Percolation Effects in Very-High-Energy Cosmic Rays

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Cosmic ray data at high energies present a number of well-known puzzles. At very high energies ($E \sim 10^{20}$ eV) there are indications of a discrepancy between ground array experiments and fluorescence detectors. On the other hand, the dependence of the depth of the shower maximum $X_{\max}$ with the primary energy shows a change in slope ($E \sim 10^{17}$ eV) which is usually explained assuming a composition change. Both effects could be accounted for in models predicting that above a certain energy showers would develop deeper in the atmosphere. In this Letter we argue that this can be done naturally by including percolation effects in the description of the shower development, which cause a change in the behavior of the inelasticity K above $E \simeq 10^{17}$ eV.

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High energy cosmic rays present a number of well-known puzzles. At very high energies ($E \sim 10^{20}$ eV) a discrepancy between ground array experiments and fluorescence detectors is usually quoted [1]. The two detection techniques are very different and so are the systematic errors involved, which have been deeply studied. In fluorescence detectors, the longitudinal profile of the shower is measured and the energy is extracted from this profile. In ground array experiments, the transverse profile of the shower is sampled at the atmospheric depth of the experiment $X_{\rm ground}$, and the estimation of the primary energy relies heavily on the Monte Carlo (MC) simulation of the shower development. Any effect causing the development of the shower deeper in the atmosphere (a shift down on the longitudinal profile) would lead to an increase of the depth of the shower maximum $X_{\max}$ and of the number of particles reaching ground at a slant depth $X_{\rm ground} > X_{\max}$. If such an effect is not included in the standard MC simulation, it may lead to an overestimation of the shower energy in ground arrays. Fluorescence detectors, on the other hand, basically derive their energy from the maximum shower size and are fairly insensitive to the height of $X_{\max}$. This could partially explain the apparent contradiction between ground and fluorescence experiments at energies near the Greisen-Zatsepin-Kusmin (GZK) cutoff [1].

On the other hand, the dependence of the depth of the shower maximum $X_{\max}$ with the primary energy shows a change in slope at $E \sim 10^{17}$ eV. This slope change can be accounted for in models predicting that above a certain energy showers would develop deeper in the atmosphere. It is usually explained by a change in the fraction of heavy nuclei in cosmic rays [see, for instance, [2]]: this fraction would be higher below the "kink" region, while above it the fraction of protons would rise. However, other effects predicting an increase of $X_{\max}$ could also explain this feature.

Both effects could be explained in a natural way by including percolation in the description of the shower development. The development of showers, hadronic and electromagnetic, in cosmic ray physics, is critically dependent on the energy carried by the fast particles produced in the first hadronic collision. In particular, the inelasticity parameter, $K \equiv 1 - x_F$, where x_F is the momentum fraction carried by the fastest particle, plays an important role.

In string percolation models [3,4] for hadron-hadron collisions, at low energy (or density) valence strings are formed, forward and backward in the center of mass, along the collision axis, containing most of the collision energy, and particles are produced from these strings. As the energy increases, additional sea strings, central in rapidity, are created, taking away part of the energy carried by the valence strings. Softer secondaries are produced, and the inelasticity increases with energy. As density increases (in the impact parameter plane all the strings look like disks, and we have to deal with a two dimension problem), strings start to overlap and merge: percolation occurs, leading to the creation of clusters of strings. From the larger percolated strings (clusters) faster particles are produced. As a consequence, the inelasticity starts to decrease with the energy.

On the contrary, most QCD-inspired models of multiparticle production predict, in hadron-hadron and nucleus-nucleus collisions at high energy, an increase with energy of the inelasticity parameter: multiple scattering model [5], dual and string models [6], and Minijet model [7]. Recently, several papers appeared studying collective and nonlinear QCD effects, based on the color glass condensate model [8], Reggeon calculus [9], and strong field string model [10,11]. These models predict large stopping power and a decrease of the momentum fraction carried by fast particles.

In the string percolation model considered in this study [3,4], two aspects are essential: while the relatively low energy regime, with K increasing, is similar to the models just mentioned, the higher energy regime, with decreasing K and the regeneration of the fast particles, is new and has some straightforward consequences in cosmic ray physics.

In percolation theory, the relevant parameter is the transverse density, η [12],

$$\eta \equiv \left(\frac{r}{\bar{R}}\right)^2 \bar{N}_s, \tag{1}$$

where r is the transverse radius of the string, $\bar{R}$ the effective radius of the interaction area. $\bar{N}_s$, the average number of strings, depends on the centrality and on the energy. The strings may overlap in the interaction area, forming clusters of N strings. If $\eta \ll 1$, the average number of strings per cluster is $\langle N\rangle \simeq 1$. If $\eta \gg 1$, $\langle N\rangle \simeq \bar{N}_s$. The average number $\langle N\rangle$ of strings per cluster is related to the average area $\langle A\rangle$, in units of r^2, occupied by a cluster [13],

$$\langle N\rangle = \langle A\rangle \frac{\eta}{1 - e^{-\eta}}, \tag{2}$$

with $\langle A\rangle$ given by [14]:

$$\langle A\rangle = f(\eta)\left[\left(\frac{\bar{R}}{r}\right)^2 (1 - e^{-\eta}) - 1\right] + 1, \tag{3}$$

where $f(\eta)$ is a percolation function,

$$f(\eta) = (1 + e^{-(\eta-\eta_c)/a})^{-1}, \tag{4}$$

$\eta_c \simeq 1.15$ is the transition point, and $a \simeq 0.85$ is a parameter controlling the slope of the curve at the transition point, with $f(\eta)$ changing from 0 to 1 at $\eta \simeq \eta_c$. We note that when $\eta \to 0$, $\langle A\rangle \simeq 1$ and when $\eta \to \infty$, $\langle A\rangle \simeq (\bar{R}/r)^2$. This kind of parametrization was tested in [13].

If $\bar{n}$ is the particle density for one string, $\bar{m}_T$ the average transverse mass produced from a single string, and there are $\bar{N}_s$ strings, one expects:

$$\frac{dn}{dy} = F(\eta)\bar{N}_s\bar{n} \quad \text{and} \quad \langle m_T\rangle = \frac{1}{\sqrt{F(\eta)}}\bar{m}_T, \tag{5}$$

with a color summation reduction factor [15,16],

$$F(\eta) \equiv \sqrt{\frac{1 - e^{-\eta}}{\eta}}, \tag{6}$$

decreasing with η. The particle density does not increase as fast as $\bar{N}_s$ [this corresponds to the saturation phenomenon [4]], and $\langle m_T\rangle$ slowly increases with energy and density. These features are seen in data [see, for instance, [17]].

Following [18], let us consider proton-proton collisions and write for the invariant s,

$$s \equiv (P_1 + P_2)^2 \simeq 4P^2 = m^2 e^{\Delta Y}, \tag{7}$$

where $\vec{P}_{1,2}$ are the momenta of the protons, $P = |\vec{P}_1| = |\vec{P}_2|$, m is the proton mass, and ΔY the length of the rapidity "plateau," which determines the maximal rapidity of the produced particles. For a string made up of two partons with Feynman-x values x_- and x_+ and assuming for simplicity a symmetrical situation around the center of mass, $x_- \simeq x_+ = \bar{x}$, the string center-of-mass energy is $s_1 = \bar{x}^2 s$, and we can write the length of the rapidity plateau for the string as:

$$\Delta y_1 = \Delta Y + 2\ln\bar{x}. \tag{8}$$

If strings overlap in the interaction region, and if $\langle N\rangle$ is the average number of strings per cluster we have, generalizing (8),

$$\Delta y_{\langle N\rangle} = \Delta y_1 + 2\ln\langle N\rangle. \tag{9}$$

At low energy or density $\langle N\rangle \simeq 1$ and only short strings are formed, not contributing to cosmic ray cascades. At high energy or density $\langle N\rangle \simeq \bar{N}_s$, percolation occurs and the situation changes.

The energy, in the center of mass, carried by the produced particles from sea strings, is given by:

$$E_{\text{CM}} = \int_{-(\Delta y_{\langle N\rangle})/2}^{+(\Delta y_{\langle N\rangle})/2} \langle m_T\rangle \cosh y \frac{dn}{dy} dy \tag{10}$$

and we obtain, making use of (5) and subtracting the 2 valence strings,

$$E_{\text{CM}} = \bar{m}_T\bar{n}\frac{1}{\sqrt{F(\eta)}}F(\eta)(\bar{N}_s - 2)[e^{\Delta y_{\langle N\rangle}/2} - e^{-(\Delta y_{\langle N\rangle})/2}]. \tag{11}$$

If we now require that asymptotically all the energy is carried by the percolating strings,

$$E_{\text{CM}}(\sqrt{s} \to \infty) = \sqrt{s}, \tag{12}$$

we obtain, from (6), (9), and (11),

$$\bar{N}_s \underset{\sqrt{s}\to\infty}{\longrightarrow} s^\lambda, \quad \text{with} \quad \lambda = 2/7. \tag{13}$$

As $\bar{N}_s$ is proportional to the high energy bare Pomeron, the value of the intercept α_p is related to λ: $\alpha_p - 1 = \lambda$. This result is consistent with results from the color glass condensate model [19].

One should notice that the bare Pomeron with the intercept $1 + \lambda$ does not determine the asymptotic behavior $\sqrt{s} \to \infty$; that is determined by a multiple scattering summation.

In order to implement the model [Eqs. (1), (2), (5), (9), and (11)], we have to establish a parametrization for N_s and to fix the parameters of the model. At some low energy threshold, $\sqrt{s_t} \simeq 10$ GeV, we have just the valence strings and $\bar{N}_s = 2$. At $\sqrt{s} \to \infty$, $\bar{N}_s \sim s^\lambda$ with λ given by Eq. (13). We then write:

$$\bar{N}_s = b + (2 - b)\left(\frac{s}{s_t}\right)^\lambda, \tag{14}$$

where the parameter $b = 1.37$ was adjusted to agree with the data on dn/dy. The remaining parameters were fixed to reasonable values: $\bar{n} = 0.65$, $\bar{m}_T = 0.78$, $r/\bar{R} = 0.2$, and $\Delta y_1 = 6$ in Eq. (11). In this way, (12) was exactly satisfied. In Fig. 1 the dn/dy data [20] are compared with the curve

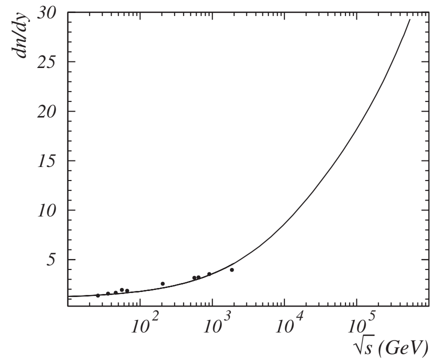

FIG. 1. Particle density as a function of $\sqrt{s}$. Data points are from [20]. The curve was obtained from (5) and (14) with $\sqrt{s_t} =$ 10 GeV and $b = 1.37$.

obtained from (5) and (14). With this parametrization for $\bar{N}_s$ we obtain that the critical density, η_c, occurs for $\sqrt{s} \simeq 10^4$ GeV.

Finally, in the eikonal limit the average number of strings $\bar{N}_s$ is related to the eikonal [see [21]] and, with the rough gray disk approximation, we obtain for the inelastic cross section $\sigma_{\text{in}} \simeq 95$ mb at $\sqrt{s} = 10^4$ GeV, in good agreement with experiment [2].

In order to have an estimate of the inelasticity K we make use of the idea that, in the fragmentation of the string, produced particles are ordered in decreasing rapidity, and the fraction of momentum carried, relative to the momentum left, is always the same [22]. At small $\sqrt{s}$, when the valence strings carry all the energy, the fastest particle (F) is the leading particle (L) and $x_F = x_L = \alpha =$ const, with $0 < \alpha \leq 1$. When sea strings are produced, carrying an energy E_{CM}, we have

$$x_L = \frac{2P_L}{\sqrt{s}} = \alpha\left(1 - \frac{E_{\text{CM}}}{\sqrt{s}}\right). \tag{15}$$

When the strings percolate, $E_{\text{CM}} \rightarrow \sqrt{s}$ and for the fastest percolating particle (P) we have:

$$x_P = \alpha \frac{\langle N \rangle}{\bar{N}_s} \frac{E_{\text{CM}}}{\sqrt{s}}. \tag{16}$$

As E_{CM} is an increasing function of $\sqrt{s}$, x_L decreases with the energy and x_P increases with energy. Thus,

$$K = \begin{cases} 1 - x_L, & \text{for } x_L > x_P \\ 1 - x_P, & \text{for } x_L < x_P \end{cases}. \tag{17}$$

In Fig. 2 we show the $\sqrt{s}$ dependence of the inelasticity K [Eq. (17)], assuming $\alpha = 0.5$. The behavior of $1 - x_L$ and $1 - x_P$ is also shown in the figure. From the combination of the two curves, K has a maximum at $\sqrt{s} \simeq 10^4$ GeV. This behavior is the required ingredient to achieve a possible explanation of the high energy cosmic ray effects mentioned above.

In the spirit of a simplified branching model, the relative shower maximum $X_{\max}$ can be expressed as:

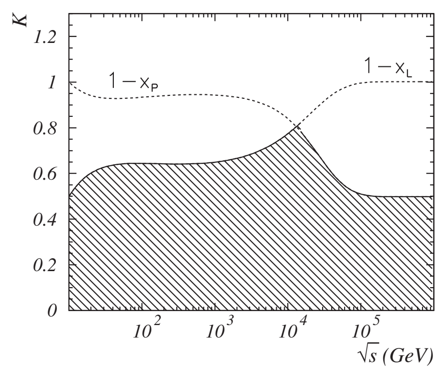

FIG. 2. The inelasticity parameter K as a function of $\sqrt{s}$, with $K = 1 - x_L$ at relatively low energies and $K = 1 - x_P$ at energies above the percolation threshold.

$$X_{\max} = X_1 + X_0 \log_{10}[(1 - K)E/E_0], \tag{18}$$

where E is the laboratory energy ($E \simeq \frac{1}{2m} s$) and K is the inelasticity (as defined in the present percolation model). $X_1 = 70$ g/cm^2, $X_0 = 60$ g/cm^2, and $E_0 = 10^7$ eV are effective parameters related to the position of the first collision, to the radiation length, and to a low energy threshold for the shower branching, respectively. The parametrization (18), showing a clear correlation between $X_{\max} - X_1$, and the inelasticity $(1 - K)$, is consistent with the analysis of [23], based on simulation using hadronic interaction generators [SIBYLL [24] and QGSJET [25], based on the dual parton model and quark gluon string model, respectively], incorporated in the CORSIKA program [26].

The dependence of $X_{\max}$, (18) on the primary energy E is shown in Fig. 3. Below the percolation threshold ($E \simeq 10^{17}$ eV) there is a decrease of the $X_{\max}(\log_{10} E)$ slope—K increases—and an increase above—K decreases. In the region where K is constant, at lower and higher energies,

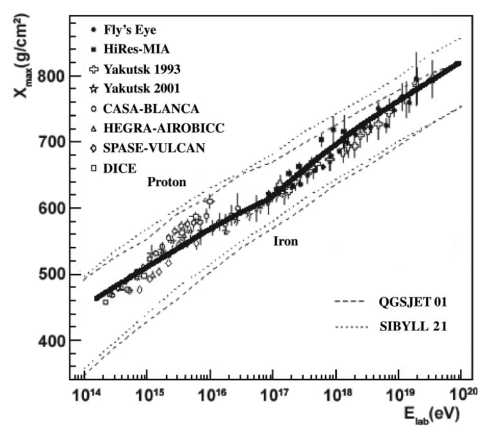

FIG. 3. The relative depth of the shower maximum as a function of the primary energy. The figure was adapted from [2], superimposing the result of the present percolation model (full line). Points are data and dashed lines show predictions of QGSJET and SIBYLL for protons and iron.

the slope becomes constant and of the order of X_0. All these features are seen in the data. We thus conclude that in percolation models, a natural explanation arises without requiring a composition change. A quantitative description of the data is beyond the scope of the present work, as it would imply a dedicated Monte Carlo simulation of proton-air interactions including percolation effects.

The possible overestimation of the energy in ground arrays in the GZK region was also studied. For showers initiated by 10^{20} eV protons, the longitudinal and transverse profiles at the atmospheric depth of the AGASA experiment were obtained with AIRES and CORSIKA. Both vertical and 45° inclined showers were studied, considering that the acceptance of ground arrays is maximal for relatively inclined showers. In order to study the effects of an increase in $X_{\max}$, the depth of the first interaction was fixed at two values, X_1 and $X_1 + 0.1X_{\max}$, and the two situations were compared. This has shown that, for inclined showers, shifting $X_{\max}$ down leads both to a larger total number of particles at the atmospheric depth of AGASA and to a larger particle density in the region 600 m away from the shower core. The effect in this particle density, which is the relevant quantity for the AGASA energy determination, is of the order of 20%. On the other hand, from the described percolation model, and using the parametrization (18), we estimate that the percolation effects in K could lead to changes of the order of 5% to 10% in $X_{\max}$, possibly explaining partially the apparent contradiction between ground arrays and fluorescence detectors at GZK energies.

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