Supersymmetric Standard Model from the Heterotic String

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We present a $\mathbb{Z}_6$ orbifold compactification of the $E_8 \times E_8$ heterotic string which leads to the (supersymmetric) standard model gauge group and matter content. The quarks and leptons appear as three **16**-plets of SO(10), whereas the Higgs fields do not form complete SO(10) multiplets. The model has large vacuum degeneracy. For generic vacua, no exotic states appear at low energies and the model is consistent with gauge coupling unification. The top quark Yukawa coupling arises from gauge interactions and is of the order of the gauge couplings, whereas the other Yukawa couplings are suppressed.

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The problem of ultraviolet completion of the (supersymmetric) standard model (SM) has been a long-standing issue in particle physics. The most promising approach is based on string theory; however, explicit models usually contain exotic particles and suffer from other phenomenological problems. The purpose of this Letter is to show that these difficulties can be overcome in the well-known weakly coupled heterotic string [1] compactified on an orbifold [2,3]. The emerging picture has a simple geometrical interpretation.

In the light cone gauge the heterotic string can be described by the following bosonic world sheet fields: 8 string coordinates X^i, $i = 1 \dots 8$, 16 internal left-moving coordinates X^I, $I = 1 \dots 16$, and 4 right-moving fields ϕ^i, $i = 1 \dots 4$, which correspond to the bosonized Neveu-Schwarz-Ramond fermions [cf. [4]]. The 16 left-moving internal coordinates are compactified on a torus. The associated quantized momenta lie on the $E_8 \times E_8$ root lattice. The massless spectrum of this 10D string is 10D supergravity coupled to $E_8 \times E_8$ super Yang-Mills theory.

To get an effective four-dimensional theory, 6 dimensions of the 10D heterotic string are compactified on an orbifold. A $\mathbb{Z}_N$ orbifold is obtained by modding a 6D torus together with the 16D gauge torus by a $\mathbb{Z}_N$ twist, $\mathcal{O} = \mathbb{T}^6 \otimes \mathbb{T}_{E_8 \times E_8'}/\mathbb{Z}_N$. On the three complex torus coordinates z^i, $i = 1, 2, 3$, the $\mathbb{Z}_N$ twist acts as $z^i \to e^{2\pi i v_N^i} z^i$, where $N v_N$ has integer components and $\sum_i v_N^i = 0$. This action is accompanied by the shifts of the bosonized fermions ϕ^i and the gauge coordinates X^I, $\phi^i \to \phi^i - \pi v_N^i$, $X^I \to X^I + \pi V_N^I$ where $N V_N$ is an $E_8 \times E_8$ lattice vector. Further, if discrete Wilson lines W_l^I are present [3], the torus lattice translations are accompanied by the shifts $X^I \to X^I + \pi n_l W_l^I$ with integer n_l. In addition to the requirement that $N V_N$ and nW, where n is the order of the Wilson line, be $E_8 \times E_8$ lattice vectors, V_N and W are constrained by modular invariance [cf. [5,6]]. At each fixed point of the orbifold, a local $\mathbb{Z}_N$ twist, composed of V_N and discrete Wilson lines, breaks $E_8 \times E_8$ to a subgroup [cf. [7]]. Given an orbifold, a torus lattice together with the shift V_N and the Wilson lines, the massless spectrum of the orbifold can be calculated. It is supersymmetric by construction and consists of the states which are invariant under the twisting and lattice translations.

In our construction, we choose a $\mathbb{Z}_6$-II orbifold based on a Lie torus lattice $G_2 \times SU(3) \times SO(4)$ (Fig. 1) with a twist vector $v_6 = (\frac{1}{6}, \frac{1}{3}, -\frac{1}{2})$ [5]. In addition to V_6, we require two Wilson lines: one of order two in the SO(4) plane, and another of order three in the SU(3) plane. In an orthonormal basis, the shift and the Wilson lines are given by

$$V_6 = (\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{3}, 0, 0, 0, 0, 0)(\tfrac{1}{3}, 0, 0, 0, 0, 0, 0, 0),$$
$$W_2 = (\tfrac{1}{2}, 0, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, 0)(-\tfrac{3}{4}, \tfrac{1}{4}, \tfrac{1}{4}, -\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, -\tfrac{1}{4}),$$
$$W_3 = (\tfrac{1}{3}, 0, 0, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3})(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, 0, 0, 0, 0).$$

The model has 12 fixed points which come in six inequivalent pairs, with the local groups

$$SO(10) \times SO(4), \quad SO(8) \times SO(6), \quad SO(12),$$
$$SO(8)' \times SO(6)', \quad SU(7), \quad SO(8)'' \times SO(6)''$$

up to U(1) factors and subgroups of the second E_8. The standard model gauge group, $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$, is obtained as an intersection of those groups. The surviving gauge group in 4D is

$$G = G_{SM} \times [SO(6) \times SU(2)] \times U(1)^8, \qquad (1)$$

where one of the U(1)'s is anomalous, and the brackets indicate a subgroup of the second E_8. The matter multiplets are found by solving the masslessness equations together with the twist- and translation-invariance conditions. The

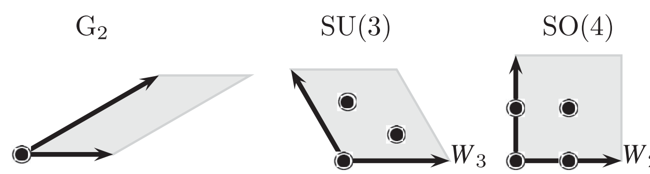

FIG. 1. $G_2 \times SU(3) \times SO(4)$ torus lattice of a $\mathbb{Z}_6$-II orbifold.

resulting spectrum includes both untwisted (U, "bulk") and twisted (T_k, "localized") states, and is given in Table I. The twisted states can belong to any of the twisted sectors T_k ($k = 1, 2, 3, 4$) depending on their string boundary conditions. There are no left-chiral superfields in the T_5 sector.

Let us now discuss some properties of the spectrum. We first note that the V_6 shift is chosen such that the local gauge symmetry at the origin is SO(10) × SO(4) × U(1) and the twisted matter at this point is a **16**-plet of SO(10) plus SO(10)-singlets. When the SU(3) × SU(2) × U(1) ⊂ SO(10) factor is identified with the SM group $G_{\rm SM}$ (with the standard GUT hypercharge embedding), the **16**-plet of SO(10) gives a complete generation of the SM matter, including the right-handed neutrino. Since there are two equivalent fixed points in the SO(4) plane (Fig. 2), there are 2 copies of the **16**-plets. Because of our choice of Wilson lines, the remaining matter has the SM quantum numbers of an additional **16**-plet plus vectorlike multiplets. This can partly be understood from the SM anomaly cancellation. Thus, we have

$$\text{matter:}\ \ 3 \times \mathbf{16} + \text{vectorlike}. \tag{2}$$

Two generations are localized in the compactified space and come from the first twisted sector T_1, whereas the third generation is partially twisted and partially untwisted:

$$2 \times \mathbf{16} \in T_1, \qquad \mathbf{16} \in U, T_2, T_4. \tag{3}$$

In particular, the up-quark and the quark doublet of the third generation are untwisted, which results in a large Yukawa coupling, whereas the down-quark is twisted and its Yukawa coupling is suppressed.

It is well known that the heterotic orbifold models have large vacuum degeneracy which gives enough freedom for realistic constructions [8–10]. There are many flat directions in the field space along which supersymmetry is preserved but some of the gauge symmetries are broken. In particular, all of the U(1) factors of Eq. (1) apart from the hypercharge are broken by giving vacuum expectation values (VEVs) along D- and F-flat directions to some of the 69 singlets s_i^0. We note that cancellation of the Fayet-Iliopoulos D term associated with an anomalous U(1) requires that at least some of the VEVs be close to the string scale. Choosing all the singlet VEVs of order the string scale, the U(1) gauge bosons are decoupled, and we have

$$G \rightarrow \mathrm{SU}(3)_c \times \mathrm{SU}(2)_{\rm L} \times \mathrm{U}(1)_Y \times G_{\rm hidden}, \tag{4}$$

with $G_{\rm hidden} = \mathrm{SO}(6) \times \mathrm{SU}(2)$. This leads to complete separation between the hidden and observable sectors.

One of the main problems of string models is the presence of exotic states at low energies. Generically, such states are inconsistent with experimental data and destroy gauge coupling unification. Even if the exotic states are vectorlike with respect to the SM, they are still harmful unless they attain large masses. However, one cannot assign the mass terms at will. They must appear due to VEVs of some singlets and be consistent with string selection rules [9,11]. In most cases, the latter prohibit many of the required couplings such that the exotic states stay light. One of the achievements of our model is that all of the exotic vectorlike states can be given large masses consistently with string selection rules.

To proceed, let us briefly summarize these rules [5]. The coupling

$$\Psi_1 \Psi_2 \ldots \Psi_n \tag{5}$$

between the states Ψ_i belonging to twisted sectors T_{k_i} (including also the untwisted sector) to be allowed, the total twist has to be a multiple of 6: $\sum_i k_i = 0 \bmod 6$. There are further restrictions on the fixed points that can enter into this product, called a space group selection rule. For example, in the SO(4) plane it amounts to $\sum_i (n^{(i)}, n^{(i)\prime}) = (0, 0) \bmod 2$, where $(n^{(i)}, n^{(i)\prime})$ are the coordinates of the fixed points in the orthonormal basis, $n^{(i)}, n^{(i)\prime} = \{0, 1\}$. Similar rules apply to the SU(3) and the G_2 planes. Further, there is a requirement of gauge invariance $\sum_i p_i = 0$, where p_i are the (shifted) momenta in the $E_8 \times E_8$ gauge lattice. Finally, the $\boldsymbol{H}$ momentum in the compact 6D space must also be conserved, $\sum_i R_1^{(i)} = -1 \bmod 6$, $\sum_i R_2^{(i)} = -1 \bmod 3$, $\sum_i R_3^{(i)} = -1 \bmod 2$, where $R_{1,2,3}$ are the $\boldsymbol{H}$ momenta associated with the G_2, SU(3), and SO(4) planes, respectively.

Based on these selection rules, we analyze allowed superpotential couplings involving vectorlike pairs of ex-

TABLE I. The $G_{\rm SM} \times [\mathrm{SO}(6) \times \mathrm{SU}(2)]$ quantum numbers of the spectrum.

Name	Representation	Count	Name	Representation	Count
q_i	$(\mathbf{3}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{1/6}$	3	$\bar{u}_i$	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{-2/3}$	3
$\bar{d}_i$	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{1/3}$	7	d_i	$(\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{-1/3}$	4
$\bar{\ell}_i$	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{1/2}$	5	ℓ_i	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{-1/2}$	8
m_i	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_0$	8	$\bar{e}_i$	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_1$	3
s_i^-	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{-1/2}$	16	s_i^+	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{1/2}$	16
s_i^0	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_0$	69	h_i	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2})_0$	14
f_i	$(\mathbf{1}, \mathbf{1}; \mathbf{4}, \mathbf{1})_0$	4	$\bar{f}_i$	$(\mathbf{1}, \mathbf{1}; \bar{\mathbf{4}}, \mathbf{1})_0$	4
w_i	$(\mathbf{1}, \mathbf{1}; \mathbf{6}, \mathbf{1})_0$	5			

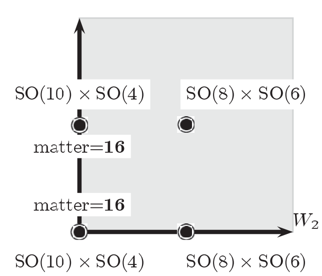

FIG. 2. Local gauge symmetries in the SO(4) plane [at the origin in the G_2 and SU(3) planes]. Two **16**-plets arise from the orbifold fixed points.

otic fields $x_i\bar{x}_j$ and the SM singlets s_a:

$$W = x_i\bar{x}_j\langle s_a s_b \ldots\rangle. \tag{6}$$

We find that all of the exotic states enter such superpotential couplings. This requires a product of up to 6 singlets. In order to decouple the exotic states one has to make sure that the corresponding mass matrices have a maximal rank such that no massless exotic states survive. An interesting feature of the model is that there are exotic states with the SM quantum numbers of the right-handed down-quarks and lepton doublets. As a result, these extra states mix with those from the **16**-plets of SO(10), such that the observed matter fields are a mixture of both [cf. [8,12]]. In particular, for down-type quarks we have the following mass matrix:

	$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$	$\bar{d}_4$	$\bar{d}_5$	$\bar{d}_6$	$\bar{d}_7$
d_1	s^5	s^5	s^5	s^5	s^5	s^3	s^3
d_2	s^1	s^1	s^3	s^3	s^3	s^3	s^3
d_3	s^1	s^1	s^3	s^3	s^3	s^3	s^3
d_4	s^6	s^6	s^6	s^3	s^3	s^6	s^6

Here s^n indicates that the coupling appears when a product of n singlets is included. Different entries with the same n generally correspond to different mass terms since they involve different singlets and Yukawa couplings. This mass matrix has rank 4 reflecting the fact that only 3 down-type quarks survive and the others have large masses, e.g., of the order of the string scale. Note that higher n does not necessarily imply suppression of the coupling: $\langle s\rangle$ can be close to the string scale and, furthermore, the combinatorial coefficient in front of the coupling grows with n.

An important phenomenological constraint on this texture comes from suppression of R-parity violating interactions. It turns out that, in order to prohibit $\bar{u}\,\bar{d}\,\bar{d}$ and similar couplings at the renormalizable level, the $\bar{d}_{6,7}$ component in the massless $\bar{d}$ quarks must be suppressed. This is achieved choosing appropriate directions in the space of singlet VEVs.

For the lepton or Higgs doublets we have

	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
$\bar{\ell}_1$	s^3	s^4	s^4	s^1	s^1	s^1	s^1	s^1
$\bar{\ell}_2$	s^1	s^2	s^2	s^5	s^5	s^3	s^3	s^3
$\bar{\ell}_3$	s^1	s^2	s^2	s^5	s^5	s^3	s^3	s^3
$\bar{\ell}_4$	s^1	s^2	s^2	s^5	s^5	s^6	s^3	s^3
$\bar{\ell}_5$	s^1	s^6	s^6	s^3	s^3	s^6	s^3	s^3

This matrix has rank 5 which results in 3 massless doublets of hypercharge $-1/2$ at low energies. In order to get an extra pair of ("Higgs") doublets with hypercharge $-1/2$ and $1/2$, one has to adjust the singlet VEVs such that the rank reduces to 4. This unsatisfactory fine-tuning constitutes the well-known supersymmetric μ problem. A further constraint on the above texture comes from the top Yukawa coupling: it is order one if the up-type Higgs doublet has a significant component of $\bar{\ell}_1$.

From the flavor physics perspective, it is interesting that only the right-handed down-type quarks and the lepton or Higgs doublets mix with the exotic states. Implications of this phenomenon will be studied elsewhere. Finally, the remaining exotic states m_i and $s_i^{\pm}$ have full rank mass matrices and can be decoupled as well.

We have checked that the above decoupling is consistent with vanishing of the D terms. This is implemented by constructing gauge invariant monomials out of the singlets [13] involved in the mass terms for the exotic states. The F-flatness condition requires a more detailed study and will be discussed elsewhere. Let us only mention that there are plenty of F-flat directions in the field space, for example, any direction in the 39-dimensional space of T_2-, T_4- and U-sector non-Abelian singlets is F flat to all orders as long as the singlets from $T_{1,3}$ have zero VEVs. This is enforced by the $\boldsymbol{H}$-momentum selection rule for the SO(4) plane. Whether the decoupling of all of the extra matter can be done using exactly flat directions or it requires isolated solutions to the $F_i = D_a = 0$ equations is currently under investigation. In any case, one can show that $F_i = D_a = 0$ can be satisfied simultaneously on certain low-dimensional manifolds in the field space, so the decoupling of extra matter can be done consistently with supersymmetry.

The string selection rules have important implications for the matter Yukawa couplings. In particular, in our setup the only large Yukawa coupling is that of the top quark. The reason is that, at the renormalizable level, the types of couplings allowed by the space group and the $\boldsymbol{H}$ momentum are UUU, $T_1T_2T_3$, $T_1T_1T_4$, UT_2T_4, UT_3T_3. The third generation up-quark and quark doublet as well as the up-type Higgs doublet (up to a mixing) are untwisted, so there is an allowed Yukawa interaction of the type UUU whose strength is given by the gauge coupling. The Yukawa couplings involving the T_3 sector vanish since there is no SM matter in that sector. The coupling UT_2T_4 is incompatible with the $SU(2)_L \times U(1)_Y$ symmetry, while the coupling $T_1T_1T_4$ is prohibited due to either gauge invariance or decoupling of the exotic down-type quarks required by suppression of R-parity violation. Therefore, all quarks and leptons apart from the top quark are massless at the renormalizable level and their Yukawa interactions appear due to higher order superpotential couplings. These are suppressed when the involved singlets have VEVs below the string scale.

An important feature of the model is that it admits spontaneous supersymmetry breakdown via gaugino condensation [14]. The SO(6) group of the hidden sector is asymptotically free and its condensation scale depends on the matter content. The **6**-plets and the **4**, $\bar{\mathbf{4}}$-plets of SO(6) can be given large masses consistently with the string selection rules. In this case, the condensation scale is in the range $\sim 10^{11}$–10^{13} GeV depending on the threshold corrections to the gauge couplings. Assuming that the

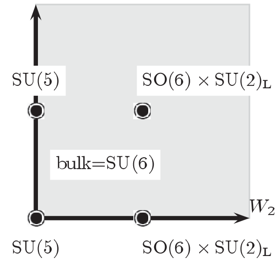

FIG. 3. A 6D orbifold GUT limit with a large SO(4)-plane compactification radius.

dilaton is fixed via the Kähler stabilization mechanism [15], gaugino condensation translates into supersymmetry breaking by the dilaton. The scale of the soft masses, $m_{3/2}$, depends on the details of dilaton stabilization and can be in the TeV range.

Since our model has no exotic states at low energies and admits TeV-scale soft masses, it is consistent with gauge coupling unification. Then a natural question to ask is what are the orbifold GUT limits [5–7] of this model. That is, what is the effective field theory limit in the energy range between the compactification scale and the string scale when some of the compactification radii are significantly larger than the others? Such anisotropic compactifications may mitigate the discrepancy between the GUT and string scales, and can be consistent with perturbativity for one or two large radii of order $(2 \times 10^{16}\ \text{GeV})^{-1}$ [16]. In our model, the intermediate orbifold picture can have any dimensionality between 5 and 10. For example, the 6D orbifold GUT limits are [up to U(1) factors]:

$$\begin{aligned} \text{SO}(4)\ \text{plane:} &\quad \text{bulk GUT} = \text{SU}(6), && N = 2,\\ \text{SU}(3)\ \text{plane:} &\quad \text{bulk GUT} = \text{SU}(8), && N = 2,\\ \text{G}_2\ \text{plane:} &\quad \text{bulk GUT} = \text{SU}(6) \times \text{SO}(4), && N = 4, \end{aligned}$$

where the plane with a "large" compactification radius is indicated and N denotes the amount of supersymmetry. In all of these cases, the bulk β functions of the SM gauge couplings coincide. This is because either G_{SM} is contained in a simple gauge group or there is $N = 4$ supersymmetry. Thus, unification may occur below the string scale. (To check whether this is the case, logarithmic corrections from localized fields, contributions from vectorlike heavy fields, and string thresholds have to be taken into account.) The SM gauge group is obtained as an intersection of the gauge groups at the different fixed points of the 6D orbifold (Fig. 3).

In this Letter, we have presented a heterotic string model which reproduces the spectrum of the minimal supersymmetric SM and is consistent with gauge coupling unification. The emerging picture has a simple geometrical interpretation. The SM gauge group is obtained as an intersection of the local E_8 subgroups at inequivalent orbifold fixed points. Two generations of quarks and leptons appear as **16**-plets localized at the fixed points with unbroken SO(10) symmetry, whereas the third "**16**-plet" involves both bulk and localized states. The Yukawa couplings do not exhibit SO(10) relations. The top quark Yukawa coupling is related to the gauge couplings, while the other Yukawa couplings are due to nonrenormalizable interactions. The model has a hidden sector which allows for supersymmetry breaking via gaugino condensation.

Finally, let us remark that although this model is very special, it is perhaps not unique. In the $\mathbb{Z}_6$-II orbifold with the V_6 shift given above, there are roughly 10^4 models (some of them may be equivalent) with the SM gauge group. $\mathcal{O}(10^2)$ of them have 3 matter generations plus vectorlike exotic matter, whereas we have so far found only one model where the vectorlike matter can be decoupled consistently with the string selection rules. We plan to investigate this issue further. It would also be interesting to understand the relation of this type of model to other phenomenologically promising constructions [17].

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