Stable Optical Trapping Based on Optical Binding Forces

Tomasz M. Grzegorczyk, Brandon A. Kemp, and Jin Au Kong
Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
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Various trapping configurations have been realized so far, either based on the scattering force or the gradient force. In this Letter, we propose a new trapping regime based on the equilibrium between a scattering force and optical binding forces only. The trap is realized from the interaction between a single plane wave and a series of fixed small particles, and is efficient at trapping multiple free particles. The effects are demonstrated analytically upon computing the exact scattering from a collection of cylindrical particles and calculating the Lorentz force on each free particle via the Maxwell stress tensor.

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Since 1970, it has been known that the motion of small dielectric particles can be controlled by laser beams [1]. The radiation pressure created by the lasers on the particles induces a scattering force along the direction of the beam propagation, and a gradient force along the gradient of the field intensity. The theoretical demonstration of the existence of a gradient force on small particles was offered in Ref. [2] using a dipole approximation in the formulation of the Lorentz force [3]. For strong intensity gradients, it was shown that the gradient force can become dominant, yielding a negative force responsible for the trapping and stable levitation reported in Refs.[4–6]. For weak intensity gradients, it was shown that a stable trap could still be realized by using two counter-propagating beams in order to balance the two scattering forces [7]. Developing upon this work, multiple trapping configurations have been proposed, such as Talbot trapping [8], trapping with diffractive optics [8], trapping with a spherical lens [9], dielectrophoresis trapping [10], and two- or three-beam trapping [11–13] which offers the control of the size of the traps typically achieved by varying the incident angles of the three beams. In all these cases, the particles are trapped either because of the gradient force, or because of the balance between the scattering forces and other external forces in the system.

In this Letter, we propose an alternative trapping regime based on optical binding forces, i.e., an alternative way to induce a negative force. Unlike the known configurations mentioned above, the present configuration can be realized with a single plane wave. Since optical binding manifests itself when multiple particles submitted to an incident electromagnetic wave interact and scatter collectively, its calculation requires us to include all the multiple interactions between all the particles. Optical binding on a system of two particles was first studied in Ref. [14] where the particle response was assimilated to a harmonic oscillator. Later, Ref. [15] used a discrete dipole approximation to evaluate the binding forces from the Maxwell stress tensor from two particles. Recently, we have proposed an exact method to compute the optical binding between an arbitrary number of cylindrical particles based on the Mie theory and the Foldy-Lax multiple scattering equations [16]. We use here this ability in order to optimize the location of a series of fixed small particles and devise a configuration where scattering and optical binding forces can be manipulated to create a new regime of particle trapping.

The configuration we shall use to begin is a generalization of the one studied in Ref. [15], where two particles are submitted to an incident plane wave propagating in the $\hat{y}$ direction, as shown in the inset of Fig. 1(a). The forces in the $\hat{x}$ and $\hat{y}$ directions (denoted F_x and F_y, respectively) computed either using the method of Ref. [15] for circular geometries or the one of Ref. [16] for cylindrical geometries exhibit a tapered oscillatory behavior, as already known from Ref. [15] and illustrated in Fig. 1(a) for the typical set of experimental parameters detailed in the caption of the figure. The oscillatory behavior of either F_x or F_y is a direct manifestation of the optical binding between the particles: the mean value of the force (dashed line in the figure) corresponds to the force on a single particle, while the amplitude of the oscillation is directly related to the strength of the binding. The latter can be explained upon considering N two-dimensional line scatterers (equivalent to infinitely thin cylinders) and writing the corresponding Foldy-Lax multiple scattering equations [17]. The total electric field $\bar{E}(\bar{r}) = \bar{E}_{\rm inc}(\bar{r}) + \bar{E}_{\rm scat}(\bar{r})$ at location $\bar{r} = \hat{x}x + \hat{y}y$ is expressed as the sum of the incident and the scattered field from the N scatterers in the system (located at positions $\bar{r}_\ell$, $\ell \in \{1, \ldots, N\}$), where $\bar{E}_{\rm inc}(\bar{r}) = \hat{z}e^{iky}$ is the incident field with unit amplitude and

$$\bar{E}_{\rm scat}(\bar{r}) = \hat{z}\sum_{j=1}^{N} i\pi f H_0^{(1)}(k|\bar{r} - \bar{r}_j|)\bar{E}_j^e(\bar{r}_j). \qquad (1)$$

The Hankel function of the first kind and zeroth order represents the two-dimensional Green's function and f is the scattering amplitude of a cylinder, related to its polarizability which can be evaluated numerically [15,18]. $\bar{E}_j^e(\bar{r}_j)$ is the total exciting field on particle j from all the

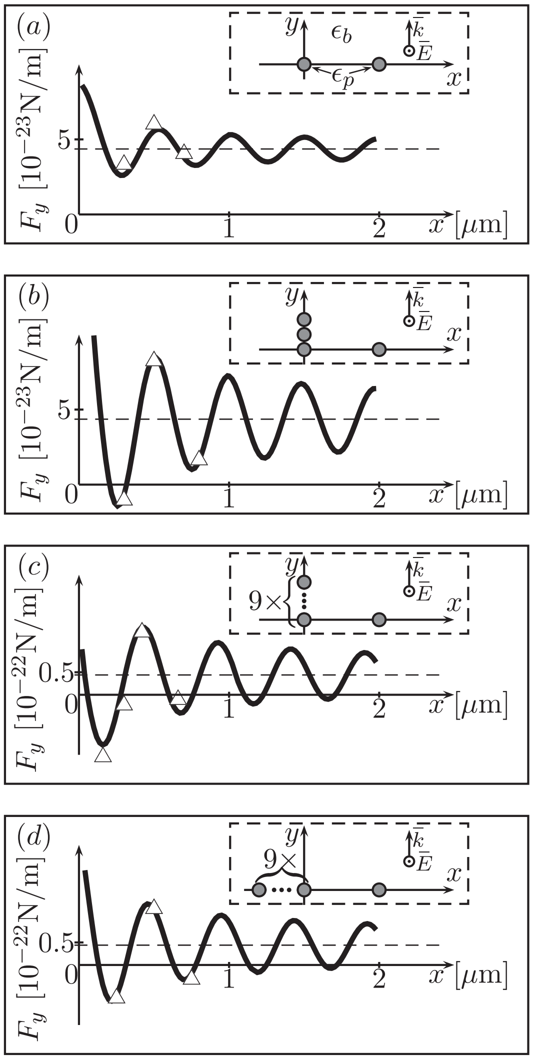

FIG. 1. Force in the $\hat{y}$ direction (F_y) felt by the right particle shown in the insets as function of its position along $\hat{x}$ due to a single plane wave incidence with $\bar{E} = \hat{z}e^{iky}$ at $\lambda_0 = 632.8$ nm. The four subplots correspond to different arrangement of fixed particles on the left: (a) single particle, (b) three particles align in $\hat{y}$, (c) nine particles aligned in $\hat{y}$, (d) nine particles aligned in $\hat{x}$. All particles have a permittivity $\epsilon_p = 2.25\epsilon_0$, are embedded in a background of $\epsilon_b = 1.69\epsilon_0$, and have a radius of 10 nm. The closely packed particles are separated by 21 nm. The dashed curve indicates the force on a single particle ($F_y \simeq 4.36 \times 10^{-23}$ N/m). The superposed triangles have been calculated by the method shown in Ref. [21] from the fields obtained from the commercial package CST Microwave Studio®.

other particles including all interactions, solved from

$$\bar{E}_j^e(\bar{r}_j) = \bar{E}_{\mathrm{inc}}(\bar{r}_j) + \sum_{\substack{\ell=1 \\ \ell\neq j}}^{N} i\pi f H_0^{(1)}(k|\bar{r}_j - \bar{r}_\ell|)\bar{E}_j^e(\bar{r}_\ell) \quad (2)$$

either iteratively or by matrix inversion. For two cylinders positioned at $\bar{r}_1$ and $\bar{r}_2$ with $y = 0$ in both cases like shown in the inset of Fig. 1(a), the total electric field can be written as

$$\bar{E}(\bar{r}) = \hat{z}\bigg[e^{iky} + \frac{i\pi f}{1 - i\pi f H_0^{(1)}(k|\bar{r}_1 - \bar{r}_2|)}(H_0^{(1)}(k|\bar{r} - \bar{r}_1|) + H_0^{(1)}(k|\bar{r} - \bar{r}_2|))\bigg]. \quad (3)$$

The magnetic field is directly obtained from Faraday's law and the force is obtained from the contour integration of the Maxwell stress tensor [19], which has been shown to be equivalent to the computation of the Lorentz force from bound currents and charges [20,21]. The leading term in the expression of the two components of the force have a dependence of the form $H_0^{(1)}(kr)$, which yields the tapered oscillatory behavior clearly visible in Fig. 1(a). The period of the oscillations is therefore directly related to the period of the Hankel function (e.g., with the parameters of Fig. 1, the Hankel function has a period of about 0.42 μm, in agreement with the oscillations shown in Fig. 1(a) obtained without approximations), while their amplitude is modulated by the scattering amplitude f and the number of particles.

The amplitude of the oscillations can therefore be increased by either increasing the size of the particles (bearing in mind that the analogy with line scatterers would become less accurate), or by increasing the number of particles in the system. It is the second approach that we exploit here: increasing the number of particles in the system increases the optical binding forces between particles, which can create a negative force. Since the number of particles and their positions are arbitrary, the resulting forces can be potentially modulated at will. In particular, we first show that using a single plane wave like in Fig. 1(a), the amplitude of the oscillation of F_y can be enhanced to actually reach zero or even negative values, indicating that optical binding forces can be made strong enough to cancel or invert the scattering force due to the incident plane wave. In order to achieve this, the proper positions of the particles have to be determined by considering their scattering characteristics. As can be seen from the parameters indicated in the caption of Fig. 1, the particles are very small compared to the wavelength in order to prevent the scattering force from dominating the binding forces (although this property has not been used in the calculations of the forces [16]), which indicates that their response can be approximated by a Rayleigh radiation from a dipole in the $\hat{z}$ direction [22]. Consequently, a strong radiation is induced in the $\hat{x}$ direction, and binding phenomena are expected to be enhanced if multiple dipoles can be induced in $\hat{z}$. This can be achieved by placing additional particles along the $\hat{y}$ axis, as shown in the insets of Figs. 1(b) and 1(c), or by aligning particles along the $\hat{x}$ axis, as shown in Fig. 1(d) and its inset. Such configurations could potentially be achieved using forces from evanescent fields, as suggested in Refs. [15,23]. We prefer the vertical configuration to the horizontal one for reasons we shall detail hereafter. The corresponding results show that as the number of particles increases, the amplitude of the oscillations of F_y increases and eventually can be such to yield a zero or a negative F_y (these results have been confirmed numerically by the commercial package CST Microwave Studio®, as explained in the caption of Fig. 1). Consequently, a system like that shown in the inset of Fig. 1(c), where nine particles are fixed and aligned along $\hat{y}$, can indeed cancel the scattering force due to the incident plane wave by the sole use of optical binding forces.

We pursue by noting that the forces shown in Fig. 1 are symmetric in x and, in the particular case of Fig. 1(c), that the first minimum has a magnitude comparable to the first maximum ($F_y \simeq -1.1 \times 10^{-22}$ N/m at $x \simeq 155$ nm and $F_y \simeq +1.5 \times 10^{-22}$ N/m at $x \simeq 430$ nm). Hence, the location of a second set of nine vertical particles can be optimized such that the independent forces tend to cancel each other by superposing their respective minima and maxima.

In order to exactly compute the force in the new configuration and optimize the location of the particles, we define an inverse problem based on the forward equations used in this Letter and presented in Ref. [16]. Although the reasoning based on independent forces from the two vertical sets of particles is not exact, it still gives a good initial guess of the initial positions to be used in the optimization scheme. The optimization is therefore run with 19 identical particles as those of Fig. 1 [18 are fixed and one spans the (xy) plane] and requires the magnitude of F_y to be less than 10% of the force on a solitary particle (indicated by the dashed lines in Fig. 1) over a wide x range at $y = 0$. The result of the optimization yields the force shown in Fig. 2 for two vertical arrangement of nine particles separated by 595 nm. It is seen that the force F_y (solid line in the figure) lies within the specified constraints over about 400 nm, which corresponds to about 67% of the range. In addition, the force F_x (dashed line in the figure) is seen to be positive for $x \in [100\text{ nm}, 297.5\text{ nm}]$ and negative for $x \in [297.5\text{ nm}, 500\text{ nm}]$, with an amplitude about 2 orders of magnitude larger than F_y. F_x therefore creates a stable equilibrium around $x = x_0 = 297.5$ nm while F_y is small but not exactly zero. A quick inspection reveals that F_y is null at $x = x_0$ and $y = y_0 = 0.4$ nm (data—not shown—are very similar to those of Fig. 2 with F_y peaking at zero at the same location as F_x is null). This configuration therefore realizes a one-dimensional trap.

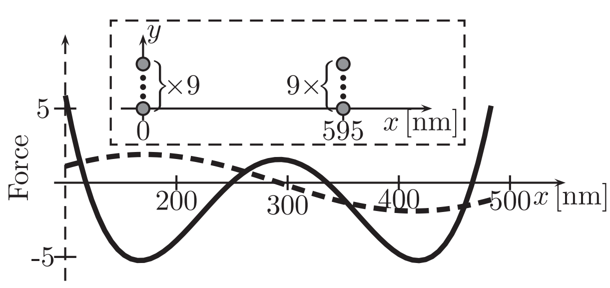

FIG. 2. Force on a free particle moving along $y = 0$ in the configuration shown in the inset. Solid line: $F_y \times 10^{-24}$ [N/m]; Dashed line: $F_x \times 10^{-22}$ [N/m]. Physical parameters are identical to those of Fig. 1.

Finally we verify that the trap is also two-dimensional, located at (x_0, y_0), as is further confirmed in Fig. 3(a) by computing the force field on a free particle spanning the (xy) plane. The force distribution clearly shows an attraction toward (x_0, y_0) induced by the scattering and optical binding forces in the system. The optical well thus created can be evaluated by computing the inverse gradient of the force, and is shown in Fig. 4. The potential energy is seen to dip sharply at the location of the trap and is thus efficient at trapping free particles. Figures 3 and 4 also show that as the number of trapped particles increases (up to four in this case), the potential well is strengthened. This is clearly seen in Fig. 4 with an increased number of trapped particles, and is also illustrated in Figs. 3(b) and 3(c) by show-

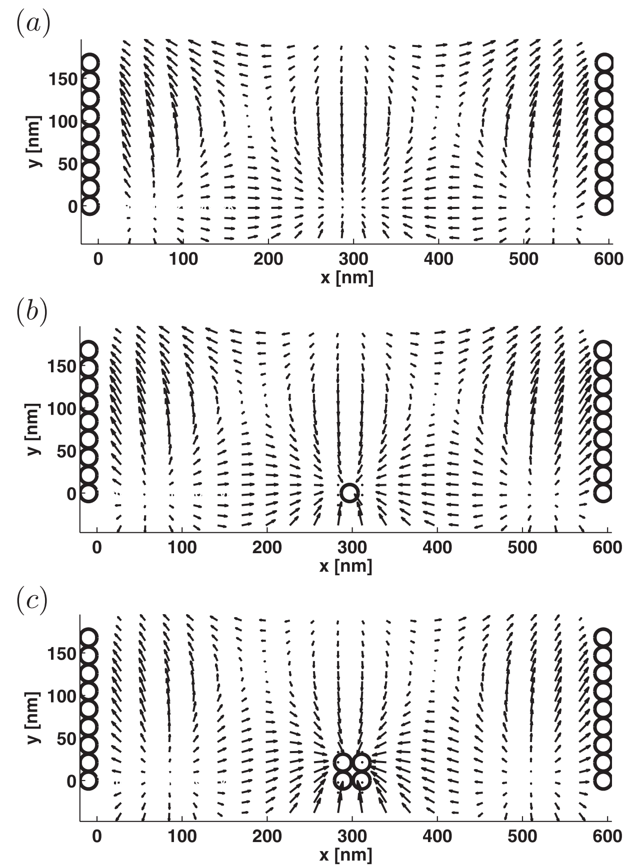

FIG. 3. Field force on a free particle in the presence of two vertical walls of particles separated by 595 nm and (a) no trapped particle, (b) a single trapped particle, (c) four trapped particles, all clustered around $(x, y) = (297.5\text{ nm}, 0.4\text{ nm})$. Physical parameters are identical to those of Fig. 1. The tail of the arrows correspond to the location of the center of the particle.

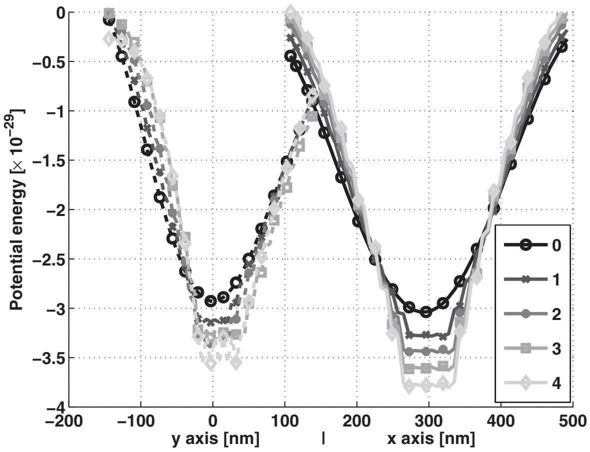

FIG. 4. Potential energy at $x \simeq 297.5$ nm for $y \in [-150, 150]$ nm and at $y \simeq 0$ for $x \in [100, 500]$ nm, for zero up to four trapped particles around $(x_0, y_0) = (297.5\ \text{nm}, 0.4\ \text{nm})$. The energy has been computed as the inverse gradient of the force field distribution shown in Fig. 3.

ing the force field in the (xy) plane for one and four trapped particles (data are not shown for two and three trapped particles as they present the same effect). In addition, Fig. 4 indicates that the well is quasiharmonic in both directions so that the irradiance necessary to maintain a given trapping accuracy despite the Brownian motion can be estimated from Refs. [24,25]. If one accepts a standard deviation of 100 nm, which corresponds to five particle diameters but still maintains the trapping property of the configuration, the necessary irradiance is $I_0 \sim 0.4\ \mu\text{W}/\mu\text{m}^2$. This irradiance is here obtained for infinitely long cylinders, i.e., two-dimensional particles, which explains the difference with the irradiance typically achieved in experimental three-dimensional configurations.

The design of the new trap, based on two sets of vertically arranged particles, is not restrictive to this geometry: a proper design of dielectric diffractive elements would achieve similar properties by molding the electromagnetic field within a controlled region of space. Multiple traps based on the optical binding force could therefore be realized, as an extension of the present work.

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[1] A. Ashkin, Phys. Rev. Lett. **24**, 156 (1970).
[2] J. P. Gordon, Phys. Rev. A **8**, 14 (1973).
[3] L. Landau and E. Lifshitz, *The Theory of Classical Fields* (Pergamon Press, New York, 1975), Vol. 2, ISBN 0-08-018176-7.
[4] A. Ashkin and J. M. Dziedzic, Appl. Phys. Lett. **19**, 283 (1971).
[5] A. Ashkin, Phys. Rev. Lett. **40**, 729 (1978).
[6] A. Ashkin, J. M. Dziedzic, J. E. Bjorkholm, and S. Chu, Opt. Lett. **11**, 288 (1986).
[7] A. Ashkin and J. M. Dziedzic, Appl. Phys. Lett. **28**, 333 (1976).
[8] J.-M. Fournier, M. M. Burns, and J. A. Golovchenko, Proc. SPIE Int. Soc. Opt. Eng. **2406**, 101 (1995).
[9] G. Roosen, B. Delaunay, and C. Imbert, J. Opt. **8**, 181 (1977).
[10] R. Hölzel, N. Calander, Z. Chiragwandi, M. Willander, and F. F. Bier, Phys. Rev. Lett. **95**, 128102 (2005).
[11] J.-M. Fournier, G. Boer, G. Delacrétaz, P. Jacquot, J. Rohner, and R. Salathé, Proc. SPIE-Int. Soc. Opt. Eng. **5514**, 309 (2004).
[12] A. Casaburi, G. Pesce, P. Zemánek, and A. Sasso, Opt. Commun. **251**, 393 (2005).
[13] S. A. Tatarkova, A. E. Carruthers, and K. Dholakia, Phys. Rev. Lett. **89**, 283901 (2002).
[14] M. M. Burns, J.-M. Fournier, and J. A. Golovchenko, Phys. Rev. Lett. **63**, 1233 (1989).
[15] P. C. Chaumet and M. Nieto-Vesperinas, Phys. Rev. B **64**, 035422 (2001).
[16] T. M. Grzegorczyk, B. A. Kemp, and J. A. Kong (to be published).
[17] L. Tsang, J. Kong, K. Ding, and C. Ao, *Scattering of Electromagnetic Waves: Numerical Simulations* (Wiley, New York, 2000).
[18] J. Venermo and A. Sihvola, J. Electrost. **63**, 101 (2005).
[19] J. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941), ISBN 0-07-062150-0.
[20] A. R. Zakharian, M. Mansuripur, and J. V. Moloney, Opt. Express **13**, 2321 (2005).
[21] B. A. Kemp, T. M. Grzegorczyk, and J. A. Kong, Opt. Express **13**, 9280 (2005).
[22] J. A. Kong, *Electromagnetic Wave Theory* (EMW, Cambridge, MA, 2005), ISBN 0-9668143-9-8.
[23] M. Nieto-Vesperinas, P. C. Chaumet, and A. Rahmani, Phil. Trans. R. Soc. A **362**, 719 (2004).
[24] S. Chandrasekhar, Rev. Mod. Phys. **15**, 1 (1943).
[25] L. Novotny, R. X. Bian, and X. S. Xie, Phys. Rev. Lett. **79**, 645 (1997).