Direct Empirical Correlation between Proton-Neutron Interaction Strengths and the Growth of Collectivity in Nuclei

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A direct correlation between experimental values of proton-neutron interaction strengths and experimental measures of the growth of collectivity in nuclei is found. In particular, differences in the *p-n* interaction strengths and differences in growth rates of collectivity in particle-particle (or hole-hole) and particle-hole regions are found to correspond.

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For over five decades the proton-neutron interaction has been considered the key ingredient in the development of configuration mixing, collectivity, and, ultimately, deformation in atomic nuclei. First suggested in 1953 by de Shalit and Goldhaber [1] and repeatedly stressed by Talmi [2–4], it is the cornerstone of the pioneering Federman-Pittel explanation [5] of the rapid onset of deformation in the $A = 100$ region, which was later extended to other mass regions [6]. Nazarewicz, Dobaczewski, and colleagues further elaborated the orbit dependence of the *p-n* interaction [7,8]. Heyde and co-workers [9,10] provided quantitative theoretical underpinning to the role of the valence *p-n* interaction and the central importance of its monopole components, and extended its applicability not only to the onset of equilibrium deformation in nuclei but also to the appearance of low-lying intruder states reflecting shape coexistence. The recent discovery [11] of first order phase transitional behavior in the equilibrium deformation has further highlighted the key role of the *p-n* interaction. Very recently Otsuka [12] and co-workers and others [13] have discussed the roles of spatial and tensor forms of the residual *p-n* interaction in $j = \ell \pm \frac{1}{2}$ configurations. Phenomenologically, the correlation of the integrated valence *p-n* interaction with the onset of collectivity and deformation has been codified in terms of the N_pN_n scheme [14], and the competition of this interaction with its nemesis, the pairing interaction, is embodied in the P factor [15].

Despite all these continuing efforts and the universally accepted importance of the valence *p-n* interaction in collectivity in nuclei, there has never, to our knowledge, been a direct, experimental correlation of empirical *p-n* strengths with experimental measures of collectivity, without the mediation of a model framework.

It is the purpose of this Letter first to point out a systematic difference in the rates of growth of collectivity in different regions that seems not to have been noted before, and then to show the first direct evidence for such an empirical correlation between collectivity and *p-n* interaction strengths, thereby providing a microscopic rationale for these different growth rates. We stress that the observables to be discussed are both obtained experimentally—one is a well-known measure of collectivity, the energy ratio $R_{4/2} \equiv E(4_1^+)/E(2_1^+)$ (in even-even nuclei), and the other an empirical extraction of the average proton-neutron interaction of the last valence proton with the last valence neutron, δV_{pn}. $R_{4/2}$ takes on values <2 near magic nuclei, is 2 for a harmonic vibrator and in the range $\sim$2–2.2 for typical vibrational nuclei, and it goes asymptotically to 3.33 for axially symmetric quantum rotors. δV_{pn} is obtained from double differences of atomic masses [16]. δV_{pn} effectively cancels out other interactions to second order and isolates that of the last valence protons and neutrons. Experimental values of δV_{pn} were extensively discussed a number of years ago (see, for example, Ref. [17], and Ref. [10] for theoretical analysis). Following the publication of the 2003 mass table [18], a new analysis [19] showed a dramatic correlation of empirical *p-n* interactions to underlying shell structure.

The present work should be of rather broad interest since collectivity, phase transitional behavior, vibrational modes, and rotation are widespread features of quantum systems [20]. Moreover, the competition of pairing, which favors higher symmetry spherical shapes, with interactions embodying lower symmetry, deformed, equilibrium shapes, is also common to other many-body systems and is closely related [21] to the ideas underpinning Landau theory and Ising-type models [22]. Indeed, in nuclei, an interacting boson approximation Hamiltonian written in an Ising form with competing spherical and deformation driving terms can describe a wide range of spectra and their dependence on system constituents very well [23,24].

Figure 1(a) shows the evolution of $R_{4/2}$ over the entire $Z = 50$–82, $N = 82$–126 major shell, plotted against N_pN_n, the product of the number of valence protons and neutrons counted to the nearest closed shell. Clearly, as expected, there is a general growth in $R_{4/2}$ from doubly magic nuclei toward midshell (largest N_pN_n values). However, closer interpretation shows an intriguing phenomenon. The $R_{4/2}$ values grow to saturation at different

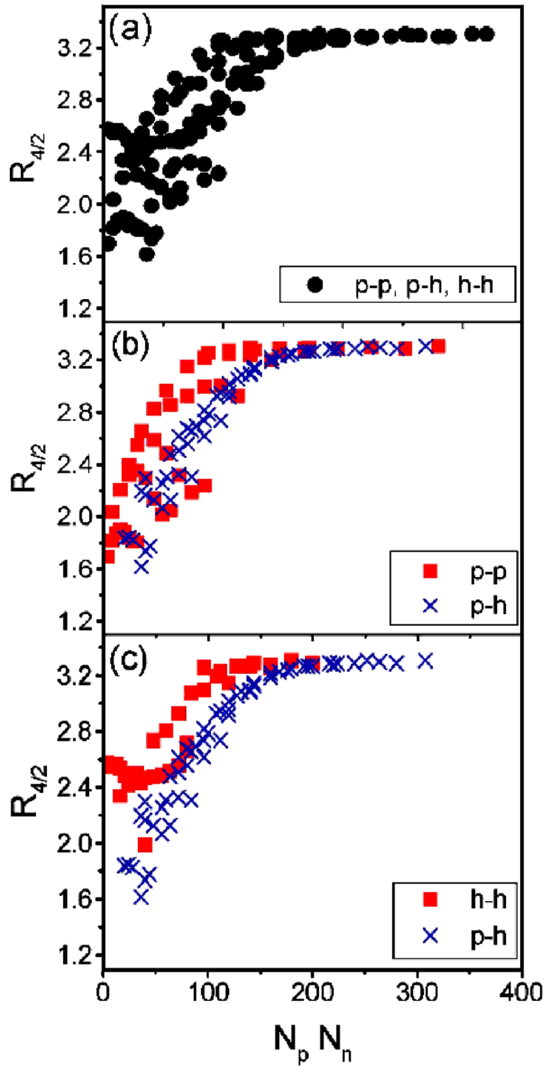
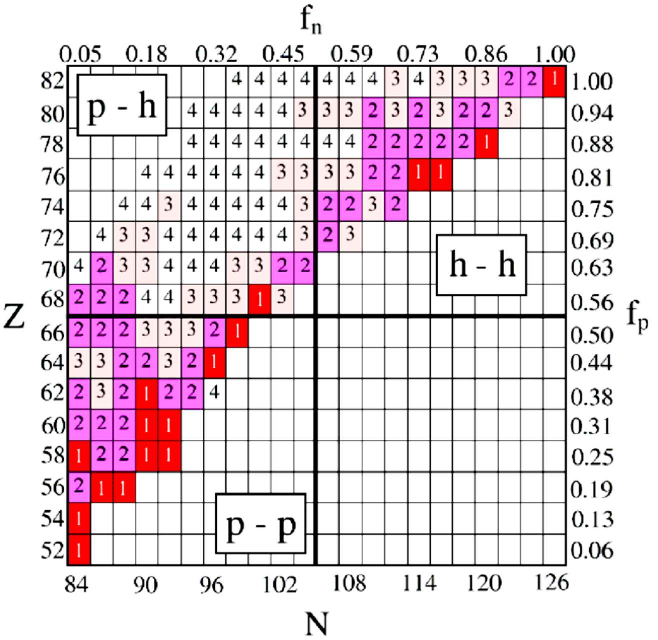


FIG. 1 (color online). $R_{4/2}$ values in the $Z = 50$–82, $N = 82$–126 region against $N_p N_n$. (a) All nuclei; (b) comparison of p-p ($Z = 50$–66, $N = 82$–104) with p-h ($Z = 68$–82, $N = 82$–104) regions; (c) comparison of h-h ($Z = 68$–82, $N = 106$–126) with the p-h region.

rates (for different $N_p N_n$ values) in different quadrants. This is seen in Figs. 1(b) and 1(c) which compare $R_{4/2}$ values in particle-particle (p-p) and hole-hole (h-h) regions with $R_{4/2}$ values in the particle-hole (p-h) region of the same major shell. See also Ref. [25] where a figure similar to Fig. 1(c) was shown in a somewhat different context. These panels in Fig. 1 show a structural evolution that, to our knowledge, has not been previously noted: collectivity grows *faster* in the p-p and h-h regions compared to the p-h region; that is, it grows faster when *both* protons and neutrons are in the first half of the shell (or both in the second half) compared to one filling below and the other above midshell.

Having pointed this out, the key point of this Letter is to show further that there is an empirical link between this behavior of nuclear collectivity and its underlying source in the *p-n* interaction, that is, that empirical valence *p-n* interaction strengths are also larger in like regions (p-p or h-h) than when protons and neutrons are filling unlike regions (p-h). To see this, Fig. 2 shows δV_{pn} (Z, N) values for the same major shells as in Fig. 1. Here, the numbers for each nucleus indicate the range of δV_{pn} according to the legend below the figure. Clearly, they are significantly larger in the p-p (lower left) and h-h (upper right) quadrants than in the p-h (upper left) regions. Thus, putting Figs. 1 and 2 together, the $R_{4/2}$ values are, on average, *larger* in the p-p *and* h-h regions than in the p-h region, *and so are the* δV_{pn} *values*. The striking correlation of experimentally extracted valence *p-n* interactions and measured $R_{4/2}$ values demonstrates the first *direct empirical* correlation between these two observables—one a measure of macroscopic collectivity and the other a measure of microscopic residual interactions. Figure 2 is for even-even nuclei. δV_{pn} values can also be constructed for odd-*A* nuclei and show similar enhancement in the p-p and h-h regions, relative to the p-h quadrant.

The effect is not only an overall visual impression; it is validated both integrally and, almost without exception, nucleus by nucleus. The *average* values of δV_{pn} in the



Z	84			90			96			102			108			114			120			126	f_p
f_n	0.05			0.18			0.32			0.45			0.59			0.73			0.86			1.00	
82								4	4	4	4	4	4	4	3	4	3	3	3	2	2	1	1.00
80						4	4	4	4	4	3	3	3	2	3	2	3	2	2	3			0.94
78						4	4	4	4	4	4	4	4	2	2	2	2	2	1				0.88
76				4	4	4	4	4	4	3	3	3	3	2	2	1	1						0.81
74			4	4	3	4	4	4	4	4	3	2	2	3	2								0.75
72		4	3	3	4	4	4	4	4	4	3	2	3										0.69
70	4	2	3	3	4	4	4	3	3	2	2												0.63
68	2	2	2	4	4	3	3	3	1	3													0.56
66	2	2	2	3	3	3	2	1															0.50
64	3	3	2	2	3	2	1																0.44
62	2	3	2	1	2	2	4																0.38
60	2	2	2	1	1																		0.31
58	1	2	2	1	1																		0.25
56	2	1	1																				0.19
54	1																						0.13
52	1																						0.06

1	> 350 keV	3	300 - 250 keV
2	350 - 300 keV	4	250 - 200 keV

FIG. 2 (color online). δV_{pn} values in the same major shell region as the nuclei shown in Fig. 1, plotted against Z and N, as well as against the major shell fractional filling f_p and f_n. The numbers in each box indicate the strength of δV_{pn}: lower numbers (1, 2) represent larger δV_{pn} values. See legend at bottom. The shading also indicates the δV_{pn} ranges. Updated from Ref. [19].

lower left (p-p) and upper left (p-h) regions of Fig. 2 are ~330 and ~250 keV, respectively. The average in the h-h region is ~300 keV. An easy way to see this correlation in more detail is to take the differences of $R_{4/2}$ values for nuclei in corresponding positions in the p-p and p-h regions (lower left and upper left quadrants in Fig. 2, or upper left and right quadrants), recalling that δV_{pn} (Z, N) gives the average p-n interaction of the $(Z - 1)$th and Zth protons with the $(N - 1)$th and Nth neutrons. This nomenclature means that δV_{pn} in, say, Te involves the first two valence protons in the $Z = 50$–82 shell, while the corresponding proton holes would involve the 81st and 82nd protons and thus be denoted by $Z = 82$, that is, Pb. Similarly, $Z = 64$ and 70 are matching proton particle-hole nuclei in this context. To see a few examples of differences of δV_{pn} values for nuclei with corresponding numbers of valence proton and neutron particles or holes in Fig. 2, consider the δV_{pn} values for the corresponding pairs of p-p and p-h nuclei, $^{148}_{58}\mathrm{Ce}_{90}$ and $^{166}_{76}\mathrm{Os}_{90}$ with $\delta V_{pn}(^{148}\mathrm{Ce}) = 370 \pm 26$ keV and $\delta V_{pn}(^{166}\mathrm{Os}) = 187 \pm 53$ keV or $\delta V_{pn}(^{158}\mathrm{Sm}) = 250 \pm 61$ keV and $\delta V_{pn}(^{168}\mathrm{Hf}) = 234 \pm 11$ keV; or the h-h, p-h pair $\delta V_{pn}(^{186}\mathrm{W}) = 340 \pm 10$ keV and $\delta V_{pn}(^{172}\mathrm{W}) = 228 \pm 13$ keV. In fact, of 18 possible p-p/p-h comparisons, where both δV_{pn} and $R_{4/2}$ are known and the uncertainties are less than the differences, all but one are larger for the p-p nuclei. For the same nuclei, only three $R_{4/2}$ values are smaller in the p-p region. One of these is the same nucleus as the δV_{pn} exception, and all three are in the region where the $Z = 64$ shell closure plays a role.

For the h-h, p-h regions, again there are 18 pairs of corresponding nuclei. Remarkably, there is a perfect correspondence: for both δV_{pn} and $R_{4/2}$, 17 values are larger in the h-h region, and the one that is not larger is for the same pair of nuclei for both δV_{pn} and $R_{4/2}$.

Figure 3 shows this in a systematic way. It compares $\Delta(\delta V_{pn})$ and $\Delta(R_{4/2})$ values for regions of p-p and p-h character. The numbers in each square indicate, for the pairs of nuclei where both δV_{pn} and $R_{4/2}$ values are known, the values of $\Delta(\delta V_{pn}) \equiv \delta V_{pn}(\text{p-p}) - \delta V_{pn}(\text{p-h})$ and of $\Delta R_{4/2} \equiv R_{4/2}(\text{p-p}) - R_{4/2}(\text{p-h})$. Clearly, there is almost a 1-1 correspondence of larger δV_{pn} values and larger $R_{4/2}$ values in p-p (or h-h) regions. It is also interesting to show some illustrative $R_{4/2}$ and δV_{pn} values across sequences of corresponding nuclei. Since N_pN_n increases fastest along diagonal trajectories from a corner of each major shell region towards the center, we follow three examples of such trajectories in Fig. 4. Once again the correlation of $R_{4/2}$ and δV_{pn} values in p-p compared to the p-h region is striking and consistent. In each panel the p-p values are larger than the p-h values for both $R_{4/2}$ and δV_{pn}. Similar results apply to the comparison of the (h-h) and (p-h) regions.

Of course, these comparisons are not quite proper. $R_{4/2}$ is an integral quantity reflecting the accumulated interactions leading to collectivity and deformation, while δV_{pn} is a differential quantity reflecting the interactions of the last two protons and neutrons. It could happen, for example, that some $R_{4/2}(Z, N)$ value is relatively small while the $\delta V_{pn}(Z, N)$ value is relatively large, if other δV_{pn} values in the same quadrant were also relatively small. Ideally, one would like to compare sums of all δV_{pn} from the doubly magic cores, giving the integrated p-n interactions. However, this is, in general, not possible because the requisite data are not available. Nevertheless, the preponderance of larger values of both δV_{pn} and $R_{4/2}$ in p-p (or h-h) regions, relative to (p-h) regions, is so systematic [compare Figs. 1(b) and 1(c) with Fig. 2] as to leave no doubt of the empirical correlation of the observables.

In summary, we have compared empirical measures of collectivity with p-n interaction strengths of the last nucleons and found a striking correlation. In particular, both are larger in regions where both protons and neutrons are filling similar (p-p or h-h) portions of major shells and smaller otherwise. The physical reason for the larger δV_{pn} interactions when both protons and neutrons are in like regions is easy to see. It is not, *per se*, the particle or hole character (the interactions are always between *particles* filling various orbits) but rather the fact, stressed in Ref. [19], that the normal parity orbits in major shells in heavy nuclei tend to fill in regular fashion, from high j-low n to low j-high n character. Hence, in like portions of a

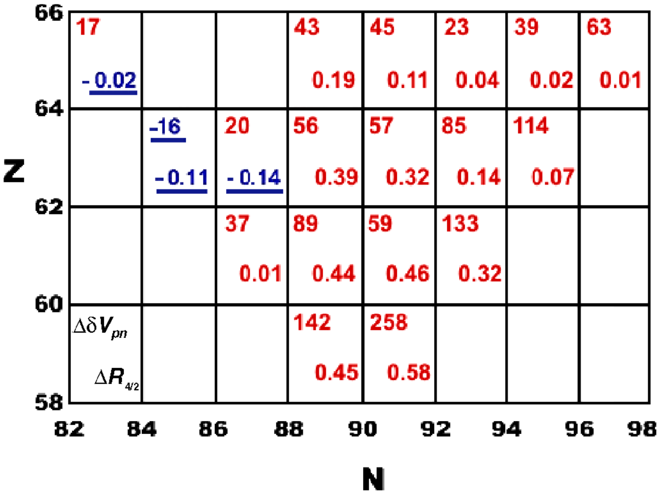


FIG. 3 (color online). Differences in δV_{pn} values (upper left in each box) and in $R_{4/2}$ (lower right in each box) for pairs of nuclei in the p-p and p-h regions of Fig. 2. The $\Delta(\delta V_{pn})$ and $\Delta(R_{4/2})$ values are defined as $\delta V_{pn}(\text{p-p}) - \delta V_{pn}(\text{p-h})$ and $R_{4/2}(\text{p-p}) - R_{4/2}(\text{p-h})$ where the pairs of nuclei in each difference have the same neutron number and have proton numbers that are symmetric about midproton shell. For example, the third entry in the second row from the top gives the differences $[\Delta(\delta V_{pn}) = 56$ keV and $\Delta R_{4/2} = 0.39]$ for the two nuclei with $N = 90$ and $Z = 64$ and 70. Negative numbers are underlined. The data are plotted in the positions of the p-p nuclei.

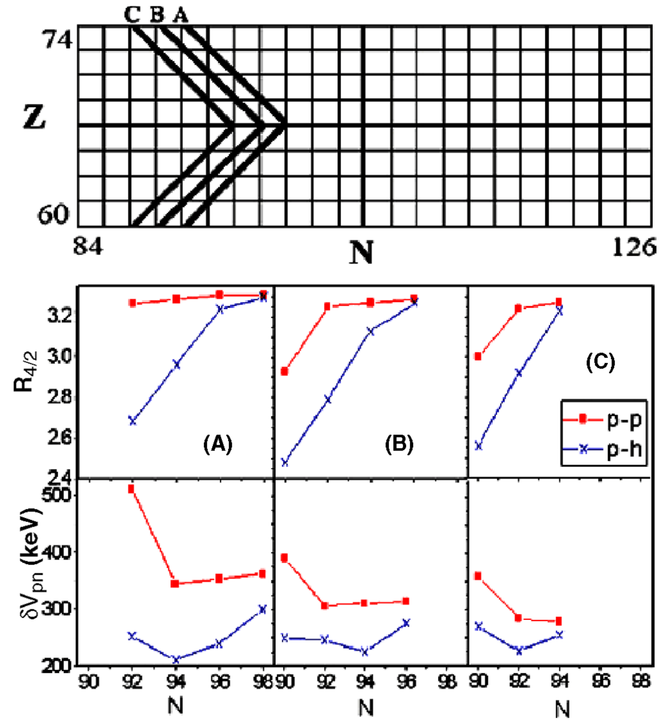


FIG. 4 (color online). $R_{4/2}$ and δV_{pn} values in sequences of p-p and p-h nuclei. The panels give values for "diagonal" sets of nuclei in p-p and p-h regions. The sets of nuclei in (A), (B), and (C) are identified in the upper part.

major shell (p-p or h-h) similar orbits are filling, leading to larger overlaps and stronger interactions, while in dissimilar portions (p-h), the orbits filling are quite different (e.g., $\pi 3s_{1/2}$ and $\nu 1h_{9/2}$ in the upper left portion of Fig. 2). Of course, like nucleons are also in similar orbits, but the like nucleon interaction is only strong in the $J = 0$ spherical-driving channel. The present result is, to our knowledge, the first direct empirical correlation of growth rates of collectivity with actual *p*-*n* interaction strengths, despite the fact that the key role of the *p*-*n* interaction in inducing configuration mixing and collectivity has been a cornerstone of our understanding of nuclear structural evolution for decades. Clearly, data on the neutron rich p-h lower right quadrant of Fig. 2 would be very valuable in further testing this correlation to complement the proton-rich p-h upper left quadrant. Finally, we note that shell structure may change far from stability and the particle and hole nature of particular Z and N values may be unknown. If the present correlation of δV_{pn} with evolution of $R_{4/2}$ values does not persist, it could point to the presence of new effects such as those involving the nearby continuum.

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