Signature of the Ground-State Topology in the Low-Temperature Dynamics of Spin Glasses

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We numerically address the issue of how the ground-state topology is reflected in the finite temperature dynamics of the $\pm J$ Edwards-Anderson spin glass model. In this system a careful study of the ground-state configurations allows us to classify spins into two sets: solidary and nonsolidary spins. We show that these sets quantitatively account for the dynamical heterogeneities found in the mean flipping time distribution at finite low temperatures. The results highlight the relevance of taking into account the ground-state topology in the analysis of the finite temperature dynamics of spin glasses.

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Spin glass models are the paradigm of disordered systems with slow dynamics [1–3]. The main ingredients which define these models are quenched disorder and an inherent frustration in the interactions. These ingredients lead to a nontrivial ground-state topology [3], and slow dynamics with spatial and dynamical heterogeneities [4–10]. Works analyzing the out-of-equilibrium properties have intuitively suggested a relation between dynamical heterogeneities and the ground-state topology [4,11,12]. However, a quantitative understanding of this precise relation still remains an open question.

In particular, recent works [8,9] have analyzed the dynamical heterogeneities found in three different heterogeneous spin models. By studying single spin dynamics different qualitative behaviors were observed. On the other hand, other studies [5–7,10] have focused on spatially coarse grained quantities and analyzed heterogeneities within a given coarse grained length.

In this work we take into account a global property, dictated by the ground-state topology, in order to analyze dynamical heterogeneities. We establish for the first time a quantitative relation between the ground-state topology and the finite temperature dynamical properties of a spin glass model. We find that the dynamical heterogeneities are well accounted for by two sets of spins characterized by their role in the ground state.

We consider, in particular, the two-dimensional $\pm J$ Edwards-Anderson (EA) spin glass model. In this model there exist clusters of spins which maintain their relative orientation for all configurations of the ground-state manifold [13–15]. We extend this ground-state information to analyze the behavior of the system at finite temperatures. In order to do this we divide the system into two sets of spins: solidary spins, i.e., the spins that form these clusters, and nonsolidary spins. The consequences of this division are twofold. On one hand it gives us a quantitative tool to establish a relation between the ground-state topology and the finite temperature dynamical properties. On the other, it gives an intuitive physical frame in which to interpret the results.

We begin our analysis considering the spin autocorrelation function, which clearly illustrates the different qualitative behaviors observed when the proposed division is taken into account. The nonsolidary spins decorrelate faster than solidary spins, which suggests a relation with the separation in fast and slow degrees of freedom. In order to address this point we analyze the time scale separation as observed in the mean persistence time and mean flipping time probability distribution functions [4,16]. We show that the observed time scale separation can be quantitatively accounted for by the two sets of spins.

We consider the two-dimensional $\pm J$ EA model for spin glasses [1–3], defined on a square lattice with periodic boundary conditions. The Hamiltonian of the model is

$$H = \sum_{\langle i,j \rangle} J_{ij}\sigma_i\sigma_j, \tag{1}$$

where $\sigma_i = \pm 1$ is the spin variable and $\langle i, j\rangle$ indicates a sum over nearest neighbors. The coupling constants $J_{ij} = \pm J$ are random variables chosen from a bimodal distribution. The time evolution of the model is governed by a standard Glauber Monte Carlo process with sequential random updates.

In this model there exists clusters of *solidary* spins which maintain their relative orientation for all configurations of the ground-state manifold [14,15]. This backbone can be detected for each sample through the identification of the diluted lattice [17,18], or its generalization, the rigid lattice [15]. The latter is formed by those bonds which are *always* satisfied or *always* frustrated in the ground-state manifold. Notice that a backbone is also present in other systems such as the K-satisfiability model [12,19].

A particular sample of size N can be characterized by recognizing all its solidary spins as shown in Fig. 1. In order to obtain a statistical average over different realizations of bond disorder, in all the results presented we have

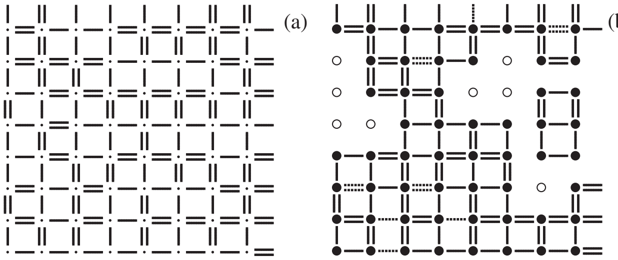

FIG. 1. (a) A particular realization of bond disorder in an 8×8 lattice. Single (double) lines indicate ferromagnetic (antiferromagnetic) bonds. (b) The corresponding rigid lattice (backbone). Full (dotted) lines indicate interactions which are always satisfied (frustrated) in the ground-state manifold. The solidary (nonsolidary) spins are indicated with closed (open) circles.

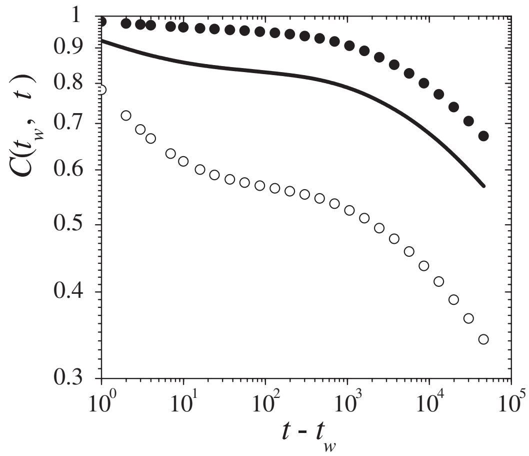

FIG. 2. Autocorrelation function C for $t_w = 10^4$ and $T = 0.5$. The full line shows the behavior of C when all spins are taken into account. Closed (open) circles correspond to the behavior of the correlation $C_s(C_{\rm ns})$ when only solidary (nonsolidary) spins are considered.

calculated the sets of solidary spins for 2000 different samples in systems with size 16×16. All mean values are obtained from averages with respect to both realizations of bond disorder and thermal histories, as in Eq. (2).

We begin our analysis of the out-of-equilibrium properties by considering the two-time autocorrelation function,

$$C(t_w, t) = \frac{1}{N} \sum_{i=1}^{N} [\langle \sigma_i(t_w) \sigma_i(t) \rangle], \qquad (2)$$

which measures the overlap of the spin configurations at times t_w and t [20]. The brackets $[\ldots]$ indicate an average over different realizations of bond disorder, while $\langle \ldots \rangle$ is a thermal average, i.e., an average over different initial conditions and realizations of the thermal noise. In each initial condition the spins take random values $\sigma_i = \pm 1$, which corresponds to a quench at $t = 0$ from $T = \infty$ to the temperature T at which the system is analyzed. It is worth stressing that usually one is interested in studying the out-of-equilibrium properties below the critical temperature T_c. However, in the two-dimensional EA spin glass model $T_c = 0$ [21,22]. Nevertheless, it is widely accepted that for low enough temperatures, the dynamics remains out of equilibrium at short times and is very similar to the one observed in three-dimensional spin glasses [12,23].

For each realization of bond disorder the division in solidary and nonsolidary spins can be taken into account rewriting the sum in Eq. (2) as $C(t_w, t) = f_s C_s + f_{\rm ns} C_{\rm ns}$, where f_s ($f_{\rm ns} = 1 - f_s$) is the fraction of solidary (nonsolidary) spins, and C_s ($C_{\rm ns}$) is the two-time autocorrelation function restricted over the solidary (nonsolidary) spins. Note that the fraction of solidary spins is approximately 67% of the total number of spins [21]. Figure 2 shows the behavior of $C(t_w, t)$ vs $t - t_w$ when $T = 0.5$ and $t_w = 10^4$. For this parameters the system is in the aging regime [23], and similar qualitative results are obtained for lower temperatures. The full line corresponds to the behavior of C when all the spins are considered. The behavior of C_s ($C_{\rm ns}$) is indicated with closed (open) circles. For short times ($t - t_w < 10$) the solidary spins are strongly correlated, i.e., they maintain their relative orientation in time. The nonsolidary spins present a qualitatively different behavior, with a faster decay of the correlation function. Only those spins which are solidary in the ground state tend to remain correlated in time at finite temperatures. For long times ($t - t_w > 10^3$) each set of spins presents the same qualitative decay as the whole system. This shows that the relaxation times of a fraction of nonsolidary spins is coupled to the ones of solidary spins thus decorrelating together at longer times. This behavior suggests a strong separation in characteristic times for the two sets of spins and a possible path to analyze dynamical heterogeneities as previously observed in Ref. [4].

One possible path to the analysis of dynamical heterogeneities is through the mean persistence time probability distribution function (PDF). This quantity depends on the time window of interest, given by t_w and t, and is defined as the time at which, in average, a given spin changes its state for the first time with respect to its state at t_w. The mean persistence time, τ_p, is obtained for every spin and the corresponding PDF is constructed, $P_p(\ln\tau_p)$. The $\ln\tau_p$ scale is preferred due the broadness of the PDF.

In Fig. 3 the behavior of $P_p(\ln\tau_p)$ is shown. The symbols are the same as in Fig. 2. The PDF of the whole system (full line) presents a sharp peak around $\ln\tau_p \sim 7$ ($\tau_p \sim 10^3$), with a pronounced shoulder for lower times. It is worth stressing that there exists a direct relation between the mean persistence time PDF and the autocorrelation function. For short times, the position of the shoulder, $\ln\tau_p \sim 2$ ($\tau_p \sim 10$), corresponds to the first decay observed in the full C, while for long times, the position of the peak coincides with the second decay observed in the full C. The division in solidary and nonsolidary spins allows for a physical interpretation of this time scale separation. In

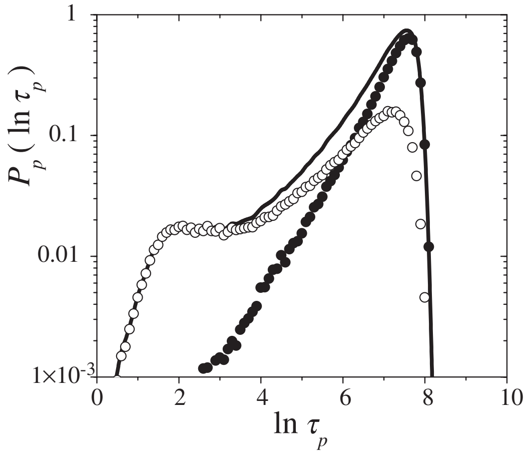

FIG. 3. Mean persistence time PDF for the time window $t - t_w = 10^4$ with $t_w = 10^4$ and $T = 0.5$. The whole distribution (full line) is divided in the PDF of the solidary (closed circles) and nonsolidary (open circles) spins.

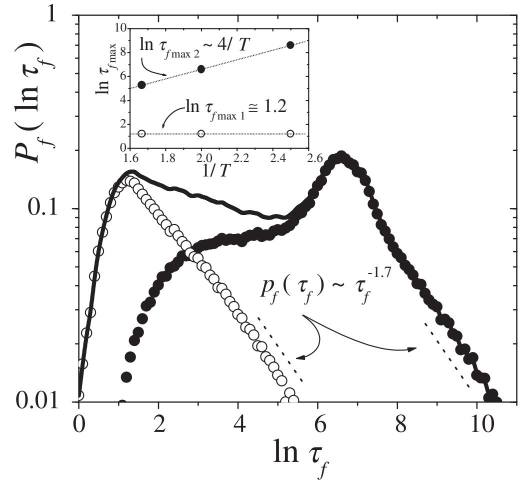

FIG. 4. Mean flipping time PDF for the time window $t - t_w = 10^4$ with $t_w = 10^4$ and $T = 0.5$. The whole distribution (full line) is divided in the PDF of the solidary (closed circles) and nonsolidary (open circles) spins. The power-law like behavior of the distribution's tails is highlighted. The inset shows the behavior of the maximum of the solidary (nonsolidary) spins PDF, $\ln\tau_{f\max 2}$ $(\ln\tau_{f\max 1})$, for three different temperatures: $T = 0.4$, 0.5, and 0.6.

Fig. 3 we show the mean persistence time PDF for solidary and nonsolidary spins separately. We observe that the shoulder found in the full PDF is given only by a contribution of the nonsolidary spins. On the other hand, both solidary and nonsolidary spins contribute to the sharp peak. The interpretation is straightforward: A fraction of nonsolidary spins decorrelate first due to their low mean persistence time. At higher times the remaining fraction of the nonsolidary spins and the solidary spins decorrelate together, both having similar mean persistence time. The same relation between the mean persistence time PDF and the autocorrelation function was observed for lower temperatures, giving support to our interpretation.

We expect this particular separation in solidary and nonsolidary spins to be reflected in other finite temperature dynamical quantities. Recently, Ricci-Tersenghi and Zecchina [4] have observed a strong time scale separation in the mean flipping time PDF, P_f, as a signature of dynamical heterogeneities. We analyze this quantity using the ground-state information. P_f is obtained by measuring the number of flips (N_{flips}) done by every spin within the time window extending from t_w to t. The mean flipping time τ_f for a given t_w and t is defined as the time window size divided by the number of flips: $\tau_f = (t - t_w)/N_{\mathrm{flips}}$ [4].

Figure 4 shows the behavior of $P_f(\ln\tau_f)$. The symbols are the same as in Fig. 2. The PDF of the whole system (full line) presents two main peaks [4], which are a manifestation of strong dynamical heterogeneities [24]. Generally speaking, these two peaks correspond to fast (left peak) and slow (right peak) spins. We also measure the mean flipping time distribution for solidary and nonsolidary spins separately. In Fig. 4 we show that the two peaks of the full PDF can be well accounted for by this separation. The slow (fast) spins at finite temperature correspond to solidary (nonsolidary) spins. At high temperatures, the two peaks collide and the strong time scale separation is no longer observable.

It is worth stressing that this separation reveals that a further internal structure is present, as can be seen in the shoulder observed in the mean flipping time PDF of the solidary spins in Fig. 4. For low temperatures we observed that the shoulder does not seem to depend on temperature. Instead, the peak of the slow spins moves to higher values in accordance with an activation process with a characteristic energy barrier (see inset in Fig. 4). This energy barrier, $4J$, corresponds to flipping a spin with only one frustrated bond. This should be contrasted with the fact that the peak of the fast spins does not move with temperature as shown in the inset. However, we must point out that a possible difference could be present in the tails of the distributions. For both fast and slow spins, the tails seem to be power-law-like, $p_f(\tau_f) \sim \tau_f^{-1.7}$.

Summarizing, we have presented a numerical study of the two-dimensional $\pm J$ EA spin glass focusing on how the information of the topology of the ground-state manifests in the finite low-temperature dynamics. We have concentrated in the preasymptotic aging regime of this particular model as representative of glassy dynamics.

In the ground state, spins can be divided in two sets, solidary and nonsolidary spins. In the EA model, this characterization is nontrivial and deserves careful and time consuming simulations [18]. Once these two sets were identified, we analyzed the contribution of each set to the finite temperature dynamics. The autocorrelation function for each set of spins behaves differently, showing a faster initial decay for nonsolidary spins. This naturally leads to the analysis of dynamical heterogeneities. First, we analyze the mean persistence time distribution, and show that it is intimately related to the two-step relaxation of the autocorrelation function. A fraction of nonsolidary spins, with lower mean persistence times, give rise to the first decay of the autocorrelation function. The decay observed at longer times corresponds to a peak in the mean persistence time distribution and is shared by solidary and nonsolidary spins.

Finally, we test the relevance of the separation in solidary and nonsolidary spins in the mean flipping time distribution, which presents strong dynamical heterogeneities. As was already pointed out in Ref. [4] this distribution presents two sharp peaks. This time-scale separation was used to dynamically define groups of slow and fast spins [4]. Here we show for the first time that this dynamical characterization is well accounted for by the ground-state characterization in solidary and nonsolidary spins. Furthermore, new interesting and promising questions arise. For instance, the mean flipping time distribution for solidary spins presented in Fig. 4 presents a clear shoulder at low mean flipping times. This shoulder corresponds to an internal structure within the set of solidary spins. This suggests that a further division into subsets could refine our results, and should be of relevance for understanding heterogeneities in EA spin glasses with continuous coupling distributions.

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