scale, $m_\phi > H$. Although moduli can certainly dissipate its energy partially even before the onset of rapid oscillation, the dissipation rate in such a slowly evolving regime would be given by a different expression which could be worked out using a formalism described in, e.g., Ref. [18]. We neglect dissipation in such a regime here. In this sense, our analysis below provides a conservative result and in reality moduli could be dissipated even more efficiently in total.

In the conventional scenario neglecting the above effect, the moduli start oscillation with the initial amplitude $\phi_i \lesssim M_G$ as the cosmic expansion rate becomes smaller than $\sim m_\phi$. For definiteness we assume the reheating after inflation has been completed by then and that the cosmic energy density consists of moduli and radiation only, which we denote by ρ_ϕ and ρ_r, respectively. We find $\rho_\phi \lesssim 2\rho_r$ at the onset of oscillation for $\phi_i \lesssim M_G$. In the above conservative spirit, we pretend that dissipation rate we have obtained, (15), is not turned on until the epoch t_i characterized by $H(t_i) = m_\phi/f$ where $f \gg 1$ is a parameter which represents the rapidness of field oscillation at that time. The energy transfer equations read

$$\frac{d\rho_\phi}{dt} = -3H\rho_\phi - \Gamma_\phi\rho_\phi, \qquad \frac{d\rho_r}{dt} = -4H\rho_r + \Gamma_\phi\rho_\phi. \tag{16}$$

Since we expect $\Gamma_\phi \gg H$, we can neglect the redshift terms in the right-hand side of the above equations for a sufficiently small time scale, $\Delta t \lesssim H^{-1}$, when total energy density $\rho_{\text{tot}} = \rho_\phi + \rho_r$ is conserved. Taking the temperature dependence of $\Gamma_\phi \propto T^5 \propto \rho_r^{5/4}$ into account, we can solve these equations in this regime, to yield

$$\rho_\phi(t) = \rho_{\text{tot}}[1-(1-Ke^{-A\rho_{\text{tot}}^{5/4}(t-t_i)})^4] \rightarrow 4K\rho_{\text{tot}}e^{-A\rho_{\text{tot}}^{5/4}(t-t_i)},$$

for $t - t_i \lesssim H^{-1}$. Here A is a constant defined by $A \equiv \Gamma_\phi\rho_r^{-5/4}$, and K is a constant of order of unity which is determined by the initial ratio $\rho_\phi/\rho_r|_{t=t_i}$. We find $K = 0.314$ for $\rho_\phi/\rho_r|_{t=t_i} = 1$.

The above result shows that the modulus decays exponentially for the period $\Delta t \lesssim H^{-1}$ with the dissipation rate $\Gamma_\phi' \equiv A\rho_{\text{tot}}^{5/4}$. As we will see below, we can naturally find $\Gamma_\phi' \gg H(t_i)$, so that the universe would be radiation dominated by $\Delta t \lesssim H^{-1}$. For $\Delta t \gtrsim H^{-1}$, redshift terms in (16) are important but the second term of the right-hand side of the second equation is already negligible. Hence we find $a^4(t)\rho_r(t)$ is conserved with $a(t) \propto t^{1/2}$ in this regime. Then we can solve the first equation to find $a^3(t)\rho_\phi(t)$ will reduce by an extra factor of $\exp(-\frac{2\Gamma_\phi'}{3H(t_i)})$ for $\Delta t \gg H^{-1}$. Thus the moduli-to-entropy ratio n_ϕ/s is lowered by a factor of $\sim\exp(-\frac{5\Gamma_\phi'}{3H(t_i)})$ including its decay in the initial regime $\Delta t \lesssim H^{-1}$. If a significant amount of entropy is created in this process, which is not likely in the present case due to the rapidness of decay unlike in the conventional scenario, the ratio could be reduced even more.

We now work out the numerical value of the dissipation rate. If there are N decay modes with the same coupling, it is given by

$$\Gamma'_\phi = A\rho_{\rm tot}^{5/4} = 1.6\times 10^{-5}\frac{N\lambda^2 g^6}{M_G^2 m_\phi^2}\left(\frac{90M_G^2 m_\phi^2}{\pi^2 g_* f^2}\right)^{5/4}$$
$$= 48\left(\frac{N}{10}\right)\left(\frac{\lambda}{2}\right)^2 g^6\left(\frac{g_*}{10^2}\right)^{-5/4}\left(\frac{f}{40}\right)^{-5/2}\left(\frac{m_\phi}{10^2\,{\rm GeV}}\right)^{1/2}{\rm GeV}, \tag{17}$$

which is to be compared with the expansion rate,

$$H = 2.5\left(\frac{f}{40}\right)^{-1}\left(\frac{m_\phi}{10^2\ {\rm GeV}}\right)\ {\rm GeV}. \tag{18}$$

Γ'_ϕ can be much larger than H if we adopt λ somewhat larger than unity or assume that ϕ has a derivative coupling to many fields. Both are naturally realized since the moduli fields are expected to be coupled with other fields universally and the coupling of the type $\lambda\frac{\phi}{M_G}(\partial\chi)^2$ arises from expanding the exponential coupling $e^{\lambda\phi/M_G}(\partial\chi)^2$ where there is no reason to restrict $\lambda < 1$. In fact, we usually find $\lambda > 1$ [19]. For perturbative analysis to be valid and to ensure oscillatory behavior of ϕ, Γ_ϕ cannot exceed m_ϕ. But within this limit, the modulus can efficiently dissipate its energy, say, by a factor of e^{-50} or so even within the above conservative analysis.

However, we cannot let the moduli decay completely, no matter how efficient our dissipation mechanism is. This is due to the fact that our mechanism relies on the thermal effects. The final abundance of moduli is not zero but its thermal value at the temperature T_f when Γ_ϕ becomes smaller than H [9]. That is, the abundance of $\boldsymbol{k}$ mode at freeze-out is given by $n_B(M_k) \simeq T_f/M_k$ with $M_k = \sqrt{m_\phi^2 + \boldsymbol{k}^2}$ for $T_f \gg M_k$. The contribution of low momentum mode $|\boldsymbol{k}| = 0 \sim m_\phi$ to the moduli-to-entropy ratio, to which we are primarily concerned, is given by

$$\frac{n_\phi}{s} \approx \frac{90}{4\pi^2 g_* T_f^3}\frac{T_f}{m_\phi}\frac{4\pi m_\phi^3}{(2\pi)^3}$$
$$\approx 10^{-17}\left(\frac{g_*}{10^2}\right)^{-5/3}\left(\frac{\lambda}{2}\right)^{4/3} g^4\left(\frac{m_\phi}{10^2\ {\rm GeV}}\right)^{2/3}, \tag{19}$$

which is comfortably small.

Note also that the tadpole diagrams generate a linear term in the effective action, which shifts the potential minimum at finite temperature by the amount $\delta\phi_{\rm min}(T)\sim\frac{\lambda T^4}{m_\phi^2 M_G}$. Hence the proposed mechanism anchors the modulus at a different field value than the zero-temperature minimum. Since the Hubble parameter is already much smaller than m_ϕ when it works, the modulus traces the change in $\phi_{\rm min}(T)$ adiabatically after being relaxed to $\phi_{\rm min}(T_f)$ at $T = T_f$ until it finally reaches the zero-temperature minimum. Therefore this shift is harmless.

In summary, we have discovered a new mechanism which dissipates coherent oscillation of moduli fields efficiently without introducing any new physics, by carefully recalculating its decay rate through the derivative coupling in the thermal background. As a result we have shown that our mechanism can dissipate the most significant part of modulus energy efficiently.

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