Density Profile of a Trapped Strongly Interacting Fermi Gas with Unbalanced Spin Populations

F. Chevy
Laboratoire Kastler Brossel, École Normale Supérieure, Paris, France
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We present a theoretical study of the density profile of a trapped strongly interacting Fermi gas with unbalanced spin populations. Making the assumption of the existence of a first order phase transition between an unpolarized superfluid phase and a fully polarized normal phase, we show good agreement with a recent experiment presented by Partridge *et al.*

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In the Bardeen-Cooper-Schrieffer (BCS) mechanism, the onset of superfluidity is associated with pairing between two fermionic species with matched Fermi levels. This scheme is relevant to systems like metal superconductors, superfluid ^{3}He, or ultracold gases, as observed in recent ground breaking experiments [1–6]. However, other physical systems, like magnetized superconductors or neutron stars, require the understanding of fermionic superfluidity in the presence of mismatched chemical potentials. The nature of superfluidity in these systems has been the subject of long-standing debate [7–9], which has been renewed by the opportunity to address this topic experimentally using gaseous samples. Various mechanisms were proposed to describe the ground state of an ensemble of fermions with mismatched Fermi levels: deformed Fermi surface [10], Fulde-Ferrell-Larkin-Ovshinnikov (FFLO) states, where Cooper pairs acquire finite momentum, and their generalization to trapped systems [11–13], interior gap superfluidity [14], or phase separation between a normal and a superfluid state through a first order phase transition [15–18]. When the strength of the interactions is varied, a complicated phase diagram mixing several of these scenarios is expected [19–21].

Extending the seminal observation of fermionic superfluidity in ultracold atom systems, two recent experiments [22,23] have started probing the regime of mismatched Fermi levels by cooling samples containing different atom numbers in each spin state. In this Letter we focus on the results presented in Ref. [23] where the authors studied the density profile of a gas of fermionic lithium when varying, in the regime of strong interactions, the population imbalance between the two trapped spin states. One of the most striking results is displayed in Fig. 1. It shows that the radius of the minority component is strongly reduced with respect to that of noninteracting gas with the same atom number. In this Letter, we propose an interpretation of this result on the basis of the existence of first order phase separation between the normal and superfluid components and the use of universality in the strong interaction regime. We show that these two ingredients are sufficient to provide a good quantitative agreement with experimental data. It is therefore complementary to recent studies of Refs. [24–27], who address the same topics using a more complicated formalism.

Our analysis is based on the assumption of the existence of a zero temperature first order phase transition between a fully polarized normal phase containing a single spin species and an "unpolarized" superfluid state composed of a balanced mixture of the two species. The existence of this phase transition was suggested by previous theoretical studies [15–17] and is based on the following qualitative argument: In the grand canonical ensemble, the chemical potentials μ_i of the two species are fixed. In the ground state, the system minimizes the grand-potential $\Xi = H - \sum_i \mu_i N_i$, where H is the Hamiltonian of the system and N_i is the population of species i. Let us now consider a situation where the chemical potentials are mismatched, with $\delta\mu = \mu_1 - \mu_2 > 0$. Promoting a particle from state 2 to state 1 decreases the grand potential by $\delta\mu$ but implies the breaking of a pair, hence increases the energy by the

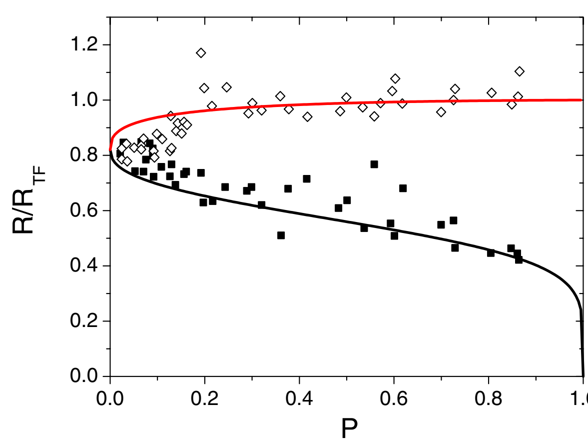

FIG. 1 (color online). Comparison between experimental data of Ref. [23] and the model presented here. $P = (N_1 - N_2)/(N_1 + N_2)$ is the imbalance between the two spin populations. The radius R of each component is normalized to the radius R_{TF} of the ideal Fermi gas containing the same atom number. Squares and diamonds correspond to experimental measurements of the radii of the minority and majority components, respectively. The full lines are the predictions of the first order transition model.

superfluid gap Δ. When $\delta\mu \lesssim \Delta$ we therefore expect the system to be unpolarized, while above this threshold a fully polarized system is obtained.

At zero temperature, the actual position of the phase transition can be found analytically in the case of a homogeneous unitary gas by matching chemical potentials and pressures in the two phases [17]. Indeed, we can write in the unpolarized phase $n_1 = n_2 = f(\mu_1, \mu_2)$. Using the thermodynamical identity $\partial_{\mu_1} n_2 = \partial_{\mu_2} n_1$, we deduce that f can actually be expressed as a function of $\mu = (\mu_1 + \mu_2)/2$ only. To obtain the exact expression for f, we then consider the case of matched Fermi surfaces, $\mu_1 = \mu_2 = \mu$. In this latter case, universality at the unitary limit allows one to write $\mu = \xi E_F$, where $E_F = \hbar^2(6\pi^2 n)^{2/3}/2m$ is the Fermi energy of a noninteracting gas of density $n = n_{1,2}$ and ξ is a universal parameter whose determination has attracted a lot of attention from both theoretical [17,28–30] and experimental groups [4,23,31–33] and is now thought to be $\xi \sim 0.45$. Using this expression for the chemical potential, we then deduce that

$$n_1 = n_2 = \frac{1}{6\pi^2}\left(\frac{m}{\xi\hbar^2}(\mu_1 + \mu_2)\right)^{3/2}. \quad (1)$$

The Gibbs-Duhem identity $dP_S = n_1 d\mu_1 + n_2 d\mu_2$ finally yields for the pressure in the superfluid phase

$$P_{\rm S} = \frac{1}{15\pi^2}\left(\frac{m}{\xi\hbar^2}\right)^{3/2}(\mu_1 + \mu_2)^{5/2}. \quad (2)$$

In the normal phase, we assume only the majority component is present. We have then an ideal gas constituted of particles of type 1 with chemical potential μ_1, thus giving

$$P_{\rm N} = \frac{1}{15\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\mu_1^{5/2}. \quad (3)$$

Equating $P_{\rm N}$ and $P_{\rm S}$ [34], we see that the two phases coexist only if μ_1 and μ_2 satisfy the condition $\mu_2/\mu_1 = \eta_{\rm c}$ with

$$\eta_{\rm c} = (2\xi)^{3/5} - 1 \sim -0.061, \quad (4)$$

a relation already found in Refs. [17,18]. In a trap, the chemical potential depends on position.

To compare with experiments, we now consider the case of a cloud of atoms trapped in a harmonic potential $V(\boldsymbol{r}) = m\sum_i \omega_i^2 x_i^2/2$. Without loss of generality, we will restrict our analysis to the case of a isotropic trap with frequency $\bar{\omega} = (\omega_x\omega_y\omega_z)^{1/3}$. We can indeed always recover the more general anisotropic case by making the scaling transform $x_i \longrightarrow \omega_i x_i/\bar{\omega}$.

To calculate the density profile of the cloud, we make use of the local density approximation, where we assume that the chemical potential of species i depends on position as $\mu_i(\boldsymbol{r}) = \mu_i^0 - V(\boldsymbol{r})$.

If we assume component 1 is the most populated, the inner superfluid region is defined by the condition $\mu_2(\boldsymbol{r})/\mu_1(\boldsymbol{r}) < \eta_c$ and is bounded by the radius R_2 defined by

$$R_2^2 = \frac{2}{m\bar{\omega}^2}\left(\frac{\mu_2^0 - \eta_{\rm c}\mu_1^0}{1 - \eta_{\rm c}}\right). \quad (5)$$

Atoms of the minority species are located in the paired superfluid phase only. We thus have

$$N_2 = \int_{r<R_2} n_2(\boldsymbol{r})d^3\boldsymbol{r} = \frac{2}{3\pi\xi^{3/2}}\left(\frac{\mu_1^0 + \mu_2^0}{\hbar\bar{\omega}}\right)^3 g(R_2/\bar{R}), \quad (6)$$

where $\bar{R}^2 = (\mu_1^0 + \mu_2^0)/m\bar{\omega}^2$ and

$$g(x) = \frac{x\sqrt{1-x^2}(-3 + 14x^2 - 8x^4) + 3\arcsin(x)}{48}. \quad (7)$$

Excess atoms of the majority species are located between $r = R_2$ and $r = R_1$ such that $m\bar{\omega}^2 R_1^2/2 = \mu_1^0$. The number of excess atoms is therefore $N_1 - N_2 = \int_{R_2}^{R_1} n_1(\boldsymbol{r})d^3\boldsymbol{r}$, hence

$$N_1 - N_2 = \frac{2}{3\pi}\left(\frac{2\mu_1^0}{\hbar\bar{\omega}}\right)^3[g(1) - g(R_2/R_1)]. \quad (8)$$

Dividing (8) by (6) yields the implicit equation for $\eta_0 = \mu_2^0/\mu_1^0$ as a function of N_1/N_2

$$\frac{N_1}{N_2} = 1 + \xi^{3/2}\frac{8}{(1+\eta_0)^3}\frac{g(1) - g(R_2/R_1)}{g(R_2/\bar{R})}. \quad (9)$$

Equation (9) is solved numerically and the value obtained for η_0 is then used to calculate the radii R_1 and R_2. The predicted evolution of the R_i versus the population imbalance $P = (N_1 - N_2)/(N_1 + N_2)$ is shown in Fig. 1. To follow Ref. [23], we have normalized each R_i to the Thomas-Fermi radius $R_{\rm TF}$ associated with an ideal gas containing N_i atoms. The agreement with the experimental data is quite good as soon as $P \gtrsim 0.1$, a remarkable result, since the model presented here contains no adjustable parameter, as soon as the value of ξ is known.

For weak population unbalance, experimental variations of the radii are flatter than predicted by the first order transition model. As proposed in Ref. [23], this suggests that the phase separation does not happen exactly at $P = 0$, but above some threshold $P_{\rm c} \sim 0.1$. This point is strengthened when one compares the theoretical and experimental density profiles. As in Ref. [23], we have represented in Fig. 2 the integrated column density $\tilde{n}_i(x) = \int dy n_i(x, y, 0)$ for $P = 0.57$ (as in Fig. 2.D of Ref. [23]). We immediately see in this figure that the transition between the two phases is very sharp, by contrast to what is observed experimentally. Finite temperature might explain this discrepancy. Indeed, for low population imbalance, the superfluid phase extends nearly throughout all the cloud, and, in particular, in regions where the density, hence the Fermi energy, is

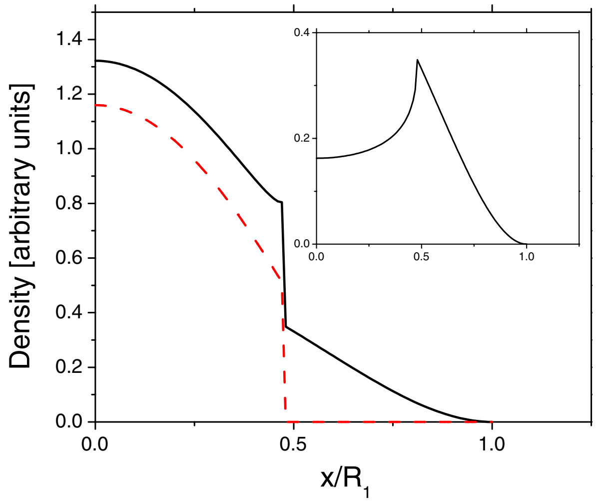

FIG. 2 (color online). Integrated column density of the majority (full line) and minority components (dashed line) for a population imbalance $P = 0.57$. Inset: excess column density $\tilde{n}_1 - \tilde{n}_2$. By contrast with the experimental results, the boundary between the two phases is here very sharp.

very low. In the unitary regime, the critical temperature for the superfluid-normal transition is given by the scaling $k_B T_c = \alpha E_F$, with $\alpha \sim 0.2$ [35]. It may then happen that near the cloud edge, the temperature T of the sample becomes locally larger than T_c. If this condition is satisfied at a radius R_c smaller than the demixing radius R_2, then the first order transition to the fully unpaired state will not happen. If we introduce the Fermi temperature $T_F = \hbar\bar{\omega}(6N_1)^{1/6}/k_B$ associated with a number of atoms N_1, the superfluid-normal and paired-unpaired transitions will happen at the same position if T/T_F satisfies the condition

$$\frac{T}{T_F} = \frac{\alpha}{2\xi}\left(\frac{R_1}{R_{TF}}\right)^2\left[1 + \eta_0 - 2\left(\frac{R_2}{R_1}\right)^2\right]. \qquad (10)$$

In Fig. 3, we have plotted as a function of temperature the evolution of the critical imbalance under which no demixing is expected. Experimentally, demixing only happens in the conditions of Ref. [23] for $P \gtrsim 0.1$, which, according to Eq. (10), corresponds to $T/T_F \lesssim 0.1$, a value compatible with experimental data. Despite a semiqualitative agreement, this finite temperature argument needs to be clarified by a more careful analysis, following, for instance, the work presented in Ref. [25]. Other scenarios can also be envisioned to explain the smooth crossover between the two phases, such as the existence of an intermediate phase—e.g., a gapless or FFLO phase as proposed in Ref. [20]—or a breakdown of the local density approximation due to the fast variation of the density profile. Nevertheless, the good agreement between theory and experiment for the data presented in Fig. 1 suggests that the crossover region between paired and unpaired phases should remain relatively narrow.

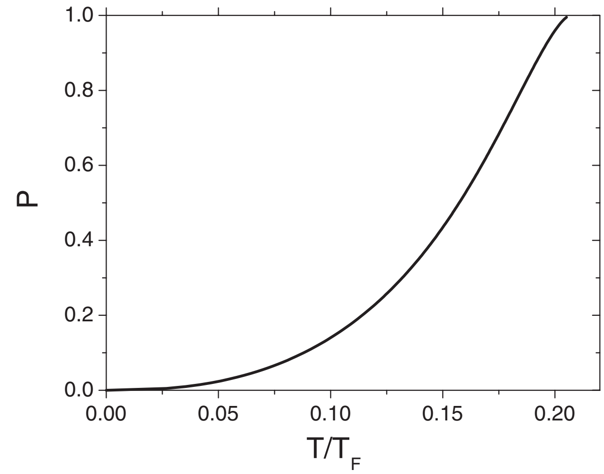

FIG. 3. Critical population imbalance under which finite temperature effects overrule demixing. The observed experimental threshold $P \sim 0.1$ corresponds to a temperature $T/T_F \lesssim 0.1$.

From the analysis presented here, it appears that the observations of Ref. [23] are consistent with a scenario of a transition between a fully paired and a fully polarized phase separated by a narrow crossover region probably related to finite temperature effects breaking the superfluid phase before demixing may occur. Interestingly, if this thermal scenario were confirmed the measurement of the critical population unbalance under which no phase separation happens could provide a useful tool for fermion thermometry. Another issue not addressed here is related to the superfluid character of the system when the population imbalanced is varied. This is especially important if one wishes to understand the results presented in Ref. [22] where it is shown that a mismatch of the Fermi surfaces by about 50% leads to a breakdown of superfluidity.

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