Noise Thermometry with Two Weakly Coupled Bose-Einstein Condensates

Rudolf Gati, Börge Hemmerling, Jonas Fölling, Michael Albiez, and Markus K. Oberthaler
*Kirchhoff-Institut für Physik, Universität Heidelberg, Im Neuenheimer Feld 227, D-69120 Heidelberg, Germany**
(Received 17 January 2006; published 6 April 2006)

Here we report on the experimental investigation of thermally induced fluctuations of the relative phase between two Bose-Einstein condensates which are coupled via tunneling. The experimental control over the coupling strength and the temperature of the thermal background allows for the quantitative analysis of the phase fluctuations. Furthermore, we demonstrate the application of these measurements for thermometry in a regime where standard methods fail. With this we confirm that the heat capacity of an ideal Bose gas deviates from that of a classical gas as predicted by the third law of thermodynamics.

DOI: 10.1103/PhysRevLett.96.130404 PACS numbers: 05.30.Jp, 03.75.−b, 05.40.−a, 74.40.+k

The generation of two independent matter-wave packets by splitting a single Bose-Einstein condensate (BEC) is a well-established technique [1–3] in the field of atom optics. New phenomena arise if the two separated parts can still coherently interact in analogy to Josephson junctions in condensed matter physics [4] and superfluid helium Josephson weak links [5]. An advantage of the realization of weakly coupled BEC in a double-well potential [6] is the possibility to observe the phase difference between the two macroscopic wave functions directly. Our experimental investigation of this relative phase reveals that it is not locked to zero but exhibits fluctuations. Two fundamental types of fluctuations are discussed in the literature—quantum fluctuations [7] and thermally induced fluctuations [8]. In this Letter we report on the experimental investigation of thermal fluctuations of the relative phase arising from the interaction of the BEC with its thermal environment, which is always present.

The essential prerequisite for the investigation of these thermally induced phase fluctuations is the ability to prepare a BEC adiabatically in a symmetric double-well potential and to adjust its temperature. In our experiments this is achieved by splitting a single ^{87}Rb BEC produced and trapped in an optical dipole trap by slowly ramping up a barrier in the center. The tunneling coupling is adjusted by the barrier height and its strength can be deduced from numerical simulations of the BEC in the trap using the model described in [9]. The temperature of the BEC is adjusted by holding the cloud in the trap, where due to fluctuations of the trap parameters energy is transferred to the atoms. Once the final temperature is reached a standing light wave is ramped up generating a barrier in the center, leading to an effective double-well trapping potential [upper part of Fig. 1(a)].

When the potential is switched off, the matter-wave packets start to expand, overlap, and form a double-slit interference pattern which depends on their relative phase as indicated in the lower part of Fig. 1(a). Repeating the interference measurements reveals that this relative phase is not constant but fluctuates around zero. The general behavior of these phase fluctuations is connected to two parameters: the temperature of the system randomizing the phase and the tunneling coupling of the two matter-wave packets stabilizing the phase. The results depicted in Fig. 1(b) show that the phase fluctuations become more pronounced as the temperature is increased since the fluctuations outweigh the stabilizing effects. From this point of view it is expected—and also experimentally observed [Fig. 1(c)]—that keeping the temperature constant and increasing the tunneling coupling leads to a reduction of the fluctuations. A measure for the fluctuations is the

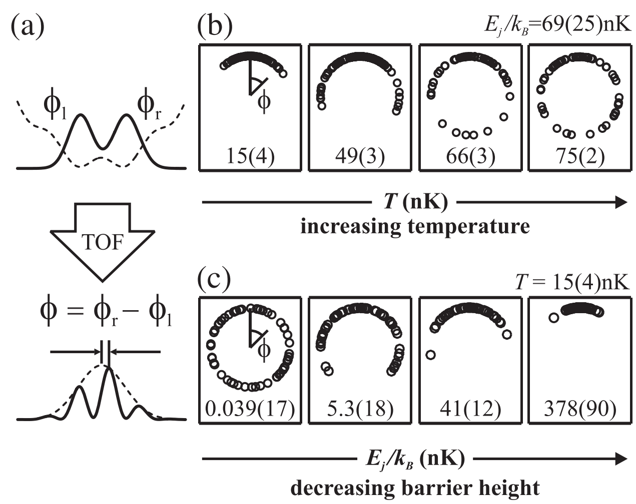

FIG. 1. Observation of thermal phase fluctuations. The experimental steps are depicted in (a). A Bose-Einstein condensate (solid line) is prepared in a double-well potential (dashed line) by adiabatically ramping up the barrier. The relative phase can be measured after a time-of-flight expansion by analyzing the resulting double-slit interference patterns (black line). (b) Polar plots of the relative phase obtained by repeating the experiment up to 60 times. The graphs show measurements for four different temperatures T at constant tunneling coupling energy E_j, i.e., constant barrier height. The phase fluctuations increase with increasing temperature. (c) Polar plots of the relative phase for a constant temperature at four different tunneling coupling energies, i.e., different barrier heights. Here the fluctuations are reduced with increasing coherent tunneling coupling showing the stabilization.

coherence factor $\alpha = \langle\cos\phi\rangle$ [8], which is directly connected to the visibility of the ensemble averaged interference fringes.

The coherence of the system can be visualized as shown in Fig. 2. For every single realization below the critical temperature the experiment reveals interference patterns with high visibility. However, the visibility is reduced by averaging over many realizations and for high temperatures it disappears completely as the mean fluctuations of the relative phase become comparable to π. The loss of coherence due to thermal fluctuations is shown in Fig. 2(a), corresponding to $\alpha = 0.046$. At the same temperature the coherence of the weakly coupled condensates can be maintained by increasing the coupling. For a tunneling coupling energy larger than the thermal energy the phase is locked to zero and the averaging reduces the visibility only slightly as shown in Fig. 2(b), where the coherence factor is given by $\alpha = 0.87$. The dependence of the coherence factor on the two parameters is the consequence of a universal scaling law which can be explained by the classical model discussed in the following.

The dynamics can be described in terms of a two mode approximation by assuming weak coupling between the localized modes of the BEC. This corresponds to treating

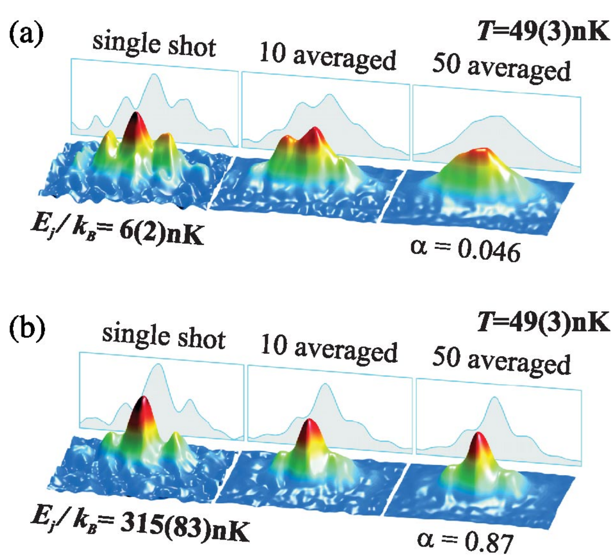

FIG. 2 (color). Loss of the coherence of the bosonic Josephson junction due to the coupling to a thermal environment. (a) The transition from coherent single realizations to incoherent ensemble averages. In the single realization a clear interference signal is observed, where the visibility is decreased due to the finite optical resolution of the imaging setup. After averaging over 10 realizations the visibility is reduced and after averaging over 50 realizations the coherence is lost ($\alpha = 0.046$). In (b) the coherent evolution of the bosonic Josephson junction is depicted. The results shown are obtained by repeating the experiment at the same temperature as above but at a stronger tunneling coupling. Here the averaging leads to only a small degradation of the visibility ($\alpha = 0.87$) showing how coherent coupling can counteract dephasing processes.

the BEC in the double-well potential as two separated matter-wave packets connected via tunneling through the barrier. In the following we will use the acronym for bosonic Josephson junction (BJJ) to describe this system. Within the two mode approximation the dynamics of the BJJ can be described by two conjugate variables, the atom number difference between the matter-wave packet on the left (l) and on the right (r) $\Delta n = (N_l - N_r)/2$ and their relative phase $\phi = \phi_r - \phi_l$ [9–12]. The Hamiltonian governing the evolution of the two conjugate variables in the limit of small Δn is given by

$$H = \frac{E_c}{2}\Delta n^2 - E_j \cos\phi, \qquad (1)$$

where E_c accounts for the atom-atom interaction in both condensates and E_j is the tunneling coupling energy resulting from the spatial overlap of the wave functions. This Hamiltonian also describes the classical motion of a particle with mass $1/E_c$ and momentum Δn at position ϕ in a periodic potential. In our experiments with temperatures $T > 10$ nK the quantum fluctuations [8,13–15] are small compared to the thermal fluctuations and therefore are neglected. Their influence can be estimated in the limit of small ϕ in which Eq. (1) can be approximated by a harmonic oscillator with the characteristic quantum mechanical energy splitting $\hbar\omega_p = \sqrt{E_c E_j}$, where ω_p is the plasma frequency, leading to the quantum mechanical fluctuations of both variables: $\langle\Delta n^2\rangle \approx \sqrt{E_j/4E_c}$ and $\langle\phi^2\rangle \approx \sqrt{E_c/4E_j}$.

The system variables E_j and E_c can be calculated from the experimental parameters. The trapping frequencies of the three-dimensional harmonic trap are $\omega_x = 2\pi \times 90(2)$ Hz and $\omega_{y,z} = 2\pi \times 100(2)$ Hz. The periodic potential of $V = V_0/2(1 + \cos(2\pi x/\lambda))$ is realized by the interference of two laser beams at a wavelength of 830 nm crossing under an angle of 10° resulting in a standing light wave with periodicity of $\lambda = 4.8(2)$ μm and is ramped up to a height of $V_0/h = 500$ to 2500 Hz. The number of atoms in the BEC fraction is chosen to be 2500(500). After the preparation of the BEC in the double-well trap, the relative phase of the two matter-wave packets is measured by analyzing the double-slit interference patterns formed after time of flight of 5 and 6 ms. The visibility of these patterns is reduced due to the short expansion time and the finite optical resolution of the imaging system. Further details of the experimental setup can be found in [16].

The relevant quantities can be calculated from these parameters using the improved two mode model [9]: E_c/k_B is on the order of 20 pK and E_j/k_B is between 30 pK and 400 nK [17], leading to $\hbar\omega_p/k_B$ being between 25 pK and 3 nK. Thus, both necessary conditions for the classical limit are fulfilled: $E_j \gg E_c$ leading to small quantum fluctuations of ϕ and $E_c \gg E_j/N^2$ (where N is the total number of atoms in the BEC) leading to small quantum fluctuations of $\Delta n/N$. Hence, our experiment can be discussed in the classical framework where the thermally induced phase fluctuations are closely analogous to the Brownian motion of a particle in a sinusoidal potential.

For a quantitative analysis in the thermodynamic limit at $k_B T \gg \hbar\omega_p$, the coherence factor [8] can be calculated by a thermal average assuming a Boltzmann distribution for the relative phases

$$\alpha = \langle \cos\phi \rangle = \frac{\int_{-\pi}^{\pi} d\phi \cos\phi \exp(E_j/k_B T \cos\phi)}{\int_{-\pi}^{\pi} d\phi \exp(E_j/k_B T \cos\phi)}. \quad (2)$$

Equation (2) points out that the relevant scaling parameter for thermal fluctuations is the ratio between thermal energy $k_B T$ and tunneling coupling energy E_j. Figure 3 shows the experimentally obtained coherence factors as a function of this scaling parameter. Every data point represents on average 40 measurements. In these experiments the temperature of the system is changed between 49 and 80 nK by evaporative cooling of the sample to the lowest temperature and subsequently increasing the temperature by holding the atoms in the trap for different times. The temperature of the sample is measured with the standard time-of-flight expansion method. The tunneling coupling energy is varied between 0.6 nK $\times k_B$ and 300 nK $\times k_B$ by adjusting the height of the potential barrier. E_j is obtained from numerical calculations using independently measured trap parameters and atom numbers. It is important to note that the recently developed improved two mode model [9] is used for these calculations because it leads to quantitative agreement between theoretical predictions and experimental measurements of dynamical quantities [16]. The solid line corresponds to the theoretical prediction of the classical model [Eq. (2)] where all parameters are determined independently. It also includes the fitting error of the relative phase which arises from the finite optical resolution and leads to a reduction of the coherence factor. As shown in Fig. 3 the general behavior of the coherence is confirmed over a 3 orders of magnitude variation of $k_B T/E_j$. These measurements reveal that the BJJ has a higher degree of coherence than expected. This deviation might possibly be explained by an increase of the tunneling coupling resulting from the excitation of transverse modes with higher energies which are neglected by the two mode approximation.

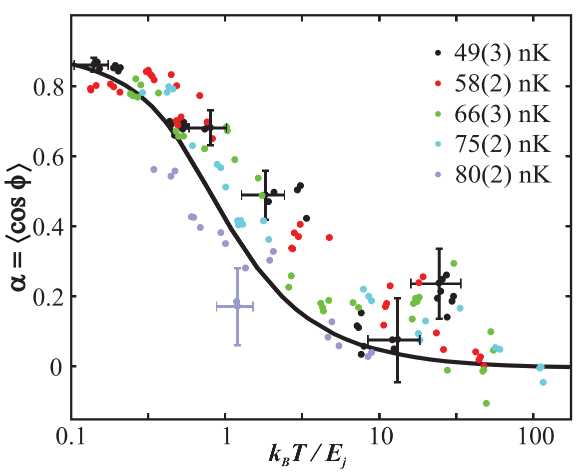

FIG. 3 (color). Scaling behavior of the coherence factor of a bosonic Josephson junction. Each point is obtained by averaging over the cosine of the phases of at least 28 (in average 40) measurements at the same experimental conditions. The coherence factor α is plotted as a function of the scaling parameter $k_B T/E_j$, which is varied over 3 orders of magnitude (49 nK $< T <$ 80 nK, 0.6 nK $< E_j/k_B <$ 300 nK). It shows good agreement with the theoretical prediction of the classical model Eq. (2) indicated by the solid line where also the uncertainty arising from the fitting error of the phase is taken into account. Typical error bars are shown which result from statistical errors and uncertainties of the experimental parameters (potential parameters, atom numbers, temperature).

Independent measurements have been performed for the lowest temperatures ($T = 15$ nK) to test for thermal equilibration. The measurements of α were compared for different E_j for two ramping schemes. The first scheme was ramping up the barrier in 1.3 s and the second scheme was holding the atoms for 1 s in the trap and then ramping up the barrier within 0.3 s. For $E_j/k_B > 1$ nK both schemes lead within the experimental errors to the same results. Thus for the fluctuation measurements the ramping in 300 ms is expected to be adiabatic with respect to the response time of the BJJ given by the inverse plasma frequency and thus ensures the thermal equilibrium.

In the following we present the application of the phase fluctuation measurements for thermometry far below the critical temperature of Bose-Einstein condensation (T_c). The temperature of the system can be directly deduced from the variance of the phase if the tunneling coupling is known. In order to apply the phase fluctuation measurements for thermometry we introduce an empirical effective tunneling coupling E_j^{eff} to account for effects beyond the classical approach. For the range of 25 nK $< E_j/k_B <$ 90 nK we deduce from the results shown in Fig. 3 that $E_j^{\mathrm{eff}} = 1.33 E_j$. The fundamental difference between this method and previous suggestions using phase fluctuations of elongated Bose-Einstein condensates for thermometry [18] is that the BJJ is not restricted to a quasi-one-dimensional situation but can be employed for all geometries. Furthermore, this method can be applied for all temperature ranges by tuning E_c and E_j such that thermal effects dominate and quantum fluctuations are negligible.

As a proof of applicability of this new type of thermometer we observe how the temperature of a BEC in a harmonic trap increases in time (see Fig. 4), which reveals clearly the effect of quantum statistics below the critical temperature. In these experiments the lowest temperatures ($T < T_c/3$) can only be measured with the phase fluctuation method since the thermal fraction is too small to be observed in time-of-flight measurements (less than 100 atoms with about 2500 atoms in the BEC fraction). For longer heating times the standard time-of-flight method

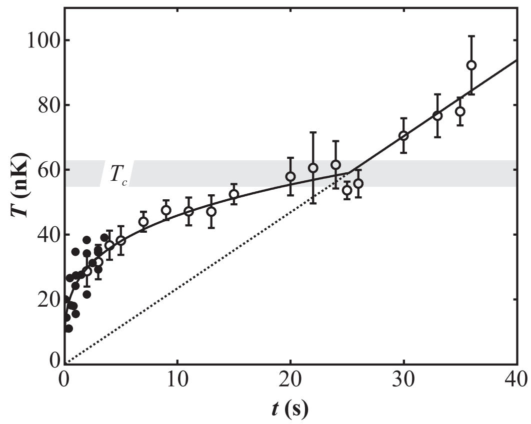

FIG. 4. Heating up of a Bose gas. The filled circles correspond to measurements employing the phase fluctuation method and the open circles to the results obtained with the standard time-of-flight method. The gray shaded region shows the critical temperature expected from the experimental parameters and their uncertainties. The solid line is a fitting function assuming a power law for the heat capacity $C \propto (T/T_c)^d$ of the Bose gas below the critical temperature $T_c = 59(4)$ nK, a constant heat capacity above the critical temperature, and a temperature independent transfer rate of energy. From this fit we deduce $d = 2.7(6)$, which is consistent with the theoretical prediction of $d = 3$ for an ideal Bose gas in a three-dimensional harmonic trap. The dashed line represents the expected behavior of an ideal classical gas for increasing temperature which makes the difference arising from quantum statistics evident.

can be applied and confirms the consistency of the two approaches in the overlap region. The solid line corresponds to a fitting function for the temperature where we assume a mean critical temperature of $T_c = 59$ nK (deduced from independent measurements), a temperature independent transfer rate of energy per particle and a power law for the temperature dependent heat capacity $C = (d+1)C_{\rm th}(T/T_c)^d$ where $C_{\rm th}$ is the heat capacity of a classical gas. The shown excellent agreement is obtained for a heating rate of 2.3(2) nK/s for a classical gas and $d = 2.7(6)$. Thus the expected exponent $d = 3$ for an ideal Bose gas in a three-dimensional harmonic trap [19] is experimentally confirmed. The expected increase of temperature of a classical gas is indicated by the dotted line and shows clearly the difference between the quantum and the classical behavior of ideal gases.

In summary, we have presented a quantitative analysis of thermally induced phase fluctuations in a bosonic Josephson junction. Our observations show that a universal scaling law describes the behavior of the coherence and its control leads to new applications. A method is presented for ultralow temperature measurements, with which we have confirmed that the heat capacity of a degenerate Bose gas vanishes in the zero temperature limit as predicted by the third law of thermodynamics [20].

We thank T. Bergeman very much for the numerical calculation of the relevant parameters and the valuable theoretical support. We would also like to thank B. Eiermann and T. B. Ottenstein for discussions and M. Cristiani, Th. Anker, and S. Hunsmann for their contributions to the experimental setup. This work was funded by Deutsche Forschungsgemeinschaft Schwerpunktsprogramm SPP1116 and by Landesstiftung Baden-Württemberg–Atomoptik. R. G. thanks the Landesgraduiertenförderung Baden-Württemberg for financial support.

*Electronic address: noisethermometry@matterwave.de

[1] M. R. Andrews *et al.*, Science **275**, 637 (1997).
[2] J. E. Simsarian *et al.*, Phys. Rev. Lett. **85**, 2040 (2000).
[3] T. Schumm *et al.*, Nature Physics **1**, 57 (2005).
[4] A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
[5] J. C. Davis and R. E. Packard, Rev. Mod. Phys. **74**, 741 (2002), and references therein.
[6] M. Albiez *et al.*, Phys. Rev. Lett. **95**, 010402 (2005).
[7] A. J. Leggett, Rev. Mod. Phys. **73**, 307 (2001).
[8] L. Pitaevskii and S. Stringari, Phys. Rev. Lett. **87**, 180402 (2001).
[9] D. Ananikian and T. Bergeman, Phys. Rev. A **73**, 013604 (2006).
[10] J. Javanainen, Phys. Rev. Lett. **57**, 3164 (1986).
[11] M. W. Jack, M. J. Collett, and D. F. Walls, Phys. Rev. A **54**, R4625 (1996).
[12] A. Smerzi, S. Fantoni, S. Giovanazzi, and S. R. Shenoy, Phys. Rev. Lett. **79**, 4950 (1997).
[13] F. Sols, Physica B (Amsterdam) **194–196**, 1389 (1994).
[14] A. Imamoglu, M. Lewenstein, and L. You, Phys. Rev. Lett. **78**, 2511 (1997).
[15] G.-S. Paraoanu, S. Kohler, F. Sols, and A. J. Leggett, J. Phys. B **34**, 4689 (2001).
[16] R. Gati *et al.*, Appl. Phys. B **82**, 207 (2006).
[17] The atom-atom interaction energy can be estimated by $E_c/k_B \approx 51\ \text{nK}(1 - \exp(\frac{-V_0/k_B - 27\ \text{nK}}{41\ \text{nK}}))/N$ and the tunneling coupling energy by $E_j/k_B \approx 196\ \text{nK} \times N \exp(\frac{-V_0/k_B - 27\ \text{nK}}{7.6\ \text{nK}})$ for $N \approx 2500$ and $24\ \text{nK} < V_0/k_B < 84$ nK, which is deduced from numerical simulations using [9].
[18] S. Dettmer *et al.*, Phys. Rev. Lett. **87**, 160406 (2001).
[19] S. R. De Groot, G. J. Hooyman, and C. A. ten Seldam, Proc. R. Soc. A **203**, 266 (1950).
[20] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1963), Vol. 1.