Temperature Square Dependence of the Low Frequency $1/f$ Charge Noise in the Josephson Junction Qubits

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To verify the hypothesis about the common origin of the low-frequency $1/f$ noise and the quantum f noise recently measured in the Josephson charge qubits, we study the temperature dependence of the $1/f$ noise and decay of coherent oscillations. The T^2 dependence of the $1/f$ noise is experimentally demonstrated, which supports the hypothesis. We also show that dephasing in the Josephson charge qubits off the electrostatic energy degeneracy point is consistently explained by the same low-frequency $1/f$ noise that is observed in the transport measurements.

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Because of their potential scalability, Josephson quantum bits are good candidates for building quantum computers [1]. To have a long decoherence time, the qubits should be well decoupled from all noise sources, in particular, the charge noise from uncontrollable charge fluctuations. Therefore, the noise and decoherence in the qubits are now the key issue of the qubit research. Although the noise has been studied in a number of works [2–7], its nature is still not yet understood.

The low-frequency noise in metallic single electron transistors (SETs) was studied intensively a while ago [8–10]. It was found that the noise is produced by charge fluctuators and its spectral density is close to $1/f$. Recently, it has been shown that the low-frequency charge noise gives the main contribution to dephasing of coherent oscillations in the Josephson charge qubits [3,4]. The high frequency quantum noise of the environment was investigated in Ref. [2] by using a qubit as a quantum spectrometer. The qubit relaxation is caused by the asymmetric quantum noise with a nonmonotonic spectrum, which tends to have a linear frequency dependence (f noise) in a wide energy range (from 2 to 100 GHz $\times$ h) like in the case of a simple ohmic environment. The quantum noise originates from absorption of the energy of excited qubits by the cold environment and, therefore, should be nearly temperature independent in the range of qubit energies higher than k_BT. Surprisingly, it turns out that the amplitude of the f quantum noise crosses the low-frequency $1/f$ noise extrapolated to the gigahertz range at a frequency $\omega_c \sim k_BT/\hbar$, which implies that both noises may have a common origin. Furthermore, if one assumes that the crossover frequency ω_c scales linearly with temperature then the strength of the $1/f$ charge noise should be proportional to temperature square (T^2 dependence) [2]. Although T^2 dependence has been observed in the critical current noise of the Josephson junction [11], and may be related to charge fluctuations in the junction, the dependence appears to be unexpected in the charge noise measurements as it contradicts to the linear temperature dependence of the $1/f$ noise in glasses [12,13]. Therefore, the T^2 dependence in the charge $1/f$ noise has to be experimentally confirmed. In addition, temperature dependence gives important information on the density of states of the fluctuators and will help to verify theoretical models of the $1/f$ noise intensively studied recently in Refs. [14–18]. Some of these works are based on the prediction of the T^2 dependence reported in Ref. [2]. Unfortunately, temperature dependence of the $1/f$ noise in SETs has not been studied in detail in earlier works. It was found that the noise increases with temperature and saturates in the low temperature range [19,20]. In Ref. [20], the quadratic temperature dependence was expected but has not been actually demonstrated.

In this work, we study temperature dependence of the $1/f$ noise in the Josephson charge qubits by measuring dc transport in the SET regime. We have found that the noise exhibits T^2 dependence at temperatures from 200 mK up to $\sim$1 K. We also study the effect of temperature on dephasing of the qubit during coherent oscillations. Decay of the coherent oscillations away from the degeneracy point of electrostatic energy is consistently explained by dephasing due to the low-frequency $1/f$ charge noise. We also briefly discuss phenomenological models of T^2 dependences from the experimental point of view.

To study temperature dependence we fabricate qubits with the same geometry and junction properties as the qubits used in Ref. [2]. The Al structure is fabricated on top of 400 nm thick Si_3N_4 insulation layer deposited on a gold ground plane. The total capacitance of the qubit island is about 600 aF (the charging energy is $E_C = e^2/2C \approx$ 30 GHz $\times$ h) and is mainly formed by its Josephson junction. Instead of the trap island used in Ref. [2], we fabricate an electrical lead connected to the qubit through a small highly resistive tunnel junction with a resistance of 10–50 MΩ (as it was in our earlier works [3,4]) to measure current through the qubit in the SET regime.

We use the qubit as an SET and measure the low-frequency charge noise, which causes the SET peak position fluctuations. Temperature dependence of the noise is measured from the base temperature of 50 mK up to 900–1000 mK. The SET is normally biased to $V_b = 4\Delta/e$ ($\sim$ 1 mV), where Coulomb oscillations of the quasiparticle current are observed. Figure 1(a) exemplifies the position of the SET Coulomb peak as a function of the gate voltage at temperatures from 50 mK up to 900 mK in increments of 50 mK. The current noise spectral density is measured at the gate voltage corresponding to the slope of the SET peak (shown by the arrow), at the maximum (on the top of the peak), and at the minimum (in the Coulomb blockade). Normally, the noise spectra in the two latter cases are frequency independent in the measured frequency range (and usually do not exceed the noise of the measurement setup). However, the noise spectra taken on the slope of the peak show nearly $1/f$ frequency dependence [see examples of the current noise S_I at different temperatures in Fig. 1(b)] saturating at a higher frequencies (usually above 10–100 Hz depending on the device properties) at the level of the noise of the measurement circuit. The fact that the measured $1/f$ noise on the slope is substantially higher than the noises on the top of the peak and in the blockade regime indicates that the noise comes from fluctuations of the peak position, which can be translated into charge fluctuations in the SET.

To obtain the $1/f$ charge noise spectral density

$$S_q(\omega) = \frac{\alpha}{\omega} \tag{1}$$

(defined for frequencies $\omega > 0$) we first take the low-frequency part of the current noise spectral density $S_I(f)$ and find the parameter A of the fitting curve A/f as $A = \langle S_I(f)\rangle/\langle 1/f\rangle$ [21]. Next, we transform the current noise into the charge noise $\alpha = A/(dI/dV_g)^2/(\Delta V_g/e)^2$ using the transfer function dI/dV_g on the slope of the peak at the measurement point, where ΔV_g is the spacing in gate voltage between two adjacent peaks (corresponding to the change of the SET charge by e). Dimensionality of α is e^2 and a typical value of α is of the order of $(10^{-3}e)^2$ at $T \leq 200$ mK.

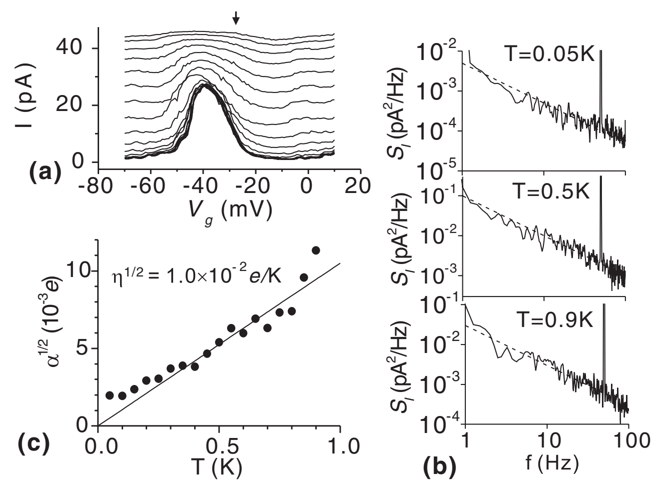

FIG. 1. (a) A Coulomb peak of the SET measured at temperatures from 50 to 900 mK with steps of 50 mK. (b) Examples of the current noise spectra S_I at different temperatures measured on the slope of the SET peak. Dashed lines are the $1/f$ dependence. (c) Temperature dependence of amplitudes $\alpha^{1/2}$ of the $1/f$ noise spectra. The solid line is $\alpha^{1/2} = \eta^{1/2}T$ with $\eta = (1.0 \times 10^{-2}\ e/\mathrm{K})^2$.

Solid dots in Fig. 1(c) represent $\alpha^{1/2}$ as a function of temperature. $\alpha^{1/2}$ saturates at temperatures below 200 mK at the level of $2 \times 10^{-3}e$ and exhibits nearly linear rise at temperatures above 200 mK with $\alpha^{1/2} \approx \eta^{1/2}T$, where $\eta \approx (1.0 \times 10^{-2}\ e/\mathrm{K})^2$ [the solid line in Fig. 1(c)]. T^2 dependence of α is observed in many samples, though sometimes the noise is not exactly $1/f$, having a bump from the Lorentzian spectrum of a strongly coupled low-frequency fluctuator. In such cases, switches from the single two-level fluctuator are seen in time traces of the current [8].

Note that at a fixed bias voltage the average current through the SET increases with temperature [see Fig. 1(a)]. However, it has almost no effect on the noise as we confirmed from the measurement of the current noise dependence. Nevertheless, to avoid possible contribution from the current dependent noise we adjust the bias voltage in the next measurements so that the average current is kept nearly constant at the measurement points for different temperatures. Figure 2(a) shows the temperature dependences of $\alpha^{1/2}$ for a different sample with a similar geometry taken in the frequency range from 0.1 to 10 Hz with a bias current adjusted to about $I = 12 \pm 2$ pA. The straight line in the plot is $\alpha^{1/2} = \eta^{1/2}T$, which corresponds to T^2 dependence of α with $\eta \approx (1.3 \times 10^{-2}\ e/\mathrm{K})^2$.

We study decay of coherent oscillations away from the electrostatic energy degeneracy point at different temperatures by measuring pulse induced current [1–3]. The Hamiltonian of our qubit written in the charge basis $|0\rangle$ and $|1\rangle$ (with and without the Cooper pair in the island) is $H = \frac{\Delta E}{2}(\sigma_z \cos\theta + \sigma_x \sin\theta)$, where $\theta = \arctan(E_J/\Delta U)$ and $\Delta E = \sqrt{\Delta U^2 + E_J^2}$ are determined by the electrostatic energy difference ΔU between the two charge states of the qubit and the Josephson energy E_J. The electrostatic energy ΔU is controlled by the qubit gate voltage. Adjusting a dc gate voltage to the point far away from the degeneracy ($\Delta U \gg E_J, \theta \approx 0$), where the ground state is nearly $|0\rangle$ we apply a rectangular voltage pulse of length t bringing the qubit in the vicinity of the degeneracy point (ΔU is of the order or smaller than E_J and $\theta \sim \pi/2$); that is, the Hamiltonian changes nonadiabatically to H_1 for time t. The coherent evolution can be presented as $\exp(-\frac{i}{\hbar} \times \int_0^t H_1 dt)|0\rangle$. Applying a sequence of identical pulses we detect a pulse induced current, which is proportional to the

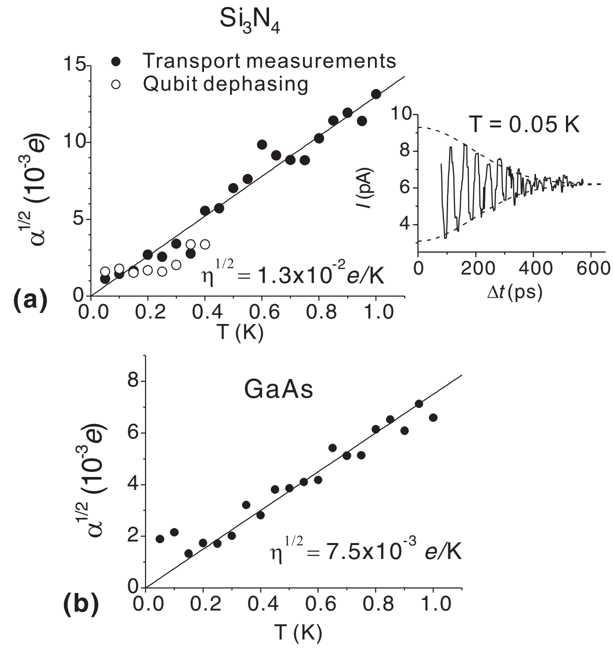

FIG. 2. (a) Solid dots show temperature dependence of $\alpha^{1/2}$ with a fixed bias current (the bias voltage is adjusted to keep the current constant). Open dots show $\alpha^{1/2}$ derived from the measurement of qubit dephasing during coherent oscillations. The coherent oscillations (solid line) as well as the envelope $\exp(-t^2/2T_2^{*2})$ with $T_2^* = 180$ ps (dashed line) are in the inset. (b) Solid dots show temperature dependence of $\alpha^{1/2}$ for the SET on GaAs substrate.

probability to find out the qubit in the state $|1\rangle$ after the pulse manipulation.

The typical current oscillation as a function of t away from the degeneracy point ($\theta \neq \pi/2$) is exemplified in the inset of Fig. 2(a). If dephasing is induced by the Gaussian noise, the oscillations decay as $\exp(-t^2/2T_2^{*2})$ with

$$\frac{1}{T_2^{*2}} \approx \frac{\cos^2\theta}{\hbar^2}\left(\frac{4E_C}{e}\right)^2 \int_{\omega_0}^{\infty} S_q(\omega)\left(\frac{2\sin(\omega t/2)}{\omega t}\right)^2 d\omega, \quad (2)$$

where $\omega_0 \approx 1/\tau$ is the low-frequency integration limit defined by the measurement time constant τ. In the case of the $1/f$ Gaussian noise of Eq. (1)

$$T_2^* \approx \frac{e\hbar}{4E_C\sqrt{\alpha \ln(\omega_1 \tau)} \cos\theta}, \quad (3)$$

where $\omega_1 \leq \pi/T_2^*$ is the effective high frequency limit.

Qubit dephasing due to the non-Gaussian noise is treated in Refs. [22,23]. For instance, in the case of a strongly coupled fluctuator, the decay is slower than Gaussian. However, importantly, Eq. (2) used for fitting the initial part of the oscillations still gives a reasonably good agreement for estimation of the amplitude of the $1/f$ noise. We have analyzed time traces of the noise and found that the noise is often close to the Gaussian, but sometimes it is clearly not, for example, in the presence of a strongly coupled fluctuator.

The solid line in the inset of Fig. 2(a) shows decay of coherent oscillations measured at $T = 50$ mK and the dashed envelope exemplifies a Gaussian with $T_2^* = 180$ ps. We derive $\alpha^{1/2}$ from Eq. (3) and plot it in Fig. 2(a) by open dots as a function of temperature. The low-frequency integration limit and the high frequency cutoff are taken to be $\omega_0 \approx 2\pi \times 25$ Hz and $\omega_1 \approx 2\pi \times 5$ GHz for our measurement time constant $\tau = 0.02$ s and typical dephasing time $T_2^* \approx 100$ ps [24].

The saturation of the $1/f$ noise at low temperatures has also been observed in earlier works [19,20]. Although its origin is not clear, we can suggest the following possible mechanisms: (1) heating of an electron system, (2) freezing out fluctuators, so that the effective number of active fluctuators decreases down to a few per decade (in this case the $1/f$ noise saturates to the level of a single fluctuator amplitude).

To collect more information about the $1/f$ noise we perform an additional experiment studying the noise in the SET fabricated on a different substrate. Although the results are not conclusive and require more systematic study we think that it is instructive to present these data here. Figure 2(b) demonstrates the $1/f$ noise temperature dependence for an SET fabricated on single-crystal GaAs. The GaAs substrate has been chosen to reduce the number of defects (as a possible origin of fluctuators) typical for amorphous materials like CVD grown Si_3N_4 or thermal oxide on top of bare silicon. Again, clear T^2 dependence is observed above 200 mK, and $\eta \approx (0.75 \times 10^{-2}\ e/\mathrm{K})^2$ is lower but close to what was measured in the case of Si_3N_4.

Below we discuss a phenomenological model explaining the properties of the noise. Our qubit is coupled to charge dipoles ed in the insulator, which, in turn, induce a charge δq in the qubit island (the fluctuators at distances smaller than the characteristic size of the island R produce $\delta q \sim ed/R$, and the matrix element of the Cooper pair transition is at most $4E_C\delta q$). Based on the phenomenology from Ref. [2], it has been pointed out in Refs. [14,16] that the T^2 dependence may originate from two-level fluctuators (characterized by bias energy ϵ and tunneling energy Δ) and linearly distributed in ϵ with an energy independent amplitude $\langle \delta q \rangle$. Summation of the Lorentzian spectra over many fluctuators with energies ϵ below k_BT gives $(k_BT)^2$ term in the noise spectrum. Such linear energy distribution, for example, appears in the models treating charge fluctuations between the superconducting island and an insulator [15,16]. The switching rate of the thermally activated fluctuators ($\epsilon < kT$) can be presented as $\gamma \sim \gamma_0(\Delta/\epsilon)^2$, where γ_0 depends on the coupling of the fluctuators to the external thermal bath, and Δ is the tunneling energy of electrons in the fluctuators [25]. The slow fluctuators, contributing to the low-frequency $1/f$ noise should have a strong suppression factor $(\Delta/\epsilon)^2 \ll 1$. On the other hand, to efficiently absorb qubit energy, the two-level systems should have Δ of order or larger ϵ. Note that the two-level systems producing the $1/f$ noise and absorbing the qubit energy are characterized by very different values of Δ. A commonly used assumption that two-level systems are distributed according to $P(\Delta) \propto 1/\Delta$ gives rise to the crossover frequency of $\omega_c \approx k_B T/\hbar$ [14].

To show relationship of the model with experiments we provide some numbers from experiments. The $1/f$ noise in this work is studied in the temperature range from 0.05 to 1 K, which means that only fluctuators with the activation energies lower than $k_B T$ (from 1 to 20 GHz $\times h$) produce the noise. Note that this thermal energy range overlaps with the qubit energy 2–100 GHz $\times h$ for which the f quantum noise has been studied in Ref. [2]. On the other hand, the $1/f$ noise is measured in the frequency range 0.1–100 Hz, which gives typical values of γ for the fluctuators contributing to the measured noise. The high frequency cutoff of the $1/f$ noise (which may give rough estimate of γ_0) is not known for our qubits. For rough estimations, we take $\gamma_0 = 1$ MHz from Ref. [4] and find that only fluctuators with $(\Delta/\epsilon)^2 \approx \gamma/\gamma_0 \sim 10^{-7}$–$10^{-4}$ contribute to the measured noise. Note that to have necessary relationship between the fluctuators and the two-level system absorbing the qubit energy, distribution $P(\Delta) \propto 1/\Delta$ should hold in a very wide range of Δ from 10^6 Hz $\times h$ or less up to 10^{11} Hz $\times h$.

Apart from the phenomenology, the most important question now is what the detailed mechanism of the $1/f$ and f noises is. A few microscopic models have been proposed in Refs. [15,16,18]. Test experiments have to be done to verify the models. For instance, behavior of the noise as a function of magnetic field and superconducting-normal state transition would be important for the theories involving superconductivity.

In conclusion, we have observed T^2 temperature dependence of the low-frequency $1/f$ noise, which supports the idea that the $1/f$ and f noises have a common origin. Typically, the noise spectral density is $(10^{-2}e)^2 \times (T/\mathrm{K})^2/\omega$ at $T > 200$ mK and saturates to the level of the order of $(10^{-3}e)^2/\omega$ at $T < 200$ mK. We demonstrated that free induction decay is consistently explained by dephasing on the low-frequency $1/f$ noise. T^2 dependence of similar amplitude is observed for two different substrate materials amorphous Si_3N_4 and single-crystal GaAs.

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