Stabilizing Superconductivity in Nanowires by Coupling to Dissipative Environments

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We present a theory for a finite-length superconducting nanowire coupled to an environment. We show that in the absence of dissipation quantum phase slips always destroy superconductivity, even at zero temperature. Dissipation stabilizes the superconducting phase. We apply this theory to explain the "antiproximity effect" recently seen by Tian *et al.* in zinc nanowires.

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The effect of dissipation on macroscopic quantum coherence is currently a subject of considerable interest [1]. In the context of quantum bits ("qubits"), a dissipative environment always increases the decoherence rate [2]. On the other hand, superconductivity in a Josephson junction is enhanced by dissipation [3]. In this Letter we examine a particularly striking example of the latter phenomenon: the stabilization of superconductivity in a nanowire by dissipation in its environment.

In one-dimensional superconducting wires, quantum phase fluctuations can destroy long-range phase coherence even at zero temperature; however, finite superfluid density can survive through the Berezinskii-Kosterlitz-Thouless (BKT) physics [4]. The quantum action for a superconducting wire at zero temperature is equivalent to that of the two-dimensional classical XY model [4] at finite temperature. The two phases of the latter correspond to the superconducting and insulating phases of the wire. In particular, BKT vortices in the XY model correspond to phase slip events in the wire in which the phase gradient unwinds by 2π . The resistance of real nanowires can display both insulating and superconducting behavior as temperature is decreased [5–7]. Both thermal [8] and quantum [6,9] phase slips play an important role in generating resistance and destroying superconductivity. However, it is not settled what determines which nanowires are superconducting [7]. In particular, it is unclear how important dissipation is in determining the low-temperature phase of nanowires [10,11].

In a recent experiment, Tian $et\ al$. [12] observed an unexpected effect when a 2 μ m long, 40 nm diameter Zn nanowire is sandwiched between two bulk superconducting electrodes. Under zero applied magnetic field, when the electrodes are superconducting, the Zn nanowire exhibits resistive behavior down to the lowest measurement temperature. However, after a sufficiently strong magnetic field B has suppressed the superconductivity of the electrodes, the nanowire becomes superconducting at about 0.8 K. Tian $et\ al$. dubbed this phenomenon the "antiproximity effect" (APE). Here we present a theory suggesting that this surprising effect is due to the dissipation at the

boundary between the nanowire and electrodes. We show that when the nanowire has a finite length, the ends of the wire are mapped onto two parallel boundary lines that can screen the vortex-antivortex interaction in the XY model. This screening destroys the superconducting phase even at T=0. When the ends of the wire are coupled to a dissipative environment, the screening becomes incomplete. As a result, for sufficiently large dissipation the superconducting phase is stabilized. The importance of the boundary dissipation has been suggested by Büchler *et al.* [10].

We begin by summarizing the experimental findings reported in Ref. [12]. Tian *et al.* prepared Zn nanowires in the pores of porous polycarbonate or porous alumina membranes [Fig. 1(a)]. They pressed In or Sn wires on each side of the membrane to form circular disks approximately 1 mm in diameter that made contact to the ends of a single nanowire or, more generally, a number of nanowires. By applying a magnetic field above the critical field of the electrodes but below the critical field of the Zn nanowire (enhanced by the small diameter [13]), they suppressed the superconductivity of the electrodes. They measured the

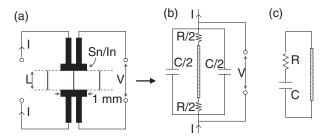


FIG. 1. Schematic diagrams of experiment by Tian *et al.* (a) Experimental configuration: pores in polycarbonate membrane contain nanowires, with In or Sn electrodes making contact to the wire(s). In this case, only one wire is contacted. Current I and voltage V are measured as shown. (b) Circuit representation showing contact resistances R/2 to each electrode and a capacitance C/2 from each half of the parallel plate capacitor, assuming the single wire is placed symmetrically. (c) Simplified model. For the purpose of damping, R and C are connected in series across the nanowire.

resistance and current-voltage (I-V) characteristics of their samples using the four-terminal arrangement indicated in Fig. 1(a). In this Letter we focus on the sample Z4, the behavior of which is shown in Fig. 3(b) of Ref. [12]. This sample had In electrodes and is believed to have contained a single nanowire with length $L=2~\mu\mathrm{m}$. When the In electrodes were driven normal by the magnetic field, the Zn nanowire exhibited a superconducting transition at a temperature that decreased as the field was further increased. In contrast, with zero applied field (hence superconducting electrodes), the resistance showed a drop of about 20 Ω as the temperature was lowered through the transition temperature of the In (about 3.4 K), but the nanowire did not go superconducting down to the lowest measurement temperature $(0.47~\mathrm{K})$.

Our model of the experiment is shown in Fig. 1(b). When the electrodes are in the normal state, the nanowire is connected to each of them via a contact resistance R/2. We estimate the resistance of the electrodes themselves in the normal state to be on the order of 1 m Ω , which is negligible for our present discussion. The nanowire and its contact resistance are in parallel with the capacitance of the parallel plate capacitor formed by the two electrodes and the intervening dielectric layer. Phase fluctuations in the nanowire induce currents through the shunting resistance and capacitor and are thereby damped. Assuming that these current fluctuations do not leak into the measurement circuitry, we use the effective circuit shown in Fig. 1(c). Using the area $A = \pi (0.5 \text{ mm})^2$ of the capacitor, the dielectric thickness $L=2 \mu m$, and a dielectric constant for polycarbonate $\epsilon = 2.9$ [14], we find $C = \epsilon \epsilon_0 A/L \approx 10$ pF.

For superconducting electrodes, we investigate the stability of a superconducting nanowire sandwiched between two superconducting leads. In this case, a supercurrent can flow between the wire and electrodes, and the contact resistances vanish. Thus for energy (frequency) less than the bulk superconducting gap of the electrodes the quantum wire is shunted by a capacitor. Later we show that under such conditions the quantum fluctuations of the superconducting phase drive the superfluid density of the zinc nanowire to zero even at zero temperature. When the superfluid density vanishes, Cooper pairs dissociate and the wire becomes normal. In that case the equivalent circuit of Fig. 1(c) becomes that of Fig. 2 where the quantum wire acts as a normal resistor with resistance R_N . This explains the Ohmic behavior at zero applied magnetic field and the constant differential resistance dV/dI in the I-V curve from zero voltage to about 500 μ V.

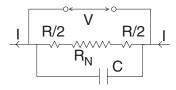


FIG. 2. Phase fluctuations ultimately drive the quantum wire normal via depairing, giving it a normal state resistance R_N .

When the electrodes are driven normal by $B \ge 30$ mT, the boundary resistances between the Zn wire and the electrodes become nonzero. In addition, if the electrodes are normal but the wire is superconducting (or vice versa), there is resistance due to charge conversion processes [15]. For sample Z4 [Fig. 3(b)] of Ref. [12], we estimate $R \approx$ 20–80 Ω . The lower bound comes from the drop in resistance when the electrodes become superconducting, and the upper bound from the residual low-temperature resistance for B = 30 mT. We note that the magnetic field dependence of the residual low-temperature resistance suggests that this resistance has a contribution from the wire as well. This estimate gives $f_0 = (2\pi RC)^{-1} \approx 0.8-0.2$ GHz and $hf_0 \approx 0.04$ –0.01 K, an order of magnitude lower than the thermal energy at the lowest measurement temperature, 0.47 K. Thus in the relevant frequency and temperature range the external circuitry behaves as a pure resistor (i.e., $1/2\pi fC \rightarrow 0$). As we show later, if this shunting resistance is smaller than the quantum of resistance $h/4e^2 \approx 6.4 \text{ k}\Omega$, it damps the superconducting phase fluctuations sufficiently to stabilize superconductivity. If the measurement temperature is lowered below hf_0 , we expect the residual quantum phase fluctuations to destroy superconductivity and cause a reentrant behavior. The fact that dissipation can stabilize superconductivity is rather similar to the behavior of the "resistively shunted (Josephson) junction" (RSJ) [3]. However, unlike the RSJ, a quantum wire can undergo depairing when the phase fluctuations are severe. As a result the normal state resistance of the nonsuperconducting wire does not have to exceed $h/4e^2$. Our theory is consistent with the observation that the APE practically vanished when one of the electrodes was replaced with a nonsuperconducting metal.

In the remainder of the Letter we present a theoretical analysis of how dissipation suppresses phase slips in a superconducting wire. Our main results are as follows: (1) Through a duality transformation, we establish the

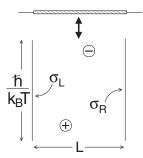


FIG. 3. A finite-length quantum wire is mapped onto a 1+1 dimensional Coulomb gas sandwiched between two metallic lines (the world line of the end points of the wire). In the absence of dissipation, the "surface" vortex densities σ_L and σ_R completely screen the logarithmic interaction between bulk vortices. As a result, vortex-antivortex pairs unbind and the wire is normal. In the presence of dissipation, the logarithmic interaction is not completely screened, and the superconducting phase is stable for sufficiently small shunt resistance.

connection between quantum phase slips and BKT vortices (instantons) in 1+1 dimensions. Our theory can be viewed as an appropriate generalization of the RSJ model to quantum wires. (2) At T=0 an isolated superconducting wire of finite length is equivalent to a classical two-dimensional electrostatic problem where bulk charges (vortices) interact with two metallic boundaries each representing the (imaginary-time) world line of the end points of the quantum wire (Fig. 3). Because of screening, vortices separated sufficiently far apart in the imaginary-time direction always unbind, and the wire is normal even at T=0. (3) With shunt resistance R, the screening is incomplete. For $0 < R < h/4e^2$, and with 1/C=0, the vortices remain bound and the quantum wire is superconducting at T=0.

The imaginary-time action describing the quantum fluctuation of the superconducting phase of a quantum wire is given by $S = \int_{-\infty}^{\infty} dx \int_{0}^{\beta} dt \mathcal{L}$, where

$$\mathcal{L} = (K/2)|\partial_x \phi|^2 + (1/2u)|\partial_t \phi|^2. \tag{1}$$

In Eq. (1) $\phi(x, t) = e^{i\theta(x,t)}$ is the phase factor of the superconducting order parameter at position x and time t, K is the superfluid density, and u is the inverse compressibility. At zero temperature ($\beta \to \infty$), depending on K and u, there are two possible phases: a superconducting phase and an insulating phase. In the superconducting phase the topological singularity in ϕ , i.e., the vortices in space and time, is bound. In the insulating phase, the space-time vortices (or instantons) proliferate. From this point of view of the quantum wire, a space-time vortex is a quantum phase slip event.

In the presence of the external circuitry in Fig. 1(c) the action in Eq. (1) acquires an extra boundary term $(S \rightarrow S + S_{\text{diss}})$ with $S_{\text{diss}} = \int_0^\beta \int_0^\beta dt dt' \mathcal{L}_{\text{diss}}$ given by

$$\mathcal{L}_{\text{diss}} = -(1/2) [\bar{\eta}(t)\partial_t \eta(t)] F(t - t') [\bar{\eta}(t')\partial_{t'} \eta(t')]. \quad (2)$$

In Eq. (2) $\eta \equiv \phi(L/2, t)\bar{\phi}(-L/2, t)$ is the relative phase factor of the two ends of the quantum wire and

$$F(t - t') = (\pi/\beta \mathcal{R}) \sum_{\omega_n} |\omega_n|^{-1} \exp[i\omega_n(t - t')].$$
 (3)

Here \mathcal{R} is the dimensionless resistance $\mathcal{R} = R/(h/4e^2)$, and $\omega_n = 2\pi n/\hbar\beta$ is the Matsubara frequency. We omit the capacitance of the external circuitry because we are interested in the temperature range $k_B T \gg h f_0 \approx 0.04$ –0.01 K where the reactance of the capacitance is negligible. Clearly, this assumption would be invalid if one were to repeat the experiment of Tian *et al.* [12] at temperatures below 0.04 K; in particular, this treatment cannot predict the expected reentrant behavior.

Next we perform the standard duality transformation [16] keeping track of the finite spatial extent of the wire to obtain $S_D = \int_{-L/2}^{L/2} dx \int_0^\beta dt \mathcal{L}_D + \int_0^\beta dt \int_0^\beta dt' \mathcal{L}_{\rm diss} + \int_0^\beta dt \mathcal{L}_{\rm ends}$, where

$$\mathcal{L}_D = \frac{1}{2K} [\partial_t \chi(x,t)]^2 + \frac{u}{2} [\partial_x \chi(x,t)]^2 - i \chi(x,t) \rho_v(x,t)$$

$$\mathcal{L}_{\text{diss}} = \frac{1}{2} [\sigma_L(t) + \sigma_R(t)] F(t - t') [\sigma_L(t') + \sigma_R(t')],$$

$$\mathcal{L}_{\text{ends}} = -i[\chi(L/2, t)\sigma_R(t) + \chi(-L/2, t)\sigma_L(t)]. \tag{4}$$

In Eq. (4) $\rho_v(x, t) = \sum_i Q_i \delta(x - x_i) \delta(t - t_i)$ is the vortex density in space-time, and

$$\sigma_{RL}(t) = \pm i\bar{\phi}(\pm L/2, t)\partial_t \phi(\pm L/2, t). \tag{5}$$

After we have integrated out χ , the first term in Eq. (4) is the standard vortex Coulomb gas action, and the second term is due to the coupling to the environment. The last term is a boundary term arising from the finite spatial extent of the quantum wire.

Equation (4) is quadratic in χ , so χ is integrated out exactly via the saddle point solution $\delta S/\delta \chi = 0$, yielding

$$(1/K)\partial_t^2 \chi + u\partial_x^2 \chi = -i\rho_v, \quad u\partial_x \chi|_{\pm L/2} = \pm i\sigma_{R.L}. \quad (6)$$

If we define $\Phi(x, t) \equiv -i\chi(x, t)$, Eq. (6) becomes

$$(1/K)\partial_t^2 \Phi + u \partial_x^2 \Phi = -\rho_v, \quad u \partial_x \Phi|_{\pm L/2} = \pm \sigma_{R,L}. \quad (7)$$

Equation (7) takes the form of a 2D electrostatic problem with bulk charge density ρ_{ν} and surface charge density $\sigma_{L,R}$. In terms of Φ , \mathcal{L}_{D} and $\mathcal{L}_{\text{ends}}$ become

$$\mathcal{L}_D = -\frac{1}{2K} [\partial_t \Phi(x, t)]^2 - \frac{u}{2} [\partial_x \Phi(x, t)]^2 + \Phi(x, t) \rho_v(x, t),$$

$$\mathcal{L}_{\text{ends}} = [\Phi(L/2, t)\sigma_R(t) + \Phi(-L/2, t)\sigma_L(t)]. \tag{8}$$

Substituting the solution $\Phi_c(x,t)$ of Eq. (7) into Eqs. (4) and (8) we obtain an action, $S_D[\sigma_L, \sigma_R, \rho_v]$, that depends only on ρ_v and $\sigma_{L,R}$:

$$S_{D} = \frac{1}{2} \int_{-L/2}^{L/2} dx \int_{0}^{\beta} dt \Phi_{c}(x, t) \rho_{v}(x, t) + \int_{0}^{\beta} dt \int_{0}^{\beta} dt' \mathcal{L}_{diss} + \int_{0}^{\beta} dt [\Phi_{c}(L/2, t)\sigma_{R}(t) + \Phi_{c}(-L/2, t)\sigma_{L}(t)].$$
(9)

To obtain a final action that depends only on the vortex density ρ_{ν} , we integrate out σ_{R} , σ_{L} . Since Eq. (9) depends on σ_{R} , σ_{L} only quadratically, we can again use the saddle point method, solving $\delta S/\delta\sigma_{L,R}=0$ and substituting the solution back into Eq. (9). In a wire of length L, the result is that vortices with spatial coordinates x_{i} ($-L/2 \le x_{i} \le L/2$) interact via

$$S_D = \sum_{i \neq j} Q_i G(x_i, x_j; t_i - t_j) Q_j.$$
 (10)

In Eq. (10) $G(x_i, x_j; t_{ij}) = \frac{1}{\hbar B} \sum_{\omega_n} (G_1 + G_2) e^{i\omega_n t_{ij}}$ with

$$G_{1} = \sqrt{\frac{K}{u}} \frac{c_{L-|x_{i}-x_{j}|} - c_{x_{i}+x_{j}}}{\omega_{n} s_{L}},$$

$$G_{2} = \sqrt{\frac{K}{u}} \frac{c_{x_{i}} c_{x_{j}}}{\omega_{n} s_{L}} \left(1 + \frac{\operatorname{sgn}(\omega_{n}) \sqrt{K} \mathcal{R} s_{L}}{4\hbar \sqrt{u} \pi s_{L/2}^{2}}\right)^{-1}.$$
(11)

Here $c_x = \cosh(\omega_n x/\sqrt{Ku})$ and $s_x = \sinh(\omega_n x/\sqrt{Ku})$. Expanding Eq. (11) for small ω_n shows that vortices with time separation much greater than L/\sqrt{uK} interact via

$$S_D = \sum_{i \neq j} Q_i Q_j [-(\hbar/2\mathcal{R}) \ln(|t_{ij}|/\tau_0) + J_{ij}(t_{ij})]. \quad (12)$$

In Eq. (12) J_{ij} is a short-range interaction (in time). It is important to note that the long-range logarithmic interaction is controlled only by the resistive dissipation. Aside from the irrelevant short-range interaction J_{ij} , Eq. (12) is identical to the phase slip action of a single RSJ (we identify Q_j with the phase slip). For $\mathcal{R} < 1$ the phase slip and antiphase slip form bound pairs, while for $\mathcal{R} > 1$ the phase slip and antiphase slip unbind [3,17]. The former corresponds to the superconducting phase of the quantum wire, while the latter corresponds to the non-superconducting phase.

For an isolated quantum wire a similar calculation leads to a short-range action for the vortices [Eq. (10)] with $G(x_i, x_j; t_i - t_j) = \frac{1}{h\beta} \sum_{\omega_n} G_1 e^{i\omega_n(t_i - t_j)}$. Since the interaction is short ranged, phase slips and antiphase slips always unbind. As a result a free, finite-length quantum wire is always nonsuperconducting, even at zero temperature. Our results are fully consistent and agree in spirit with those of Büchler *et al.* [10], who derived the phase slip interaction from phenomenological boundary conditions. In the current work, we derive the boundary effects and the phase slip interaction in the presence of dissipation *exactly*. Consequently, we have an explicit phase slip interaction, valid at both long and short time scales as is required for a quantitative understanding of the phase slip physics of a quantum wire.

As emphasized earlier, a piece of physics of the quantum wire not present in the RSJ is the fact that as phase fluctuations suppress the superfluid density to zero, Cooper pair disassociation will take place before the wire becomes a Cooper pair insulator. Once electrons depair, the wire can no longer be described by the phase-only action given in this Letter. This is quite similar to the case of superconductor to nonsuperconductor quantum phase transitions in homogeneous films, where electron tunneling always finds a closing energy gap at the phase transition [18]. Thus, the normal state resistance of the wire is not directly related to the shunt resistance R used in our purely bosonic model and which determines the fate of the wire. Finally, we note that the asymptotic interaction between vortices far separated in time is valid when $\hbar/k_BT \gg$ L/\sqrt{uK} . For nonzero temperatures, a sufficiently long nanowire acts as an infinite wire. When the BKT vortices are bound, the nanowire exhibits the transport properties of an attractive Luttinger liquid [19] and hence exhibits superconductinglike characteristics.

To conclude, in this Letter we present a theory for the quantum phase slips of a superconducting nanowire, and the effect of environmental dissipation on them. We apply this theory to explain the antiproximity effect recently observed by Tian *et al.*. We attribute the recurrence of superconductivity when the electrodes are driven normal by a magnetic field to the onset of dissipation from the boundary resistance between the quantum wire and the electrodes. This dissipation suppresses phase fluctuation in the wire and stabilizes superconductivity.

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