Quasiadiabatic, Nonfocusing Transition-Energy Crossing

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(Received 22 November 2005; published 7 April 2006)

A quasiadiabatic nonfocusing transition-energy crossing is proposed for suppressing any nonadiabatic and undesired features in a longitudinally separated function-type accelerator, in which particles are confined by an radio-frequency voltage with an adiabatic reduction of the amplitude and accelerated by a step voltage. This new method has been examined, both theoretically and experimentally.

DOI: 10.1103/PhysRevLett.96.134801 PACS numbers: 29.27.Bd, 29.27.Fh, 41.85.Ct

The transition energy is a singular point in the acceleration of a proton synchrotron. Most proton synchrotrons consisting of a normal FODO lattice (an array of magnets where F is focusing, D is defocusing, and O is the drift space between magnets) have their transition energy in the middle of acceleration. More or less, these synchrotrons have faced various undesired features when a proton beam crosses the transition energy. The bunch shape in a radio-frequency (rf) bucket is remarkably deformed by an inherently nonadiabatic nature in the synchrotron oscillation in the vicinity of a transition energy in a synchrotron. This is well known to be caused by a linearly rapid change of the slippage factor in time [1]. The rf bunch is shortened in time space and stretched in momentum space. The former leads to a growth of the line density, inducing undesired coherent instabilities, such as the microwave instability [2], as well as increasing transverse space-charge effects. The latter may restrict the momentum aperture of an accelerator ring. In addition, nonlinear kinematic effects (Johnsen effect [3]) in the synchrotron oscillation should be relatively enhanced [4,5]. Mismatching of the rf bunch shape to the rf bucket before and after a transition crossing, which is caused by the nonlinear kinematic term, is inevitable, because this effect is not time reversible. Accordingly, the emittance growth becomes significant. It is reported that, in the relativistic heavy ion collider at Brookhaven National Laboratory, the short bunch at the transition energy induces a serious electron-cloud instability [6]. It is insisted that these awkward situations result in beam loss, which limits the operational capability of an accelerator.

A novel transition crossing method to completely eliminate the nonadiabatic motion strongly affected by the rf field gradient was proposed by Griffin [7] for an rf synchrotron. The novel method in an rf synchrotron has already been demonstrated in the Fermilab Main Ring [8], where the field gradient of the fundamental harmonic is eliminated by imposing the third harmonics during the transition crossing. Meanwhile, a similar method was introduced for an induction synchrotron [9]. The method is called a focusing-free transition crossing, where a proton bunch passes through the transition energy with no longitudinal focusing forces, but its acceleration is assured with flat step voltages, which are generated with induction acceleration devices [9]. The bunch deformation of shortening in time and stretching in momentum should disappear unless particles existing in the bunch see any of the confinement voltage, such as the rf or barrier voltages, for a short time period before and after the transition energy.

In this Letter, a transition crossing method to introduce an adiabatic longitudinal motion is proposed. An rf bunch can be stretched in time and shortened in energy space by adiabatically decreasing the rf voltage. Thus, if the adiabatic reduction of the rf voltage is performed for a long time period around the transition energy, the nonadiabatic feature mentioned above should be suppressed. This is called a quasiadiabatic, nonfocusing transition-energy crossing (QNTC) in this Letter. In a longitudinally separated function-type accelerator [9], in which particles are confined by an rf voltage or burrier voltages and accelerated by a step voltage, the confinement voltage can be arbitrarily manipulated as long as the particles do not diffuse, while a strict acceleration voltage is necessary for the orbit of a charged particle to be balanced in the radial direction. The introduction of QNTC is most suitable for this type of accelerator. In this Letter, theoretical analyses of the QNTC are given. In addition, this idea has been demonstrated in the KEK proton synchrotron (KEK-PS), which is being operated as a hybrid synchrotron employing the existing rf for the confinement and induction system for acceleration [10]. Those results are reported here.

The discrete synchrotron equations without any longitudinal space-charge effects and the nonlinear kinematic terms, which show the turn-by-turn evolution of the particle energy E and time difference $\Delta\tau$ from the arrival time of a synchronous particle at the acceleration device, are described as follows:

$$E_{m+1} = E_m + e\{(V_c)_m + (V_a)_m\}, \quad (1)$$

$$\Delta\tau_{m+1} = \Delta\tau_m + \eta_{m+1}T_{m+1}\frac{E_{m+1} - (E_s)_{m+1}}{(\beta_{m+1})^2(E_s)_{m+1}}, \quad (2)$$

where $V_a = \rho C_0 dB/dt$ (ρ, curvature; C_0, circumference; B, bending magnetic field) is the accelerating voltage, β and T are the relativistic beta and the revolution period of the synchronous particle, respectively, and $\eta = 1/\gamma_t^2 - 1/\gamma^2$ (γ_t, the transition gamma) is the slippage factor. V_c represents the confinement voltage, which varies in time according to

$$V_c(t) = \pm V_{\rm rf}\left|\frac{t}{t_0}\right|^n \sin\{\omega_{\rm rf}(t)t\} \quad (3)$$

for a finite time period of $2t_0$, in which $t = 0$ corresponds to the transition energy; n is a real number, and n is 0 for the nominal transition crossing (NTC) in a conventional rf synchrotron and $n > 0$ for the QNTC. $\|$ denotes the absolute value. $V_{\rm rf}$ and $\omega_{\rm rf}/(2\pi)$ are the amplitude and frequency of the rf voltage, respectively. The sign of the voltage is changed at the transition crossing to maintain the phase stability. A synchronous particle never sees V_c.

Assuming $\eta/E_s = a_1 t$ and $a_1 = 2eV_a/(\gamma_t^4 m_0^2 c^4 T)$ near γ_t [1], from Eqs. (1) and (2), the following differential equation for the small-amplitude time difference averaged over a turn is obtained:

$$\frac{d^2\Delta\tau}{dt^2} - \frac{1}{t}\frac{d\Delta\tau}{dt} \mp K\alpha_n a_1 |t|^n t\Delta\tau = 0, \quad (4)$$

where $K = 2\pi e/T^2$ is a constant and $\alpha_n = V_{\rm rf}/t_0^n$. Since the transition energy is sufficiently high in most of the accelerators, $\beta = 1$ and $T =$ constant are supposed in Eq. (4). We focus our concern on $t > 0$, because Eq. (4) is symmetric below and above γ_t. The solution to Eq. (4) is written as

$$\Delta\tau(t) = AtJ_{2/(n+3)}(\phi(t)) + BtN_{2/(n+3)}(\phi(t)), \quad (5)$$

$$\phi(t) = \frac{2}{n+3}\sqrt{K\alpha_n a_1}\, t^{(n+3)/2}, \quad (6)$$

where J_ν and N_ν are Bessel and Neumann functions of order ν. A and B are the coefficients, which are determined by the initial condition for large t. The solution [Eq. (5)] strictly has a constant amplitude only for $n = 1$. This suggests that the bunch length is constant during passing through γ_t. Meanwhile, the amplitude for $0 \leq n < 1$ and $n > 1$ increases and decreases as a function of t, respectively. From the intrinsic nature of longitudinal beam dynamics, an increment and a decrement in the time difference are always associated with decreasing and increasing in the energy deviation from that of a synchronous particle.

In order to delineate the longitudinal motion for the QNTC in a realistic manner, particle tracking based on Eqs. (1) and (2) was carried out. In this calculation, the machine parameters of the KEK-PS with the 8 GeV ramping pattern were assumed (see Table I). The patterns of V_a and V_c are shown in Fig. 1, in which the beginning of the injection porch, the beginning of the acceleration period, and the end of acceleration are called $P1$, $P2$, and $P3$, respectively. "P" is the abbreviation of "pattern timing." For the QNTC, t_0 was set to 0.125 sec. An initial beam distribution measured at injection was used as the initial condition.

TABLE I. Machine parameters for the KEK-PS.

γ_t	6.63
C_0 (m)	339
h	9
Injection energy (GeV)	0.5
Extraction energy (GeV)	8

The temporal evolutions of the root-mean squares of the bunch length in time and energy space are shown in Fig. 2. As expected, the simulation results for the NTC indicate that Δt shrinks and $\Delta E = E - E_s$ increases in the vicinity of γ_t. This is a well-known nonadiabatic feature associated with the transition crossing. As expected for the QNTC with $n > 1$, the bunch length is stretched in time and intensely shortened in energy space. In the case of $n = 1$, the bunch shape is maintained to be almost constant during the time period of $-t_0 \leq t \leq t_0$. Though the motion dominated by Eq. (4) is symmetric below and above γ_t as mentioned above, the results for the QNTC ($n = 1$) are slightly asymmetric against γ_t. The difference from linear theory is attributed to the asymmetry in the actual η. It has been numerically confirmed that $\eta/E_s = a_1 t$ is a good approximation for $t_0 = 24$ msec.

The QNTC experiment was carried out in the present KEK-PS, which has been operated as a hybrid synchrotron.

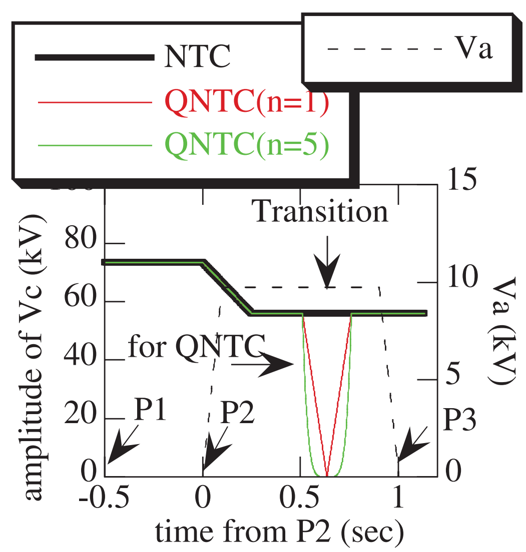

FIG. 1 (color). Pattern of confinement and acceleration voltage.

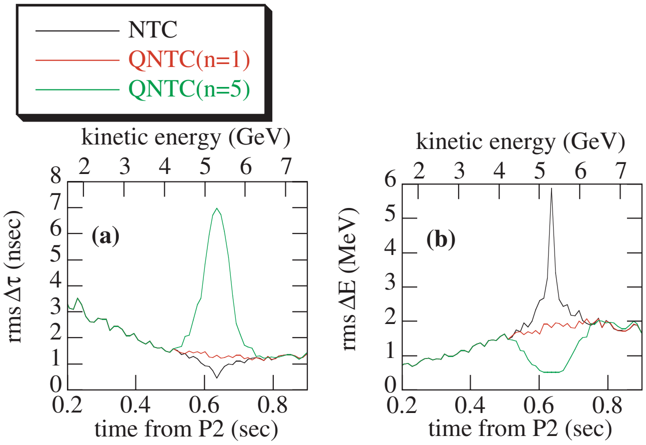

FIG. 2 (color). Temporal evolution of the rms bunch length (a) in the time domain and (b) in energy space.

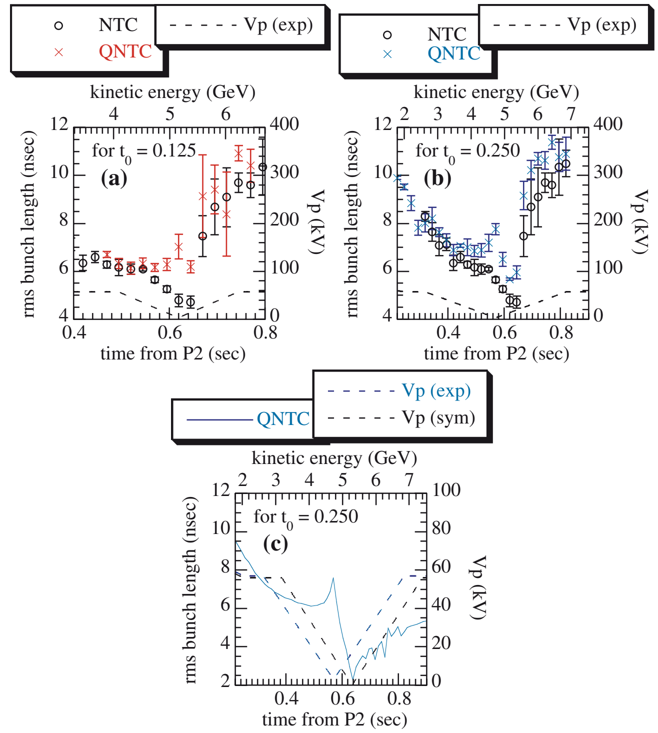

FIG. 4 (color). Rms bunch length for (a) NTC and QNTC for $t_0 = 0.125$ sec (experiment), (b) NTC and QNTC for $t_0 = 0.250$ sec (experiment), and (c) QNTC for $t_0 = 0.250$ sec (simulation).

The QNTC with $n = 1$ was employed because it was the unique solution for maintaining a constant bunch size. The amplitude of the rf voltage, which has to obey the programmed voltage (V_p), is controlled by an automatic voltage controller [11]. A triangular pulse with a negative amplitude was superimposed on the existing V_p to generate the rf voltage for the QNTC. Two cases of the triangular pulse ($t_0 = 0.125$ and 0.250 sec) were prepared in an acceleration time period of 1 sec (see Fig. 3).

The bunch length in the vicinity of γ_t, measured by the wall current monitor, is shown in Figs. 4(a) and 4(b), in which V_p(exp) means the amplitude of the rf voltage used in the experiment. The bunch length for the QNTC with a longer t_0 became longer than that for the case with a shorter t_0 below γ_t, as predicted. For both cases, the bunch length was confirmed to be nearly constant below γ_t. However, there are sudden changes of the bunch length at around 0.6 sec from $P2$, especially in Fig. 4(b), which is just below γ_t. This seems to be induced by bunch rotation that originated from mismatching. Since it is technically difficult to control the rf amplitude matching to an accurate timing of the transition crossing, the rf voltage profile became more or less asymmetric below and above γ_t. The simulation results with the asymmetric V_p(exp) of Fig. 4(b) is shown in Fig. 4(c), in which V_p(sym) is the symmetric one for the reference. The tracking qualitatively reproduced the experimental result. It is notable that the constant bunch length in Fig. 2 has been obtained assuming V_p(sym). This strongly suggests that the amplitude of the rf voltage should be symmetric to the timing of the transition crossing for keeping the constant bunch length.

The temporal evolution of the beam intensity measured by the slow intensity monitor is shown in Fig. 5, in which the beam was injected at 50 msec before $P2$, accelerated for 1 sec, and then extracted. The beam loss concerning the

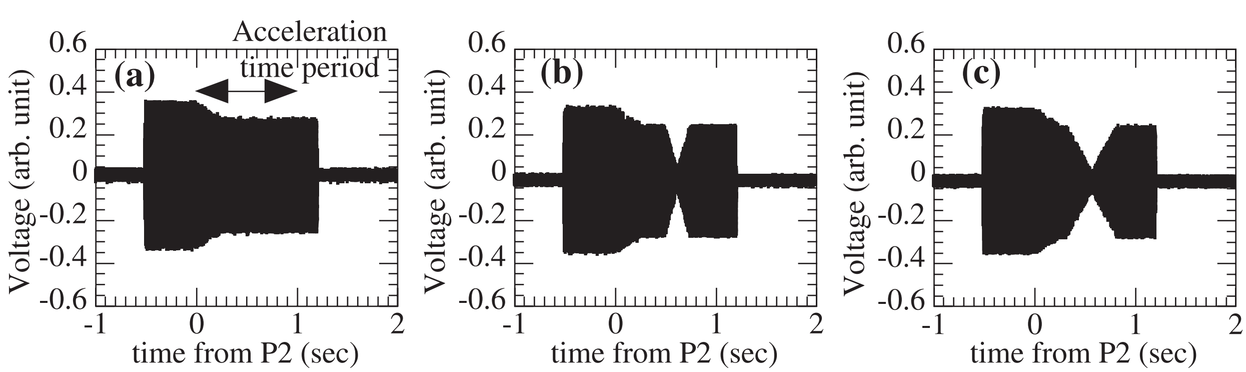

FIG. 3. rf voltage for (a) NTC and QNTC for (b) $t_0 = 0.125$ sec and (c) $t_0 = 0.250$ sec (measured).

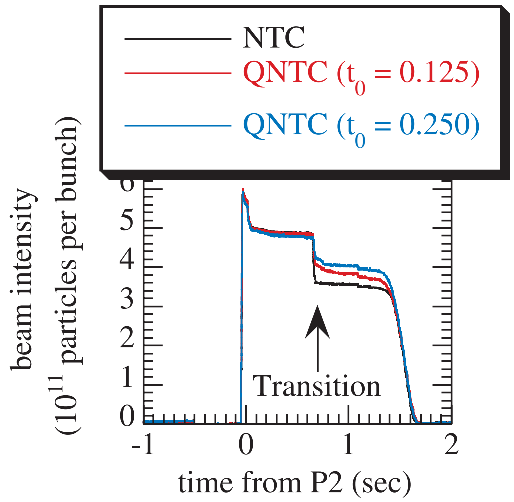

FIG. 5 (color). Beam intensity for NTC, QNTC for $t_0 = 0.125$ sec and QNTC for $t_0 = 0.250$ sec.

transition crossing was shown around 0.635 sec from $P2$. For both cases, the beam loss was substantially suppressed, compared to that for the NTC, although the amplitude of the rf voltage was deformed by 1/4–1/2 of the accelerating time. While the bunch length of Fig. 4(b) indicated the significant change with time, it was still longer than for NTC. So it seems that the beam loss around the transition energy became lower than for NTC. A beam instability may cause this beam loss, and the detailed study will be done in the next step.

It is insisted that the experiment has manifested a big figure of merit that the separation of acceleration and longitudinal confinement provides. This is one example of big freedoms of beam handling that the induction synchrotron concept [9] introduces in circular accelerators. The present quasiadiabatic, nonfocusing transition-energy crossing technique is available for an rf synchrotron, in which a higher harmonic rf is utilized for the acceleration [8] as well as the induction step voltage.

S. Ninomiya and S. Takano provided much information concerning the KEK-PS rf system. One of the authors (Y. S.) acknowledges T. Toyama for comments on his work. He also thanks members of the superbunch group for supporting this study. The present research was financially supported by a Grant-In-Aid for Scientific Research for Creative Scientific Research (No. KAKENHI 15GS0217).

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