Superradiant Spin-Flip Radiative Emission of a Spin-Polarized Free-Electron Beam

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Radiative emission from the magnetic moments of the spins of an electron beam has never been observed directly, because it is fundamentally much weaker than the electric charge emission. We show that the detectivity of spin-flip and combined spin-flip–cyclotron-resonance-emission radiation can be substantially enhanced by operating with ultrashort spin-polarized electron beam bunches under conditions of *superradiant* (coherent) emission. The proposed superradiant spin-flip radiative emission scheme can be used for noninvasive diagnostics of polarized electron or positron beams. Such beams are of relevance in important scattering experiments off nucleons in nuclear physics and off magnetic targets in condensed matter physics.

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All electron beam radiation sources, in a spectral range stretching from the microwave to x rays, are based on interaction of the free-electron charge with the electric field of an electromagnetic (EM) wave. These include spontaneous emission schemes such as Cherenkov and synchrotron radiation, as well as stimulated emission schemes such as microwave tubes and free-electron laser (FEL) [1,2].

Electrons (as well as positrons and other particles) are also endowed with magnetic moment due to their spin, and therefore can emit EM radiation by interaction with a magnetic field. This interaction is weak relative to the electric interaction, and therefore spontaneous spin-flip radiative emission by *free electrons* has never been observed directly. Its occurrence is manifested when electrons (or positrons) are subjected to magnetic force in a storage ring, where its cumulative effect polarizes (transversely) the spin state of the circulating charged particles beam [3,4]. However, there is no fundamental restriction for its *direct* observation.

In the present Letter the characteristics and the conditions for observation of spontaneous free-electron spin-flip emission of radiation (FESFER) from a transversely polarized electron beam are presented. We show that if the polarized e beam is bunched, the emission of the FESFER radiation can be significantly enhanced by the process of superradiance (SR) [5–8]. Besides the fundamental interest in observing this effect, it may be useful for noninvasive diagnostics of spin-polarized e beams.

Recent progress in the development of spin-polarized photocathode e-gun injectors [9] for RF-LINAC accelerators led to important measurements and discoveries in nuclear physics (e.g., [10,11]). The electron beams, produced in such photocathode e guns, are photoemitted from a crystalline semiconductor cathode illuminated by a picosecond laser. They preserve their pure quantum spin states ($\pm 1/2$) relative to the axial (emission) direction during the emission and acceleration. Net polarization levels $(P_\uparrow - P_\downarrow)/(P_\uparrow + P_\downarrow)$ as high as 80% may by achieved [9]. The axial polarization of the spins can be subsequently rotated and transformed to transverse polarization by a combination of static magnetic and electric field deflectors. In a separate publication we have suggested that this polarization rotation can also be done with an external electromagnetic wave at the electron spin resonance (ESR) condition, without diverting the axial propagation transport line of the beam [12]. It is shown here that the superradiant radiation pulses, which are emitted by such transversely polarized e beam bunches upon traversal through an axial magnetic field, can be observable and possibly can be used to indicate the polarization level of the electron beam.

The classical magnetic moment $\boldsymbol{\mu}_j'$ of a particle j with spin, propagating on an axis (z) parallel to a uniform magnetic field $\mathsf{B} = B_0\hat{\mathbf{e}}_z$ with velocity $\boldsymbol{\beta} = \beta\hat{e}_z$ and energy γmc^2 precesses in its relativistic rest frame at the ESR frequency [13,14] (note $B_z' = B_z$):

$$\boldsymbol{\omega}_{s0}' = \frac{ge}{2m}\mathbf{B}_0 \tag{1}$$

$$\boldsymbol{\mu}_{\perp j}'(t) = \mathrm{Re}[\mu_{s\perp j}'\hat{\mathbf{e}}_+ e^{i\omega_{s0}(t-t_{0j})+i\varphi_{s0j}}]. \tag{2}$$

Here $g = 2.002\,3193$ is the Lande g factor, $\omega_{s0} = \omega_{s0}'/\gamma$, $\hat{\mathbf{e}}_+ = (\hat{\mathbf{e}}_+ + i\hat{\mathbf{e}}_y)/\sqrt{2}$, t_{0j} is the entrance time of particle j to the magnetic field region, φ_{s0j} its initial precession phase, $\mu_{s\perp j}' = \mu_s \sin\Psi_j$, $\mu_{szj}' = \mu_s \cos\Psi_j = \mathrm{const}$, and

$$\mu_s = \frac{\hbar ge}{2m}S = \mu_B gS, \tag{3}$$

where μ_B is the Bohr magneton. For an electron or a positron $S = 1/2$, and $\mu_s = 9.2848 \times 10^{-24}$ J/T. The SI units system is used throughout.

Assuming a negligible quantum recoil effect, the electron spin, precessing at frequency ω_{s0}', emits an EM radiation wave (a waveguide mode or a free space plane wave) at the same central emission frequency ω_{s0}' in its relativistic rest frame:

$$\mathbf{E}'(\mathbf{r}', t') = \mathrm{Re}\{\tilde{\mathbf{E}}(\mathbf{r}_\perp')e^{ik_{z0}'z'-i\omega_{s0}'t'}\}. \tag{4}$$

The frequency of this wave is seen in the laboratory frame as

$$\omega_r(\Theta) = \frac{\omega'_{s0}/\gamma}{1 - \beta\cos\Theta}, \quad (5)$$

where $\cos\Theta = k_z/k$ ($k = \omega_r/c$). For $\Theta = 0$ (forward emission of a TEM wave), the radiation is Doppler upshifted to $\omega_{r0} = \omega_r(0) = (1 + \beta)\gamma\omega'_{s0}$. With commonly available magnetic fields -0.5 Tesla (for normal magnets) or 10 Tesla (with superconducting magnets)—and moderate acceleration to $E = 100$ MeV, the forward FESFER emission can be in the IR up to the visible regime ($\lambda_{r0} = 15 - 0.75$ μm).

To analyze the spin-flip radiative emission of free electrons in a finite length axial magnetic field, we use a modal expansion of Maxwell equations in the frequency domain:

$$\{\breve{\mathbf{E}}(\mathbf{r}, \omega), \breve{\mathbf{H}}(r, \omega)\} = \sum_q \breve{C}_q(z)\{\tilde{\mathbf{E}}_q(\mathbf{r}), \tilde{\mathbf{H}}_q(\mathbf{r})\}, \quad (6)$$

where $\{\tilde{\mathbf{E}}_q(\mathbf{r}), \tilde{\mathbf{H}}_q(\mathbf{r})\} = \{\tilde{\mathsf{E}}_q(\mathbf{r}_\perp), \tilde{\mathsf{H}}_q(\mathbf{r}_\perp)\}e^{ik_{zq}z}$ is a set of eigenmodes of the structure (waveguide or free space) in which radiation emission takes place, and $\breve{f}(\omega) \equiv \int_{-\infty}^{\infty} e^{i\omega t} f(t)dt$.

Extending the formulation of [7] to include magnetic currents, the total spectral radiative energy emitted from a bunch of N electrons is

$$\frac{dW_q}{d\omega} = \frac{1}{8\pi \mathsf{P}_q}\left|\sum_{j=1}^{N}(\Delta\breve{W}^e_{qj} + \Delta\breve{W}^m_{qj})\right|^2, \quad (7)$$

where P_q is the normalization power of mode q and

$$\Delta\breve{W}^e_{qj} = -e\int_{-\infty}^{\infty} \mathbf{v}_j(t)\cdot\tilde{\mathbf{E}}^*_q[\mathbf{r}_j(t)]dt, \quad (8)$$

$$\Delta\breve{W}^m_{qj} = \int_{-\infty}^{\infty} \boldsymbol{\mu}_j(t)\cdot\tilde{\mathbf{H}}^*_q[\mathbf{r}_j(t)]dt. \quad (9)$$

Concentrating now on the FESFER term [second term in (7) squared], using (2) and (4) in (9) and setting $z_j(t) = v(t - t_{0j})$, one obtains an explicit expression for the average FESFER radiation spectral energy per mode q:

$$\frac{dW^m_q}{d\omega} = \frac{1}{8\pi\mathsf{P}_q}|\Delta\breve{W}^m_q|^2\left\langle\left|\sum_{j=1}^{N} e^{i\omega_{s0}t_{0j} + i\varphi_{s0j}}\sin\Psi_j\right|^2\right\rangle_{\perp s}. \quad (10)$$

Assuming all particles have the same trajectories (staying on the magnet axis $0 < z < L$), their common magnetic work function squared is:

$$|\Delta\breve{W}^m_q|^2 = \frac{1}{2}\left(\frac{\omega'_{s0}L}{\gamma v}\mu_0\mu_s|\tilde{\mathsf{H}}_{q+}(x_e, y_e)|\right)^2 \mathrm{sinc}^2(\theta_s L/2), \quad (11)$$

$$\theta_s \equiv \frac{\omega - \omega'_{s0}/\gamma}{v} - k_z(\omega) = 2\pi\frac{\omega - \omega_r(\Theta)}{\Delta\omega(\Theta)}, \quad (12)$$

where $\tilde{\mathsf{H}}_{q+}$ is the transverse right-hand circular polarization component of mode $\tilde{\mathbf{H}}_q$, and $\omega_r(\Theta)$ is given by (5). The finite interaction length homogeneous broadening linewidth of (12) at a fixed direction is:

$$\Delta\omega(\Theta)/\omega_r(\Theta) = \gamma\beta c/f_{s0}{}'L. \quad (13)$$

The radiation pattern of (11) as function of Θ is shown in Fig. 1. It is valid for emission in free space or in a waveguide. In a waveguide the "zigzag" angles of the radiation modes are discretized: $\cos\Theta_q = k_{zq}(\omega)/k = (k^2 - k^2_{c0q})^{1/2}/k$, where k_{c0q} is the cutoff wave number of mode q. The linewidth expression (13) is valid only when waveguide dispersion is negligible (away from zero slippage), otherwise the linewidth is wider [8]. In free space one can either use a set of discrete modes like the Hermit-Gauss set [15] or extend the modal expansion (6) to integration over continuous radiation modes (plane waves) [16]: $\{\tilde{\mathsf{E}}_q(\mathbf{r}_\perp), \tilde{\mathsf{H}}_q(\mathbf{r}_\perp)\} \propto \exp(i\mathbf{k}_\perp\cdot\mathbf{r}_\perp)$. This would lead to an expression for the optical radiant intensity $dW/d\omega d\Omega$ [instead of (10)], which is also proportional to the radiation pattern (11) displayed in Fig. 1.

The polar coordinates radiation pattern of Fig. 1 indicates that most of the radiation is emitted in the forward direction. For a relativistic beam most of the total emission in a wide frequency bandwidth is into a cone of $\Delta\Theta = 2/\gamma$ opening angle. The monochromatic spatially coherent radiation is emitted into a smaller angle cone of $\Delta\Theta_{\rm coh} = 2\sqrt{\lambda_{r0}/L}$ (see shaded section in Fig. 1). The number of coherent photons emission in the forward direction (spatially coherent emission into a single mode $q = 0$ and a pulse Fourier-transform limited bandwidth $|\omega - \omega_{r0}| < \Delta\omega/2$) is found from (10), (11), and (13) for $\theta_s \cong 0$:

$$(N_{\rm ph})_{\rm coh} = \frac{1}{\hbar}\frac{dW_0}{d\omega}\Delta\omega = \frac{\pi}{\hbar}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\mu_s^2 f'_{s0}L}{A_{\rm em}c^3\gamma}\langle\ldots\rangle_{\perp s}. \quad (14)$$

Here we used the definition $A_{\rm em} \equiv \mathsf{P}_q/[1/2\sqrt{\mu_0/\varepsilon_0}\times|\tilde{\mathsf{H}}_{q+}(x_e, y_e)|^2]$. In free space the diffraction limited area of the fundamental Gaussian mode [7] is $A_{\rm em} = \lambda_{r0}L/4$. The approximate wide-band total emission (into a $\Delta\Theta = 2/\gamma$ opening angle cone in free space) is calculated by

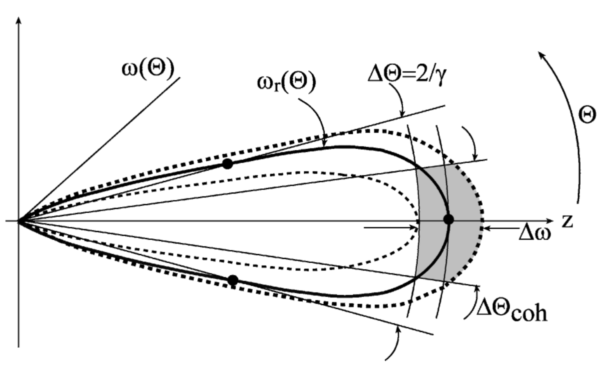

FIG. 1. The FESFER radiation pattern. The shaded section represents the phase-space region of coherent emission.

multiplying (14) by a factor $\frac{1}{4}(\Delta\Theta)^2/(\Delta\Theta_{\rm coh}^2) = Lf'_{s0}/4\gamma$:

$$(N_{\rm ph})_{\rm tot} \cong \frac{2\pi}{\hbar}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\mu_s^2 f'^3_{s0} L}{c^5\gamma}\langle\ldots\rangle_{\perp s}. \qquad (15)$$

The factor $\langle\ldots\rangle_{\perp s}$ is the average resultant transverse magnetic moment of all the particles in the bunch. It is maximal when the individual magnetic moments are all polarized transversely ($\Psi_j = \pi/2$). It is further significantly enhanced if the precession phases φ_{s0j} and entrance times t_{0j} are correlated. In this case the entire bunch can emit coherently intense superradiant radiation [5–7].

If the particles enter the magnet at random initial times t_{0j} or random precession phases φ_{s0j}, the mixed terms in $\langle\ldots\rangle_{\perp s}$ cancel out and, consequently,

$$\langle\ldots\rangle_{\perp s} = N\langle\sin^2\Psi_j\rangle = \tfrac{2}{3}N. \qquad (16)$$

If we consider a case where the electron beam is tightly bunched: $\omega\langle(t_{0j} - t_0)^2\rangle^{1/2} \ll 2\pi$, and all particles have the same initial precession phase $\langle(\varphi_{s0j} - \varphi_0)^2\rangle^{1/2} \ll 2\pi$, then their FESFER emissions add up in phase (superradiant emission [7]), and, consequently, $\langle\ldots\rangle_{\perp s} = N^2$. In practice the electron beam is partially polarized: $N_\uparrow$ electrons are emitted from the photocathode at axial "spin-up" pure quantum state, $N_\downarrow$ at "spin-down," and N_r—at random spin orientation. After rotation to transverse-spin polarization, the corresponding first two groups of pure spin-state electrons appear as two transverse-spin giant magnetic dipoles $|\boldsymbol{\mu}_\uparrow| = N_\uparrow\mu_s$, $|\boldsymbol{\mu}_\downarrow| = N_\downarrow\mu_s$ of opposite orientation: $\varphi_{s0\uparrow} = \varphi_{s0\downarrow} - \pi$, and then

$$\langle\ldots\rangle_{\perp s} = N^2(P_\uparrow - P_\downarrow)^2 + \frac{2}{3}NP_r, \qquad (17)$$

where $P_\uparrow = N_\uparrow/N$, $P_\downarrow = N_\downarrow/N$, and $P_r = N_r/N$. Equations. (14) and (15) with $P_\uparrow = P_\downarrow = 0$, $P_r = 1$ in (17), are, respectively, the expressions for coherent and total *spontaneous* (shot noise) FESFER radiation emission [17]. The same equations with $P_r = 0$, are the corresponding expressions for *superradiant* FESFER. They are enhanced then by a significant factor of $3N(P_\uparrow - P_\downarrow)^2/2$.

The spontaneous FESFER power is miniscule. Taking an example of $B_0 = 10$ T, $L = 1$ m, $\gamma = 10$ ($\lambda_{r0} = 8.6$ μm), a high charge electron bunch of $q = 1$ nC ($N = 6.25 \times 10^9$), and a bunch repetition rate of 1 GHz, we obtain for the coherent and total *classical* spontaneous photon emissions, respectively (14)–(16): $(N_{\rm ph})_{\rm coh}^{\rm sp} = 1.2 \times 10^{-8}$ ph/bunch, $(N_{\rm ph})_{\rm tot}^{\rm sp} = 7.2 \times 10^{-6}$ ph/bunch, $d/dt(N_{\rm ph})_{\rm coh}^{\rm sp} = 12$ ph/sec, $d/dt(N_{\rm ph})_{\rm tot}^{\rm sp} = 7200$ ph/sec.

The FESFER emission is enhanced by a big factor X $3N/2$, when all electrons emit superradiantly. However, this requires that the electron bunch duration will be short relative to the radiation period: $f_{r0}t_b < 1$. With the present technological state of the art, the available bunch duration is $t_b = 0.1$–1 ps, therefore, we consider an example of low frequency FESFER emission $f_{r0} = 1$ THz ($\lambda_{r0} = 300$ μm), which can be attained with $B_0 = 0.5$ T, $\gamma = 6$. Setting now in (14) and (15) $\langle\ldots\rangle_{\perp s} = N^2$, one obtains $(N_{\rm ph})_{\rm coh}^{\rm SR} = 0.19$ ph/bunch, $(N_{\rm ph})_{\rm tot}^{\rm SR} = 0.56$ ph/bunch, $d/dt(N_{\rm ph})_{\rm coh}^{\rm SR} = 1.9 \times 10^7$ ph/sec, $d/dt(N_{\rm ph})_{\rm tot}^{\rm SR} = 5.6 \times 10^8$ ph/sec.

The calculated flux of FESFER photons emission may be detectable, especially with SR enhancement. However the real obstacle for direct observation of FESFER is the concurrent occurrence, at a higher emission rate of cyclotron resonance emission (CRE) [18] by electrons that enter the axial magnetic field section with any transverse velocity $\beta_{\perp j}$. Direct calculation of $\Delta\check{W}^e_{qj}$ (8) for an electron entering the uniform axial magnetic field section at time t_{0j}, with the initial gyration phase φ_{c0j} results in

$$\Delta\check{W}^e_{qj} = |\Delta\check{W}^e_q| e^{i(\omega_{c0}t_{0j} + \varphi_{c0j})}, \qquad (18)$$

$$\omega_{c0} = \omega'_{c0}/\gamma, \qquad \omega'_{c0} = e/mB_0. \qquad (19)$$

The magnetic and electric work functions can be shown to be related by simple proportion:

$$\alpha = |\Delta\check{W}^m_q/\Delta\check{W}^e_q| = \hbar\omega'_{c0}/2\gamma{\rm mc}^2 \ll 1. \qquad (20)$$

Keeping both terms of (7), we can write

$$\frac{dW_q}{d\omega} = \frac{1}{8\pi\mathcal{P}_q}|\Delta\check{W}^e_q|^2_{\max}\langle|\sum_j e^{i\omega t_{0j}}[\beta_{\perp j}e^{i\varphi_{c0j} + i\theta_c L/2}{\rm sinc}(\theta_c L/2) + \alpha\sin\Psi_j e^{i\varphi_{s0j} + i\theta_s L/2}{\rm sinc}(\theta_s L/2)]|^2\rangle, \qquad (21)$$

where $|\Delta\check{W}^e_q|_{\max} = (eL/\sqrt{2}\beta)|\tilde{\mathbf{E}}^*_{q\perp}\cdot\hat{\mathbf{e}}_+|$. θ_c is given by (12) and (5) with $\omega_{c0}{}'$ (19) substituting $\omega_{s0}{}'$ (1).

The CRE center frequency [corresponding to $\theta_c(\omega_r) = 0$] is given by (5) with $\omega_{c0}{}'$ substituting $\omega_{s0}{}'$. Unfortunately the difference between the FESFER and CRE frequencies $\delta\omega/\omega_r = (\omega'_{s0} - \omega'_{c0})/\omega'_{c0} = g/2 - 1 = 1.16 \times 10^{-3}$, is much smaller than the emission frequency linewidth (13) for practical magnet lengths L. Consequently, it is hard to separate the emission lines of the pure CRE [first term in (21) squared] and the FESFER (second term squared) by frequency filtering.

If the CRE and FESFER wave phases are uncorrelated, the mixed CRE/FESFER term resulting from squaring the brackets in (21) vanishes. The FESFER term (10) is then much smaller than the pure CRE term:

$$(dW^m_q/d\omega)/(dW^e_q/d\omega) = \alpha^2\langle\ldots\rangle_{\perp s}/\langle\ldots\rangle_{\perp c} \ll 1. \qquad (22)$$

It can be substantially enhanced if the FESFER is *superradiant*, $[\langle(\varphi_{s0j} - \varphi_{s0})^2\rangle^{1/2}$, $\omega_r t_b \ll 2\pi]$ but the CRE is *spontaneous* (namely the entrance gyration phases φ_{c0j} are random). In this case $\langle\ldots\rangle_{\perp s}/\langle\ldots\rangle_{\perp c} = N/\langle\beta^2_\perp\rangle$. But realizing the condition for random φ_{c0j} means a requirement for very good alignment of the beam and the magnetic field, and attaining the maximal enhancement factor requires small beam angular spread ($\langle\beta_\perp^2\rangle$). Thus technological limitations make it difficult to obtain dominant FESFER emission [namely, (22) bigger than 1].

One can consider an alternative way for measuring FESFER that circumvents the difficulty of the CRE background, and turns it into a helpful means. The mixed term in (21), being proportional to α, is much larger than the FESFER term, which is proportional to α^2 (22). If one injects the electron beam bunch into the magnet at conditions of superradiant emission of *both* CRE and FESFER, the mixed term signal can be measured with proper signal processing.

Since the CRE is by nature synchronous with the FESFER, and coherently related to it (as long as $\omega t_b < 2\pi$), it can serve as a "local oscillator" in a heterodyne detection scheme of the FESFER radiation (in this case it is desirable to enhance the CRE emission by slight angular beam deflection, so that $\langle\ldots\rangle_{\perp c} = \beta_{\perp 0}^2 N^2$). To explore this possibility it is necessary to transform (6) to the time domain. Following [19] one obtains

$$\mathbf{E}(\mathbf{r}_\perp, L, t) \propto N|\tilde{\mathsf{E}}_{q+}(\mathbf{r}_\perp)| \cdot f(t - t_0 - L/v) \times \mathrm{Re}\{[\beta_{\perp 0} e^{i\omega_{rc0}(t-t_0-L/v)+i\varphi_{c0}} + i\alpha(P_\uparrow - P_\downarrow)e^{i\omega_{rs0}(t-t_0-L/v)+i\varphi_{s0}}]\hat{\mathbf{e}}_+\}, \quad (23)$$

where $f(t) \cong \mathrm{rect}(t/t_{\mathrm{sl}})$ is the wave packet envelope function and $t_{\mathrm{sl}} = 2\pi/(\Delta\omega)$ is the radiation slippage time. When this field is detected by a square-law detector, the measured signal C will be proportional to

$$|\mathbf{E}(\mathbf{r}_\perp, L, t)|^2 \propto N^2|\tilde{\mathsf{E}}_{q+}(\mathbf{r}_\perp)|^2 f^2(t - t_0 - L/v)\{\beta_{\perp 0}^2 + \alpha\beta_{\perp 0}(P_\uparrow - P_\downarrow)\sin[\delta\omega(t - t_0 - L/v) + \varphi_{c0} - \varphi_{s0}]\}. \quad (24)$$

Since for practical parameters, $\delta\omega t_{\mathrm{sl}} = 2\pi\delta\omega/\Delta\omega \ll 2\pi$, there will not be CRE-FESFER beat oscillation in a single pulse. Yet, similar beat waveforms are expected in all pulses, if good beam stability can be maintained. Therefore the beat signal [second term in (24)] can be distinguished from the first term after processing and averaging over many pulses. For example, consider adjusting the initial cyclotron gyration and spin resonance precession phases to be in phase: $\varphi_{c0} = \varphi_{s0}$, or out of phase: $\varphi_{c0} = \varphi_{s0} - \pi$. If $\delta\omega t_{\mathrm{sl}} \ll 2\pi$, the first order Taylor expansion of the second term in (24) is $\pm\alpha\beta_{\perp 0}(P_\uparrow - P_\downarrow)\delta\omega(t - t_0 - L/v)$. Differentiation of the signal (24) will null the contribution of the first term and leave the second term. Signal averaging over many pulses, along with modulation of the cyclotron or spin phase or amplitude, and corresponding correlated processing, can reveal the FESFER/CRE beat signal out of random noise.

In conclusion, FESFER emission is weak, but its observation is not fundamentally prohibited. It can be substantially enhanced at superradiance emission conditions. A promising method for detecting and measuring FESFER is by heterodyne detection of its beat with the concurrent CRE radiation which slightly deviates in frequency because of the gyromagnetic factor g. The FESFER measurement may be used for noninvasive diagnosis of the spin polarization state of polarized electron beams. Finally, it is pointed out that the expressions for superradiance, derived here classically, are consistent with Dicke's classical limit for $N \gg 1$ [5]. However, the expressions for spontaneous FESFER emission require a quantum electrodynamic correction.

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