Parity Doubling and $SU(2)_L \times SU(2)_R$ Restoration in the Hadron Spectrum

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We construct the most general nonlinear representation of chiral $SU(2)_L \times SU(2)_R$ broken down spontaneously to the isospin SU(2), on a pair of hadrons of same spin and isospin and opposite parity. We show that any such representation is equivalent, through a hadron field transformation, to two irreducible representations on two hadrons of opposite parity with different masses and axial-vector couplings. This implies that chiral symmetry realized in the Nambu-Goldstone mode does not predict the existence of degenerate multiplets of hadrons of opposite parity nor any relations between their couplings or masses.

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For massless up and down quarks, QCD has an exact $SU(2)_L \times SU(2)_R$ chiral symmetry realized in the Nambu-Goldstone mode. The axial $SU(2)_A$ is not a symmetry of the vacuum, and is instead manifested by the appearance of massless Goldstone bosons, the pions. The unbroken isospin subgroup, $SU(2)_V$, is realized explicitly in the Wigner-Weyl mode, by degenerate isospin multiplets in the mass spectrum. Small chiral symmetry violating corrections due to the u and d quark masses can be calculated in chiral perturbation theory. An analogous situation holds for three massless quarks (u, d, and s), albeit with larger symmetry breaking corrections. In this Letter we treat the two flavor case, and, for clarity, assume that the u and d quarks (and the pions) are exactly massless.

The implications of chiral symmetry can be obtained in a systematic way using effective chiral Lagrangians organized in a derivative expansion [1–4]. The operators in the chiral Lagrangian are the most general chiral invariants at each order in power counting, constructed from pion and matter (hadron) fields, which transform nonlinearly under chiral transformations.

It has been suggested in the literature that, although chiral symmetry is broken spontaneously, it can be "restored" in certain sectors of the theory, specifically among highly excited baryons and mesons. Because the symmetry includes pseudoscalar ("axial") charges, restoration would imply that hadrons of opposite parity form (approximately) degenerate multiplets. This suggestion has been offered as an explanation of "parity doubling," the tendency for hadrons of the same spin and isospin and opposite parity to have similar masses (for recent incarnations of these ideas; see Ref. [5]).

Similar ideas have been applied to the quartet of heavy mesons constructed by combining a charm quark with a light quark system of quantum numbers $s_\ell^{p_\ell} = \frac{1}{2}^+$ and $\frac{1}{2}^-$, respectively. In these pictures chiral symmetry is not realized explicitly in the spectrum, but states appear in pairs of opposite parity and their axial-vector couplings are predicted to be related [6,7].

Explanations of parity doubling as manifestations of chiral symmetry restoration assume, either explicitly or implicitly, the existence of representations of $SU(2)_L \times SU(2)_R$ which include states of opposite parity, and are thus larger than the irreducible representations of the unbroken group $SU(2)_V$. In this Letter we study the most general nonlinear realizations of chiral symmetry on a pair of hadron states with opposite parity. The questions we address are

(i) Do nontrivial representations of chiral symmetry exist, which encompass hadron states of opposite parity?

(ii) Assuming that chiral symmetry *is* realized linearly in a sector of the hadron spectrum, does the symmetry imply nontrivial relations among hadron properties, such as masses or couplings?

The answer to both these questions is in the negative. Once chiral symmetry is broken spontaneously with the appearance of massless pions, it does not require states of opposite parity to be related in any way. It is possible to write down representations involving states of opposite parity that transform into one another linearly under chiral transformations. However, chirally invariant operators in the effective Lagrangian destroy any relations between their masses and coupling constants. These operators may be suppressed for other, dynamical reasons, restoring the relations between coupling constants and masses. However, the relations are not a consequence of chiral symmetry, but rather a consequence of the dynamical assumptions that suppressed the offending operators.

Our results agree with the classic analysis of Coleman *et al.* [8] of nonlinear representations of a Lie algebra. However, we believe that a very explicit solution is instructive, and throws light on the physics of the problem, which is somewhat obscured in the formal treatment of Ref. [8] and by some claims made in the literature [5]. We will follow closely the treatment and notations of Weinberg [3,4].

We conclude that parity doubling cannot be a consequence of the $SU(2) \times SU(2)$ symmetry of QCD alone. If it occurs, it must be a manifestation of additional dynamics beyond chiral symmetry. In a companion paper [9] we take a close look at the data on parity doubling among the baryons. We find that the data do confirm a pattern of parity doubling among excited nonstrange baryons (the evidence is much weaker among strange and doubly strange baryons, where the data are poorer). Also in Ref. [9] we examine the possibility that restoration of the $U(1)_A$ symmetry of QCD (broken explicitly by the anomaly) is the dynamical origin of parity doubling.

We consider the most general realization of chiral symmetry on a pair of hadrons B, B' with the same isospin I and unspecified spin. For definiteness, we take $B(B')$ to be parity even (odd). A given realization is identified uniquely through the action of the generators on the hadron states. The action of the isospin operators on the hadron states is given by an isospin rotation $[T^a, B_i] = -t^a_{ij}B_j$, $[T^a, B'_i] = -t^a_{ij}B'_j$, where t^a are $2I+1$ dimensional matrices giving a representation of SU(2), and i, j are indices labelling the isospin state of the hadron.

The action of the *axial* charges X^a on the hadron states is more complicated. When acting on the pion field, the effect of an axial rotation is written as [3]

$$[X^a, \pi^b] = -if^{ab}(\pi) = -i[\delta^{ab}f(\pi^2) + \pi^a\pi^b g(\pi^2)] \tag{1}$$

where $\pi^2 = \pi^a\pi^a$ and we have chosen units such that $f_\pi = 1$, making our π^a equivalent to π^a/f_π in conventional units. Fixing the functions f, g defines a particular choice for the pion field. Weinberg [3] chooses $f(x) = \frac{1}{2}(1-x)$ and $g(x) = 1$, which we also find convenient. Our argument does not depend on a particular convention and will be formulated by keeping f, g completely general.

We turn next to hadron fields other than pions. We begin with the most general form of an axial rotation on B and B' that conserves parity. Without loss of generality it can be taken to act homogeneously on their sums and differences, $S = B + B'$ and $D = B - B'$. The fields S and D do not have definite parity, but are instead transformed into each other, $PSP^\dagger = D$. The action of an axial rotation on these states has the most general form

$$[X^a, N_i] = \{\pm h_1\delta^{ab} \pm h_2\pi^a\pi^b + v\varepsilon_{abc}\pi^c\}t^b_{ij}N_j \tag{2}$$

where $N = S, D$, the $\pm$ refers to S and D, respectively, and (h_1, h_2, v) are functions of π^2 that can be different for different hadrons. We neglected here a possible term of the form $w(\pi^2)\pi^a N_i$, which can be eliminated by a particularly simple field redefinition, $N \to \Phi(\pi^2)N$.

It is clear at this point that we are allowing representations of the chiral algebra where parity is embedded in a nontrivial way, i.e., for which axial rotations take a hadron B into a *different* hadron, B', of opposite parity, in addition to creating pions. Particular cases of transformations include

(i) $(h_1, h_2, v) = (1, 0, 0)$. This is a linear representation, the $(I_L, 0)$ and $(0, I_R)$ representations for S and D respectively. Expressed in terms of the fields with definite parity,

$$[X^a, B_i] = t^a_{ij}B'_j, \qquad [X^a, B'_i] = t^a_{ij}B_j. \tag{3}$$

(ii) $(h_1, h_2, v) = [0, 0, v_0(x)]$. This is the standard nonlinear (SNL), parity-conserving realization of chiral symmetry on the two states B and B' separately,

$$[X^a, N_i] = v_0(\pi^2)\varepsilon_{abc}\pi^c t^b_{ij}N_j, \qquad N = B, B'. \tag{4}$$

The form of the $v(x)$ function in this case is fixed uniquely by chiral symmetry [3] as $v(x) = v_0(x) \equiv \frac{1}{x}[\sqrt{f^2(x)+x} - f(x)]$. With our definition of the pion field, this is $v_0(x) = 1$.

We first construct the most general form of the chiral symmetry realization (h_1, h_2, v) compatible with the Lie algebra of the chiral group. We then show that, through appropriate field transformations (which possibly mix the hadrons of opposite parity B, B' and pions), all such realizations are physically equivalent to two decoupled nonlinear realizations on two isospin multiplets of opposite parity $\tilde{B}, \tilde{B}'$. This proves the absence of nontrivial realizations, linear or nonlinear, of the chiral symmetry which mix matter states of opposite parity.

The most general form for the transformation of the hadron fields Eq. (2) under an axial rotation is specified by the set of three functions $(h_1(x), h_2(x), v(x))$ subject to the consistency conditions

$$\begin{aligned} 2vf + h_1^2 + xv^2 &= 1, \\ vg + 2v'(f + xg) + h_1h_2 &= v^2, \end{aligned} \tag{5}$$

where the x dependence of f, g, h_1, h_2, and v has been suppressed. These equations follow from the Jacobi identity $[X^a, [X^b, B]] - [X^b, [X^a, B]] = i\epsilon_{abc}[T^c, B]$. The first of Eq. (5) requires that at fixed x, h_1, and v lie on an ellipse for arbitrary f and g. We write the general solutions of these equations in terms of an arbitrary function $\theta(x)$, chosen such that $\theta = 0$ corresponds to the SNL solution, Eq. (4),

$$\begin{aligned} h_1(x) &= \sqrt{1 + f^2(x)/x}\,\sin\theta(x), \\ v(x) &= \frac{1}{x}\sqrt{f^2(x) + x}\,\cos\theta(x) - f(x)/x, \end{aligned} \tag{6}$$

and $h_2(x)$ is determined from the second of Eq. (5).

The space spanned by the solutions to Eq. (6) is shown in Fig. 1 for the particular pion representation used here. Notice that the surface in Fig. 1 is simply connected, extends all the way from $x = 0$ to infinity and it is diffeomorphic to a cylinder. Any curve $(v(x), h_1(x))$ lying on the surface of Fig. 1 and having a unique intersection with each of the constant-x ellipses is an allowed nonlinear representation of $SU(2)_L \times SU(2)_R$, which mixes the baryons of opposite parity. The linear realizations, Eq. (3), correspond to the vertical lines $h_1 = \pm 1$, $v = 0$, or $\theta(x) = \pm\arctan(x/f(x))$, and the SNL realization is the vertical line $h_1 = 0$, $v = 1$, or $\theta(x) = 0$.

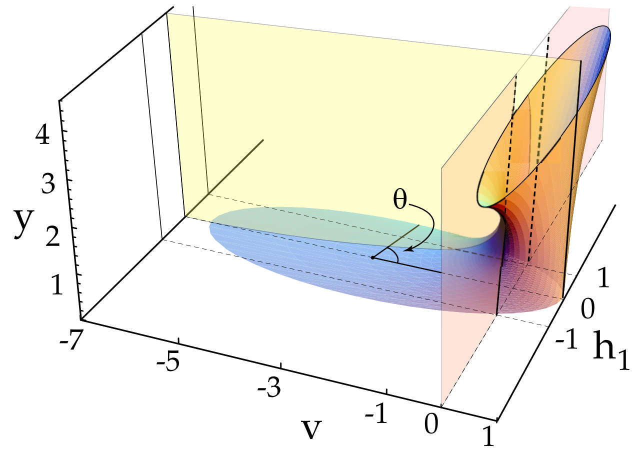

FIG. 1 (color online). Solutions space for representations of chiral symmetry that appear to be nontrivial in parity. h_1 and v are plotted as functions of $y = 2\ln(1+x)$, with the origin of the y axis at $y_0 > 0$, $y_0 \ll 1$. At each y, h_1 and v lie on an ellipse. Any curve $(h_1(y), v(y))$ on this surface corresponds to one particular realization of the chiral symmetry on a pair of states of opposite parity. The vertical lines $h_1 = \pm 1$, $v = 0$ and $h_1 = 0$, $v = 1$ lie on the surface. They are the linear and SNL representations discussed in the text.

In the SNL the masses and coupling constants of different multiplets of definite isospin and parity are unrelated. So the questions posed at the outset reduce to: Is there a redefinition of the fields B, B' that maps *any* curve $(v(x), h_1(x))$ on the surface of Fig. 1 to the SNL realization $\theta(x) = 0$? The redefinition we seek is given by a transformation of the form

$$\tilde{N}_i = [\exp(2i\pi^a t^a \Theta_N(x)/\sqrt{x})]_{ij} N_j, \qquad N = S, D \quad (7)$$

with $\Theta_S = -\Theta_D \equiv \Theta$ a real function of x. It is easy to show that the new fields $\tilde{S}, \tilde{D}$ are still a representation of the group with their own functions $(\tilde{h}_1, \tilde{h}_2, \tilde{v})$ satisfying Eq. (5). Hence this solution can be parameterized by a single angle $\tilde{\theta}$, which is furthermore given by $\tilde{\theta} = \Theta + \theta$. If we choose $\Theta(x) = -\theta(x)$, then we have $\tilde{\theta}(x) = 0$. Thus any solution $(h_1(x), h_2(x), v(x))$ can be mapped to the parity-conserving standard nonlinear realization of Eq. (4), corresponding to $\theta(x) = 0$. Because the surface in Fig. 1 is simply connected the function Θ is continuous and the field redefinition is always allowed.

We illustrate this formal result by quoting the explicit form of the field redefinition which takes the baryon fields transforming in the linear representation Eq. (3) into two decoupled nonlinear transforming fields $\tilde{B}, \tilde{B}'$ with opposite parity

$$\tilde{B} = \frac{B - 2i\pi^a t^a B'}{\sqrt{1+\pi^2}}, \qquad \tilde{B}' = \frac{B' - 2i\pi^a t^a B}{\sqrt{1+\pi^2}}. \quad (8)$$

Finally, we examine the way that the typical physical consequences of a linear representation of chiral symmetry—relations among masses and coupling strengths—are undone by the presence of massless pions, and what dynamical assumptions are needed to restore them. We consider the simplest case—two hadron fields, B and B', postulated to obey the linear transformation law, Eq. (3), such that B and B' lie in the $(I, 0) \pm (0, I)$ representations of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, respectively. Under axial rotations they transform into one another, independently of the pion field, as if the symmetry were realized in the Wigner-Weyl mode. Of course, the pions transform nonlinearly, by Eq. (1). The most general effective Lagrangian invariant under Eqs. (1) and (3), containing only operators of dimension $d \leq 4$, is

$$\begin{aligned}\mathcal{L} &= \bar{B} i\not{\partial} B + \bar{B}' i\not{\partial} B' - m_0(\bar{B}B + \bar{B}'B') \\ &\quad + m_1\left(\bar{B}\frac{1-\pi^2}{1+\pi^2}B - \bar{B}\frac{4i\pi^a t^a}{1+\pi^2}B' - (B \leftrightarrow B')\right) \quad (9)\end{aligned}$$

Without loss of generality we assume here and in the following that B, B' are spin-$1/2$ baryons. If the second line of Eq. (9) were ignored, the hadrons described by B and B' would be degenerate. However, the term proportional to m_1, allowed by the nonlinear transformations of Eqs. (1) and (3), breaks the degeneracy of B and B'. The actual physical content of $\mathcal{L}$ can more easily be seen by going over to the nonlinearly transforming fields $\tilde{B}, \tilde{B}'$ defined in Eq. (8). Expressed in terms of these fields, the Lagrangian Eq. (9) assumes the form

$$\begin{aligned}\mathcal{L} &= \bar{\tilde{B}}(i\not{\partial} - \varepsilon^{abc}\pi^a \not{\partial}\pi^b t^c)\tilde{B} - \bar{\tilde{B}}(\not{D}\pi^a)t^a\tilde{B}' \\ &\quad + (\tilde{B} \leftrightarrow \tilde{B}') - (m_0 - m_1)\bar{\tilde{B}}\tilde{B} - (m_0 + m_1)\bar{\tilde{B}}'\tilde{B}' \quad (10)\end{aligned}$$

where $D_\mu \pi^a = 2\partial_\mu \pi^a/(1+\pi^2)$ is the covariant derivative of the pion field. Note that $\tilde{B}$ and $\tilde{B}'$, the mass eigenstates, are not degenerate.

Invariance of the Lagrangian Eq. (10) under axial transformations leads also to predictions concerning the axial charges of B and B'. The conserved axial Noether current is

$$A^a_\mu = \bar{\tilde{B}}\gamma_\mu t^a \tilde{B}' + \bar{\tilde{B}}'\gamma_\mu t^a \tilde{B} + (\text{pion terms}), \quad (11)$$

so the axial charges of B and B' vanish, and the off-diagonal BB' axial charge is unity. These predictions are not disturbed by the term proportional to m_1. However, they are invalidated by further chirally invariant terms involving the covariant derivative of the pion field that can be added into the Lagrangian. These can be written in terms of B and B' and are invariant when they transform linearly [see Eq. (3)] and the pion transforms according to Eq. (1). There are three possible terms invariant under parity, and linear in $D_\mu \pi$. It is easiest to write them in terms of the redefined fields, $\tilde{B}$ and $\tilde{B}'$,

$$\begin{aligned}\delta\mathcal{L}_2 &= c_2[\bar{\tilde{B}}(\not{D}\pi^a)\gamma_5 t^a\tilde{B} + \bar{\tilde{B}}'(\not{D}\pi^a)\gamma_5 t^a\tilde{B}'] \\ &\quad + c_3[\bar{\tilde{B}}(\not{D}\pi^a)t^a\tilde{B}' + \bar{\tilde{B}}'(\not{D}\pi^a)t^a\tilde{B}] \\ &\quad + c_4[\bar{\tilde{B}}(\not{D}\pi^a)\gamma_5 t^a\tilde{B} - \bar{\tilde{B}}'(\not{D}\pi^a)\gamma_5 t^a\tilde{B}']. \quad (12)\end{aligned}$$

The resulting Noether axial current becomes

$$A^a_\mu = (c_2 + c_4)\bar{\tilde{B}}\gamma_\mu\gamma_5 t^a \tilde{B} + (c_2 - c_4)\bar{\tilde{B}}'\gamma_\mu\gamma_5 t^a \tilde{B}' + (1 - c_3)(\bar{\tilde{B}}\gamma_\mu t^a \tilde{B}' + \text{H.c.}) + (\text{pion terms}) \quad (13)$$

and can accommodate any values of the axial matrix elements between the states $\tilde{B}, \tilde{B}'$. At each order in the derivative expansion new operators appear, allowed by chiral symmetry, that contribute to the $\tilde{B} - \tilde{B}'$ mass splitting and change their couplings.

Finally we comment on the generalized Goldberger-Treiman (GT) relations that relate the axial charges, pion couplings, and mass differences of states with opposite parity. Regardless of whether they are degenerate or are related by chiral transformations, two baryons of opposite parity ($\tilde{B}$ and $\tilde{B}'$) always obey a GT relation

$$g_\pi^{+-} = G_A^{+-}(M_{\tilde{B}'} - M_{\tilde{B}})/f_\pi \quad (14)$$

where g_π^{+-} and G_A^{+-} are the pion coupling (defined by $\mathcal{L}_{BB'\pi} = ig_\pi^{+-}\bar{\tilde{B}}'\pi^a t^a \tilde{B} + \text{H.c.}$) and transition axial charge for $\tilde{B}$ and $\tilde{B}'$. This follows simply from requiring axial current conservation $q^\mu\langle\tilde{B}'|A^a_\mu|\tilde{B}\rangle = 0$ and has nothing to do with the transformation properties of $\tilde{B}$ or $\tilde{B}'$ or the "restoration" of chiral symmetry. Note that the GT relation does require that the s-wave coupling (g_π^{+-}) vanishes if $\tilde{B}'$ and $\tilde{B}$ are degenerate, as discussed, for example, in Ref. [7]. In our approach Eq. (14) is simply a restatement of the connection between the axial current, Eq. (13), and the parameters of the Lagrangian, with $G^A_{+-} = 1 - c_3$. Equation (14) can be used to extract the axial coupling G^A_{+-} from the $\tilde{B}' \to \tilde{B}\pi$ data.

Previous work on parity and chiral doubling neglects one or more of the constants $\{m_1, c_{2-4}\}$, which can result in considerable predictive power. However these predictions are not consequences of chiral symmetry. Instead they are the result of whatever (often unstated) dynamical assumptions enabled the authors to ignore the terms that would have invalidated the predictions. Note that, if c_{2-4} are arbitrarily set to zero, the nonrenormalization of the conserved Noether axial current implies that they will not be induced by loop corrections to all orders [9]. This is confirmed by the one-loop results obtained in Ref. [10] using chiral perturbation theory.

This brings us to the main conclusion of our Letter: if one attempts to realize chiral symmetry in a linear way on a subset of states in a world with spontaneous symmetry breaking and massless pions, the chiral symmetry in fact gives no relations among the properties of these states, such as masses and couplings. Such predictions, which are typical of a symmetry realized in the Wigner-Weyl mode, would hold only if certain chirally invariant operators are dynamically suppressed.

We close by noting that the arguments in this Letter do not preclude the restoration of chiral symmetry at high temperature or high chemical potential. In both cases the restoration occurs if QCD undergoes a phase transition to a Wigner-Weyl phase, where there are no massless Nambu-Goldstone bosons. As mentioned in the introduction, there is some evidence for parity doubling, at least in the spectrum of nonstrange baryons. In Ref. [9] we examine this evidence and speculate on possible explanations (including those of Refs. [11,12]), other than restoration of $SU(2)_L \times SU(2)_R$, which as we have shown, cannot occur in the presence of massless pions.

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