Dipole Induced Transparency in Drop-Filter Cavity-Waveguide Systems

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(Received 11 November 2005; published 18 April 2006)

We show that a waveguide that is normally opaque due to interaction with a drop-filter cavity can be made transparent when the drop filter is also coupled to a dipole, even when the vacuum Rabi frequency of the dipole is much less than the cavity decay rate. The condition for transparency is simply achieving large Purcell factors. We describe how this effect can be useful for designing quantum repeaters for long distance quantum communication.

DOI: 10.1103/PhysRevLett.96.153601 PACS numbers: 42.50.−p, 03.67.Hk

The field of semiconductor cavity quantum electrodynamics (CQED) has seen rapid progress in the past several years. One of the main reasons for this is the development of high quality factors optical microcavities with mode volumes that are less than a cubic wavelength of light [1]. These high-Q cavities allow previously unattainable interaction strengths between a cavity mode and a dipole emitter such as a quantum dot.

There are a large number of applications that require strong interactions between a cavity and dipole emitter. These include methods for conditional phase shifts on single photons [2], atom number detection [3], and nonlinear optics [4]. One important property of cavity-dipole interaction is that, under appropriate conditions, the dipole can flip the cavity from being highly transmissive to being highly reflective. This can result in entanglement between the dipole and reflected field. Such entanglement was used in Ref. [2] with a single-sided cavity configuration to achieve a quantum phase gate.

It has long been believed that, in order for a dipole to fully switch the cavity, the vacuum Rabi frequency of the dipole, often denoted g, must exceed both the cavity and dipole decay rates. We refer to this regime as the high-Q regime. In this Letter, we show that in the "bad cavity" limit, defined as the regime where the cavity decay rate is much bigger than the dipole decay rate, the cavity can be switched almost perfectly, even when g is much smaller than the cavity decay rate. We consider a single cavity that is coupled to two waveguides and behaves as a resonant drop filter. Such systems are mathematically equivalent to driving a double-sided cavity with an incident field. The reflection properties of a single-sided cavity have been investigated elsewhere [5]. Drop filtering has been experimentally demonstrated in a variety of semiconductor systems including photonic crystals [6] and microdisks coupled to ridge waveguides [7].

When an optical input field is resonant with the cavity, the drop filter would normally transmit all the field from one waveguide to another. Hence, the waveguide would appear opaque at the cavity resonance. We show that, if one places a dipole in the drop-filter cavity, the waveguide becomes highly transparent even when g is much smaller than the cavity decay. In the high-Q regime, this result is clear, because the cavity mode is split into a lower and an upper polariton by more than a linewidth (normal modes splitting). In the low-Q regime, where g is less than the cavity decay rate, this result is surprising because the incident field can still drive both the cavity modes. Transparency in this regime is instead caused by destructive interference of the cavity field, which is analogous to the destructive interference of the excited state of a 3-level atomic system in electromagnetically induced transparency [8]. For this reason, we refer to this effect as dipole induced transparency (DIT).

The fact that switching can be observed without the high-Q regime is extremely important for the field of semiconductor CQED. Although the high-Q regime has been achieved in atom cavity QED [3], it is extremely difficult to achieve using semiconductor technology. Semiconductor implementations of cavity QED systems, such as photonic crystal cavities coupled to quantum dots, usually suffer from large out-of-plane losses, resulting in short cavity lifetimes. Things become even more difficult when one attempts to integrate these cavities with waveguides. The cavity-waveguide coupling rate must be sufficiently large that we do not lose too much of the field out of plane. At the same time, leakage into the waveguide introduces additional losses, making the high-Q regime even more difficult to achieve. Our result relaxes the constraint on using the high-Q regime, allowing complete switching in a more practical parameter regime for semiconductors. To demonstrate the application of DIT, we conclude this Letter by showing how it can be used to share entanglement between spatially separated dipoles and to perform a full nondestructive Bell measurement on two dipoles. These operations are extremely useful for building quantum repeaters [9,10].

Figure 1 shows a schematic of the type of system we are considering. A cavity containing a single dipole emitter is evanescently coupled to two waveguides. The cavity is assumed to have a single mode that couples only to the forward propagating fields. The dipole may be detuned by δ from cavity resonance, denoted ω_0, while g is the vacuum Rabi frequency of the dipole. Both waveguides

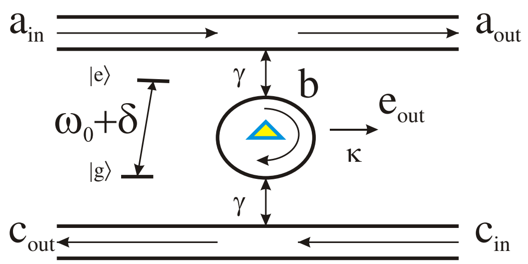

FIG. 1 (color online). Cavity-waveguide system for quantum repeaters.

are assumed to have an equal coupling rate into the cavity. This condition is known as critical coupling and results in the input field from one waveguide being completely transmitted to the other when $\gamma \gg \kappa$ [11].

We begin with the Heisenberg operator equations for the cavity field operator $\hat{\mathbf{b}}$ and dipole operator σ_-, given by [12]

$$\frac{d\hat{\mathbf{b}}}{dt} = -(i\omega_0 + \gamma + \kappa/2)\hat{\mathbf{b}} - \sqrt{\gamma}(\hat{\mathbf{a}}_{\rm in} + \hat{\mathbf{c}}_{\rm in}) - \sqrt{\kappa}\hat{\mathbf{e}}_{\rm in} - ig\sigma_-, \tag{1}$$

$$\frac{d\sigma_-}{dt} = -\left(i(\omega_0 + \delta) + \frac{1}{2\tau}\right)\sigma_- + ig\sigma_z\hat{\mathbf{b}} - \hat{\mathbf{f}}. \tag{2}$$

The operators $\hat{\mathbf{a}}_{\rm in}$ and $\hat{\mathbf{c}}_{\rm in}$ are the field operators for the flux of the two input ports of the waveguide, while $\hat{\mathbf{e}}_{\rm in}$ is the operator for potential leaky modes due to all other losses such as out-of-plane scattering and material absorption. The bare cavity has a resonant frequency ω_0 and an energy decay rate κ (in the absence of coupling to the waveguides). This decay rate is related to the cavity quality factor Q by $\kappa = \omega_0/Q$. The parameter γ is the energy decay rate from the cavity into each waveguide. Similarly, the dipole operator σ_- has a decay rate $1/2\tau$, and $\hat{\mathbf{f}}$ is a noise operator which preserves the commutation relation. The output fields of the waveguide $\hat{\mathbf{a}}_{\rm out}$ and $\hat{\mathbf{c}}_{\rm out}$ are related to the input fields by $\hat{\mathbf{a}}_{\rm out} - \hat{\mathbf{a}}_{\rm in} = \sqrt{\gamma}\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}_{\rm out} - \hat{\mathbf{c}}_{\rm in} = \sqrt{\gamma}\hat{\mathbf{b}}$ [12]. Our analysis works in the weak excitation limit, where the quantum dot is predominantly in the ground state. In this limit, $\langle\sigma_z(t)\rangle \approx -1$ for all time, and we can substitute $\sigma_z(t)$ with its average value of -1. Assuming the cavity is excited by a weak monochromatic field with frequency ω, the waveguide outputs can be solved and are given by

$$\hat{\mathbf{a}}_{\rm out} = \frac{-\gamma\hat{\mathbf{c}}_{\rm in} + \left(-i\Delta\omega + \frac{\kappa}{2} + \frac{g^2}{-i(\Delta\omega-\delta)+1/2\tau}\right)\hat{\mathbf{a}}_{\rm in} - \sqrt{\kappa\gamma}\hat{\mathbf{e}}_{\rm in}}{-i\Delta\omega + \gamma + \kappa/2 + \frac{g^2}{-i(\Delta\omega-\delta)+1/2\tau}}, \tag{3}$$

$$\hat{\mathbf{c}}_{\rm out} = \frac{-\gamma\hat{\mathbf{a}}_{\rm out} + \left(-i\Delta\omega + \frac{\kappa}{2} + \frac{g^2}{-i(\Delta\omega-\delta)+1/2\tau}\right)\hat{\mathbf{c}}_{\rm out} - \sqrt{\kappa\gamma}\hat{\mathbf{e}}_{\rm out}}{-i\Delta\omega + \gamma + \kappa/2 + \frac{g^2}{-i(\Delta\omega-\delta)+1/2\tau}}, \tag{4}$$

where $\Delta\omega = \omega - \omega_0$.

Consider the case where the dipole is resonant with the cavity, so that $\delta = 0$. In the ideal case, the bare cavity decay rate κ is very small and can be set to zero. In this limit, when the field is resonant with the cavity and $g = 0$, we have $\hat{\mathbf{a}}_{\rm in} = -\hat{\mathbf{c}}_{\rm out}$, as one would expect from critical coupling. In the opposite regime, when $2\tau g^2 \gg \gamma + \kappa/2$, we have $\hat{\mathbf{a}}_{\rm in} = \hat{\mathbf{a}}_{\rm out}$, so that the field remains in the original waveguide. This condition can be rewritten as $F_p = 2\tau g^2/(\gamma + \kappa/2) \gg 1$, where F_p is the Purcell factor (the ratio of the dipole decay rate into the cavity to the bare dipole decay rate). In order to make the waveguide transparent (i.e., decouple the field from the cavity), we need to achieve large Purcell factors. However, we do not need the full normal mode splitting condition $g > \gamma + \kappa/2$. When $1/\tau \ll \gamma + \kappa/2$, we can achieve transparency for much smaller values of g.

Figure 2 plots the probability that $\hat{\mathbf{a}}_{\rm in}$ transmits into $\hat{\mathbf{a}}_{\rm out}$ and $\hat{\mathbf{c}}_{\rm out}$. Assuming that the initial field begins in mode $\hat{\mathbf{a}}_{\rm in}$, we define $\hat{\mathbf{a}}_{\rm out}/\hat{\mathbf{a}}_{\rm in} = \sqrt{T_a}e^{i\Phi_a}$ and $\hat{\mathbf{c}}_{\rm out}/\hat{\mathbf{c}}_{\rm in} = \sqrt{T_c}e^{i\Phi_c}$ and use cavity and dipole parameters that are appropriate for a photonic crystal cavity coupled to a quantum dot. We set $\gamma = 1$ THz, which is about a factor of 10 faster than κ for a cavity with a quality factor of $Q = 10\,000$. We set $g =$ 0.33 THz, a number calculated from finite difference time domain simulations of cavity mode volume for a single

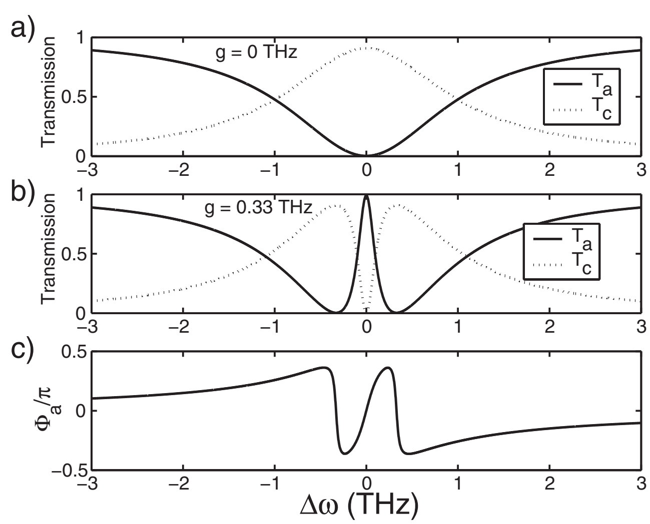

FIG. 2. Probability for field in $\hat{\mathbf{a}}_{\rm in}$ to transmit into $\hat{\mathbf{a}}_{\rm out}$ and $\hat{\mathbf{c}}_{\rm out}$, respectively. (a) Transmission with no dipole in cavity. (b) Transmission with a dipole in the cavity. (c) Phase imposed on the transmitted field $\hat{\mathbf{a}}_{\rm out}$ when a dipole is present in the cavity.

defect dipole cavity in a planar photonic crystal coupled to a quantum dot [1]. The dipole decay rate is set to $1/\tau = 1$ GHz, taken from experimental measurements [13].

Figure 2(a) considers the case where the cavity does not contain a dipole. In this case, $g = 0$, representing a system where two waveguides are coupled by a drop-filter cavity, whose transmission width is determined by the lifetime of the cavity. When a dipole is present in the cavity, the result is plotted in Fig. 2(b). In this case, a very sharp peak in the transmission spectrum appears at $\Delta\omega = 0$, with a width of approximately 0.1 THz. Because g is 3 times smaller than the cavity linewidth, this peak is not caused by normal mode splitting but rather by destructive interference of the cavity field.

In the bad cavity limit, where $1/\tau$ is very small, a simple expression for the width at full width at half maximum of the dip can be calculated in the limit that $\gamma \gg \kappa/2$ (cavity losses are dominated by waveguides). In this limit, the width of the dip is found to be $\sqrt{\gamma^2 + 4g^2} - \gamma$. In the high-$Q$ regime where $g \gg \gamma$, the transmission dip is equal to $2g$ as expected by normal mode splitting. In the low-Q regime where $g \ll \gamma$, the spectral width of the transmission peak is equal to $2g^2/(\gamma) = 2/\tau_{\mathrm{mod}}$ (where τ_{mod} is the modified spontaneous emission lifetime of the dipole). This difference in functional behavior, i.e., quadratic vs linear dependence in g, is indicative of the fact that transmission in the low-Q regime originates from a different physical process than the high-Q regime. The width of the transmission peak is important because it places a bandwidth limitation on the incoming pulse. In the low-Q regime, this bandwidth limitation means that the incoming pulse must be longer than the modified spontaneous emission lifetime of the dipole, while in the high-Q regime it must be longer than the Rabi oscillation period $1/g$.

In Fig. 2(c), we plot Φ_a for the case where $g = 0.33$ THz. The region near zero detuning exhibits very large dispersion, which results in a group delay given by $\tau_g = (\gamma + \kappa/2)/g^2$. One can show that the group velocity dispersion at zero detuning vanishes, ensuring that the pulse shape is preserved.

We now consider the effect of detuning the dipole. The transmission spectrum for a dipole detuned by $\delta = 0.4$ THz is plotted in Fig. 3. Introducing a detuning in the dipole causes a shift in the location of the transmission peak, which occurs at the dipole resonant frequency. Thus, we do not have to hit the cavity resonance very accurately to observe DIT. We need only to overlap the dipole resonance within the cavity transmission spectrum.

The fact that one can switch the transmission of a waveguide by the state of a dipole in the low-Q regime can be extremely useful for quantum information processing. As one example, we now present a way in which DIT can be applied to engineering quantum repeaters for long distance quantum communication. Quantum repeaters can be implemented all optically [14,15], as well as using atomic systems [10]. One of the main problems with these proposals is that it is difficult to implement the full Bell measurement required for swapping entanglement. This leads to a communication rate that is exponentially decaying with the number of repeaters. More recent proposals incorporate interaction between nuclear and electron spins to implement the full Bell measurement [16]. Here we propose a method for implementing entanglement, as well as a full Bell measurement on an atomic system using only interaction with a coherent field. This leads to an extremely simple implementation of a quantum repeater.

In Fig. 4(a), we show how DIT can be used to generate entanglement between two spatially separated dipoles. A weak coherent beam is split on a beam splitter, and each port of the beam splitter is then sent to two independent cavities containing dipoles. The waveguide fields are then mixed on a beam splitter such that constructive interference is observed in ports $\hat{\mathbf{f}}$ and $\hat{\mathbf{h}}$. Each dipole is assumed to have three relevant states: a ground state, an excited state, and a long lived metastable state, which we refer to as $|g\rangle$, $|e\rangle$, and $|m\rangle$, respectively. The transition from ground to excited state is assumed to be resonant with the cavity, while the metastable to excited state transition is well off resonance from the cavity and is, thus, assumed not to

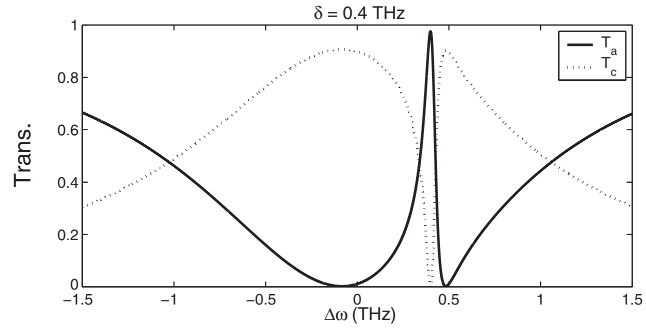

FIG. 3. Transmission of waveguide for $\delta = 0.4$ THz, the detuning of the dipole from the cavity.

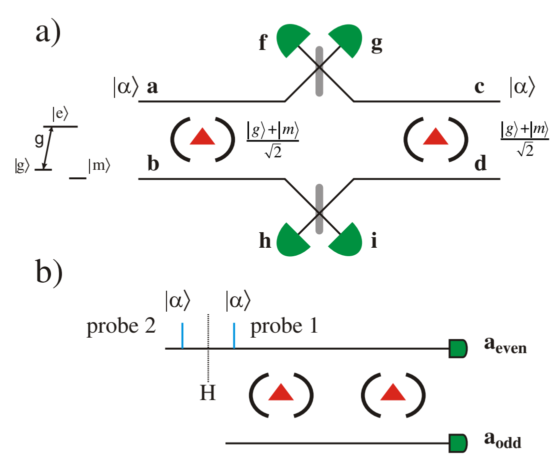

FIG. 4 (color online). Application of DIT to quantum repeaters. (a) A method for generating entanglement between two dipoles using DIT. (b) A nondestructive Bell measurement.

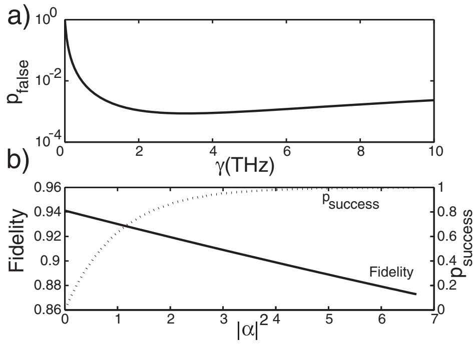

FIG. 5. (a) Probability of detecting even parity for an odd parity state as a function of γ. (b) The solid line plots the fidelity of the state $(|gg\rangle \pm |mm\rangle)/\sqrt{2}$ after a parity measurement. The dotted line plots the probability that the measuring field contains at least one photon for detection.

couple to state $|e\rangle$. The states $|g\rangle$ and $|m\rangle$ represent the two qubit states of the dipole.

When the dipole is in state $|m\rangle$, it does not couple to the cavity, which now behaves as a drop filter. Thus, we have a system that transforms $\hat{\mathbf{a}}_{\rm in}^{\dagger}|g\rangle|0\rangle \rightarrow \hat{\mathbf{a}}_{\rm out}^{\dagger}|g\rangle|0\rangle$ and $\hat{\mathbf{a}}_{\rm in}^{\dagger}|m\rangle|0\rangle \rightarrow -\hat{\mathbf{c}}_{\rm out}^{\dagger}|m\rangle|0\rangle$. This operation can be interpreted as a C-NOT gate between the state of the dipole and the incoming light. When the dipole is in a superposition of the two states, this interaction generates entanglement between the path of the field and the dipole state. After the beam splitter, this entanglement will be transferred to the two dipoles. If the state of both dipoles is initialized to $(|g\rangle + |m\rangle)/\sqrt{2}$, it is straightforward to show that a detection event in ports $\hat{\mathbf{g}}$ or $\hat{\mathbf{i}}$ collapses the system to $(|g, m\rangle - |m, g\rangle)/\sqrt{2}$.

Another important operation for designing repeaters is a Bell measurement, which measures the system in the states $|\phi_{\pm}\rangle = (|gg\rangle \pm |mm\rangle)/\sqrt{2}$ and $|\psi_{\pm}\rangle = (|gm\rangle \pm |mg\rangle)/\sqrt{2}$. Figure 4(b) shows how to implement a complete Bell measurement between two dipoles using only cavity-waveguide interactions with coherent fields. The two cavities containing the dipoles are coupled to two waveguides. A coherent field $|\alpha\rangle$ is initially sent down the upper waveguide, and each dipole will flip the field to the other waveguide if it is in state $|m\rangle$ and will keep the field in the same waveguide if it is in state $|g\rangle$. Thus, a detection event at ports $\hat{\mathbf{a}}_{\rm even}$ and $\hat{\mathbf{a}}_{\rm odd}$ corresponds to a parity measurement. A Bell measurement can be made by simply performing a parity measurement on the two dipoles, then a Hadamard rotation on both dipoles, followed by a second parity measurement. This is because a Hadamard rotation flips the parity of $|\psi_+\rangle$ and $|\phi_-\rangle$ but does not affect the parity of the other two states.

The performance of the Bell apparatus is analyzed in Fig. 5. Figure 5(a) plots the probability that an odd parity state will falsely create a detection event in port $\hat{\mathbf{a}}_{\rm even}$, as a function of γ. The probability becomes high at large γ due to imperfect transparency. It also increases at small γ because of imperfect drop filtering. The minimum value of about 10^{-3} is achieved at approximately 3 THz. In Fig. 5(b), we plot both the fidelity and the success probability of a parity measurement as a function of the number of photons in the probe field. The fidelity is calculated by applying the Bell measurement to the initial state $|\psi_i\rangle = (|g, g\rangle \pm |m, m\rangle)/\sqrt{2}$ and defining the fidelity of the measurement as $F = |\langle\psi_f|\psi_i\rangle|^2$, where $|\psi_f\rangle$ is the final state of the total system which includes the external reservoirs. The probability of success is defined as the probability that at least one photon is contained in the field. The fidelity is ultimately limited by cavity leakage, which results in ''which path'' information being leaked to the environment. This information leakage depends on the strength of the measurement which is determined by the number of photons in the probe fields. Using more probe photons results in a higher success probability but a lower fidelity. To calculate this tradeoff, we use previously described values for cavity and reservoir losses and set the coupling rate γ to 4 THz, which is where the probability of false detection is near its minimum. At an average of three photons, a fidelity of over 90% can be achieved with a success probability exceeding 95%. These numbers are already promising, and improved cavity and dipole lifetimes could lead to even better operation.

This work was funded in part by the MURI center for photonic quantum information systems (ARO/DTO Program No. DAAD19-03-1-0199), and a Department of Central Intelligence postdoctoral grant.

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