Hindrance of Heavy-Ion Fusion due to Nuclear Incompressibility

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We propose a new mechanism to explain the unexpected steep falloff of fusion cross sections at energies far below the Coulomb barrier. The saturation properties of nuclear matter are causing a hindrance to large overlap of the reacting nuclei and consequently a sensitive change of the nuclear potential inside the barrier. We report in this Letter a good agreement with the data of coupled-channels calculation for the ^{64}Ni + ^{64}Ni combination using the double-folding potential with Michigan-3-Yukawa-Reid effective $N - N$ forces supplemented with a repulsive core that reproduces the nuclear incompressibility for total overlap.

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In recent years a new phenomenon observed in the fusion of several medium-heavy dinuclear systems [1–5] has presented a challenge to the theoretical understanding of the reaction mechanism at deep sub-barrier energies. From the inspection of capture cross sections for various combinations of colliding systems such as ^{58}Ni + ^{58}Ni, ^{64}Ni + ^{64}Ni, ^{60}Ni + ^{89}Y, ^{64}Ni + ^{100}Mo, ^{90}Zr + ^{90}Zr, ^{90}Zr + ^{89}Y, and ^{90}Zr + ^{92}Zr, an unexpected hindrance was observed at sub-barrier bombarding energies below a certain threshold E_s that varies from system to system. The authors of Ref. [5] remarked that the exploration of the hindrance phenomenon is only in its initial stage and an identification of the underlying physical cause is still missing. The standard theoretical approach to treat capture reactions, i.e., the quantum tunneling through the relative barrier of the dinuclear system coupled to different low-lying collective degrees of freedom (vibrations and rotations) was unable to explain the steep falloff in the cross sections. The coupled-channels (c.c.) approach failed to describe the dynamics of the fusion process below a certain value of the bombarding energy. The source of this phenomenon is not only interesting for the understanding of the reaction mechanism but may have essential consequences for nuclear reactions that occur in stars [5]. It could imply that the synthesis of heavy elements is hindered below a certain energy threshold.

To resolve the enigma several hypotheses have been proposed. An issue debated by several authors was the large diffuseness needed to fit high-precision fusion data [6,7]. A suggestion that is very close in spirit to the present work was put forward in [8]. The authors made the observation that potentials such as the Winther-Akyüz [9] provide reliable barriers but the fact that they cannot reproduce the data far below the barrier is a signature that the ion-ion potential has another form in the inner part of the barrier. Using simple arguments (no channel coupling, WKB calculation of the transmission coefficient) they pointed out that the steep falloff in the tunneling probability is related to the disappearance of the classically allowed region below a certain energy. If this is true, it would mean that we are confronted with the existence of a shallow pocket of the potential inside the barrier.

Another interesting observation [2] made in relation to the data was that the astrophysical S factor, as defined by Burbidge *et al.* [10], develops a maximum for the systems enumerated above, which, again, cannot be reproduced by c.c. calculations using the deep Winther-Akyüz potential. For the systems mentioned earlier the maximum occurs at an energy 20–30 MeV above the compound nucleus ground state.

We advocate in this work the idea that in order to analyze the fusion data the double-folding potentials may avoid the systematic failure of other potentials. To reach this goal we have to make some amendments to the schemes that are usually employed; see, for example, Ref. [11]. First of all, we need to take into account the saturation of the nuclear matter. Second, the neutron and proton content of the various dinuclear combinations have to be included in the potential, a fact which is often overlooked or only indirectly accounted for in the Woods-Saxon parametrization. We show that by properly addressing these issues light can be shed on the hindrance in sub-barrier fusion.

There are several facts pointing toward the existence of shallow pockets in the relative heavy-ion potential used to describe various nuclear reactions. The first evidence came from the discovery of resonant structures in collisions of light nuclei, the best known example being the sharp peaks in the bombarding energy dependence of the excitation curves measured in the ^{12}C + ^{12}C scattering by Bromley *et al.* [12]. These resonances would then resemble states in a quasimolecular potential well.

It became clear in the last two decades that the resonant behavior observed not only in ^{12}C + ^{12}C but also in ^{12}C + ^{16}O, ^{12}C + ^{13}C, ^{16}O + ^{16}O, ^{16}O + ^{24}Mg, is not an isolated phenomenon occurring only in a few lighter systems. It persists even in heavier colliding systems, such as the ^{24}Mg + ^{24}Mg [13] and ^{28}Si + ^{28}Si [14]. Manifestation of clusterization in relation to quasimolecular pockets is also known for various heavy nuclear systems such as cluster radioactivity [15], or cold fission of actinides [16].

To simulate the appearance of shallow pockets several recipes have been proposed in the past: (a) a Gaussian added to the conventional Woods-Saxon potential was used in Refs. [17,18] in order to fit the reaction cross sections observed in $^{12}C + ^{12}C$, $^{12}C + ^{16}O$, and $^{16}O + ^{16}O$; (b) potentials computed within the density functional method [19,20], and (c) the well-known proximity potential [see Ref. [21] for the 1977 and [22] for the 2000 versions].

A repulsive core can be also simulated by folding two nuclear density distributions with an effective nucleon-nucleon ($N - N$) interaction. Double-folding potentials, without repulsive cores, introduced by Satchler and Love [23], are accurate only in the tail region of the nucleus-nucleus potential where the density distributions are only gently overlapping and thus the assumption of ''frozen density'' is less questionable. However, this assumption ignores any readjustment due to the mutual excitations of the nuclei or the Pauli exclusion principle for strong overlap. Density dependent interactions have been used in recent times to simulate the saturation of nuclear matter for α + nucleus scattering [see Ref. [24] and references therein]. For the target-projectile combinations used in the experiments with the ATLAS facility at Argonne [1–4] these double-folding potentials do not necessarily apply because the resulting pockets are still too deep and the barriers too thin below a certain energy. For these reasons another approach is considered to incorporate the effect of the nuclear incompressibility.

Since we are interested in performing dynamical calculations we consider two heavy ions with one-body deformed densities ρ_1 and ρ_2, subjected to vibrational fluctuations, and center of masses separated by the distance $\boldsymbol{R}$. Then the interaction between these two ions can be evaluated as the double-folding integral of these densities

$$V(\boldsymbol{R}) = \int d\boldsymbol{r}_1 \int d\boldsymbol{r}_2 \rho_1(\boldsymbol{r}_1)\rho_2(\boldsymbol{r}_2)v(\boldsymbol{r}_{12}, \rho_1, \rho_2), \qquad (1)$$

where $\boldsymbol{r}_{12} = \boldsymbol{R} + \boldsymbol{r}_2 - \boldsymbol{r}_1$. In the above formula there is allowance for a density dependence of v as discussed previously. The central part of the effective $N - N$ potential v contains a direct part that also depends on isospin since in all the cases of interest $N \neq Z$, and an exchange part that takes into account the effect of antisymmetrization under exchange of nucleons between the two nuclei. For the density independent part of the effective nucleon-nucleon force we use the Michigan-3-Yukawa(M3Y)-Reid parametrization [24–26]. To compute the double-folding potential we use the Fourier technique as expounded in a previous publication [25].

The density independent M3Y interactions are correctly predicting the potential for peripheral collisions. However, reactions sensitive to the potential at smaller distances are not reproduced [24]. To cure this deficiency the ion-ion potential should also contain a repulsive core. For the repulsive part we take a double-folding integral as in Eq. (1) but use densities with smaller diffusivity ($a \leq 0.4$) and the $N - N$ interaction is modeled by a zero-range form with strength V_{core}, following the suggestion of Ref. [27]. In what follows we refer to the used potential (including the Coulomb part that is also calculated via the double-folding procedure) as the M3Y + repulsion.

To derive the properties of the short-range repulsive core, we note that an overlapping region with doubled nucleon density is formed once the distance R between the nuclei becomes less than $R_p + R_t$, where R_p and R_t are the nuclear radii along the collision axis. The doubling of the density increases the energy of the nucleons in the overlapping region. In the case of complete overlap (for $R = 0$) the increase of the interaction energy per nucleon is, up to quadratic terms in the normal density,

$$\frac{\Delta V}{A_p} = \rho_0^2 \left(\frac{\partial^2 B(\rho)}{\partial \rho^2} \right) \Bigg|_{\rho=\rho_0} = \frac{K}{9}, \qquad (2)$$

where in the last equality we use the proportionality of the incompressibility K of normal nuclear matter to the curvature of the energy per nucleon [28]. Since K is usually not measured directly but deduced from isoscalar giant monopole or dipole resonances, and since there are conflicting results coming from the cross sections of the corresponding experimental data [see [29] and references therein] we use instead of a universal value the predictions of the Thomas-Fermi model [30] as a function of the relative neutron excess $\delta = (\rho_n - \rho_p)/\rho$ of the compound (fused) nucleus. Eventually the strength of the repulsive core, V_{core}, is determined by assuming that ΔV must be identified with the value of the heavy-ion interaction potential at the coordinate origin $R = 0$.

The parameters of the Fermi-Dirac density distribution entering in the double-folding potential (1) are calculated as follows: for the proton density distribution we use the parameters resulting from a compilation of elastic electron scattering data [31], i.e., $r_{0p} = 1.065$ fm and $a_p = 0.575$ fm. For the neutron distribution we took the liberty to vary the parameters under reasonable limits, taking into account the following constraints: (a) we deal with a moderately neutron rich nucleus and (b) the barrier of the M3Y + repulsion potential should be close to the one predicted by the Winther-Akyüz potential, since this potential gives a good description of the data in the barrier and high-energy region [3]. Thence we take $r_{0n} = 1.085$ fm and $a_n = 0.534$ fm. To obtain the incompressibility of $K = 228$ MeV, predicted by the Thomas-Fermi model for the compound nucleus ^{128}Ba [30], we use a strength of $V_{\text{core}} = 496$ MeV, and a diffuseness of the repulsive part of the density distribution of $a \approx 0.4$ fm for both protons and neutrons.

To illustrate the shape of this potential for a case of interest we compare in Fig. 1 the spherical heavy-ion potential for the symmetric dinuclear system ^{64}Ni + ^{64}Ni to different potentials that have been used in the past to

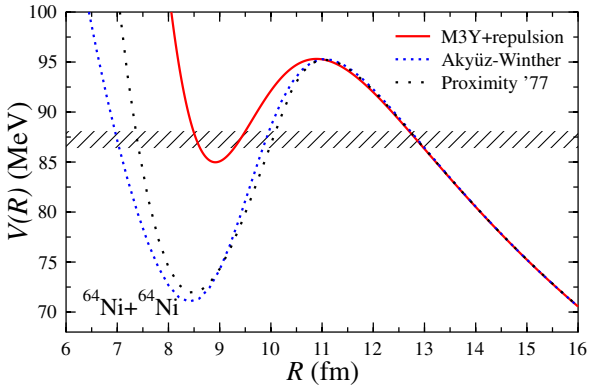

FIG. 1 (color online). Various spherical ion-ion potentials for ^{64}Ni + ^{64}Ni. The solid curve is the potential employed in the present work. The curve with small dashes is the Winther-Akyüz potential used in [1–5]. The dashed strip corresponds to experimental boundaries of the threshold energy E_s.

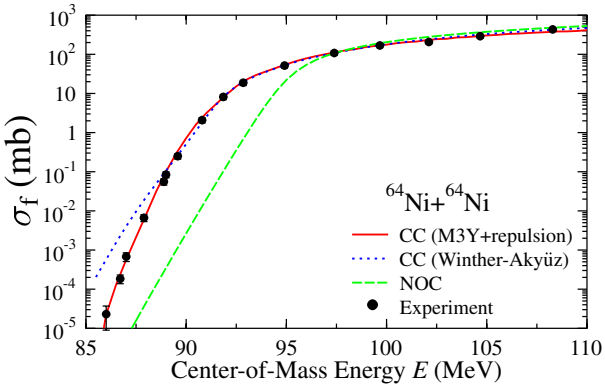

FIG. 2 (color online). Experimental fusion excitation function for the system ^{64}Ni + ^{64}Ni [3] is compared to various calculations described in the text.

explain fusion data [Winther-Akyüz [9]] or fusion barriers [Proximity '77 [21]]. The thickness of the dashed region in Fig. 1 reflects the uncertainty in the energy E_s, where the experimental S factor has a maximum.

As a study case we consider the system ^{64}Ni + ^{64}Ni and treat the fusion dynamics within the c.c. method with linear and quadratic couplings in the quadrupole and octupole vibrational amplitudes [32]. The c.c. equations, which include couplings to the low-lying 2^+ and 3^- states, mutual excitations, and two-phonon quadrupole excitations, are solved in the rotating frame approximation with the usual outgoing-wave boundary conditions at large distances. Ingoing-wave boundary conditions are imposed at the radial separation where the heavy-ion potential develops a local minimum, $V_{\min} \approx 85$ MeV. As a result of this scheme, the calculated cross section will vanish for center-of-mass energies $E \leq V_{\min}$. Naturally one would expect a nonzero cross section even below $V_{\min}$, since the compound nucleus can in principle be formed down to a threshold energy of $E \approx 49$ MeV.

In Fig. 2 the experimental excitation function of the fusion reaction ^{64}Ni + ^{64}Ni $\rightarrow$ ^{128}Ba is compared with the results using the Winther-Akyüz potential as in Ref. [3] (dotted line) and with the M3Y + repulsion potential (solid line), in both cases using the same recipe for the c.c. calculations. The cross section for no couplings is also shown (dashed curve) when the Winther-Akyüz potential is used. It is seen in Fig. 2 that the agreement with the data, when using the M3Y + repulsion potential, is much better than the one provided by the Winther-Akyüz potential. The "M3Y + repulsion" excitation function has the right shape, not only because the potential attains a higher-lying minimum but also because the curvature of the barrier is different, producing a thicker classically forbidden region. Taking all the experimental data points into account we obtain a minimum χ^2 per point of $\chi^2/N = 10.1$ for an overall energy shift $\Delta E = 0.9$ MeV of the calculated cross sections when the Winther-Akyüz potential is used. For the M3Y + repulsion we get a much better result with a minimum $\chi^2/N = 0.86$ for an energy shift of $\Delta E = 0.16$ MeV.

The S-factor representation of the ^{64}Ni + ^{64}Ni data (solid points) is compared in Fig. 3 with the two c.c. calculations, based on the M3Y + repulsion potential (solid curve) and the Winther-Akyüz potential (dashed curve). The clear maximum in the S factor is reproduced only by the M3Y + repulsion potential. At this point one should recall the experience gained in the sixties on molecular resonances. As shown in [33] the S factor exhibits a sequence of quasimolecular resonances for lighter systems.

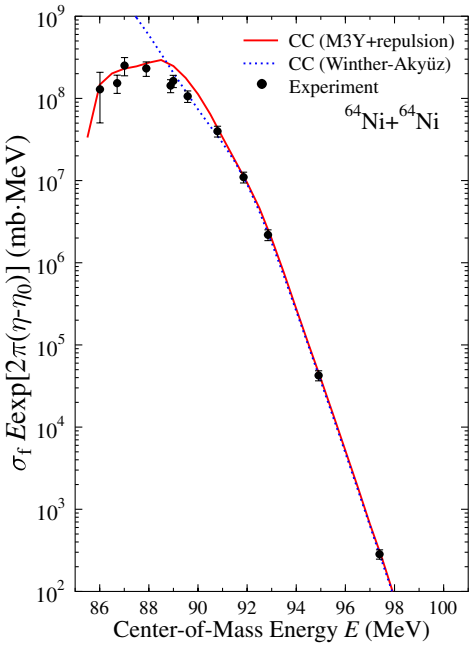

FIG. 3 (color online). The experimental S factor of ^{64}Ni + ^{64}Ni [3] (solid points) is compared to the coupled-channels calculations performed with the M3Y + repulsion (solid curve) and the Winther-Akyüz (dashed curve) potential.

In the present case we get a maximum that is too broad to be assigned to a resonance, the curvature in the S factor being explained by the shallow pocket in the potential. The maximum of the theoretical curve corresponds approximately to the maximum of the data points.

A similar study was performed on another case, ^{58}Ni + ^{58}Ni, for which data are available from an older experiment with the smallest cross sections in the range of mb [34]. Even in this case we confirm the trend of the excitation functions, namely, a faster decrease than expected due to the existence of a repulsive core.

Thus, the understanding of the experimental data, at least for the case ^{64}Ni + ^{64}Ni, requires a modified shape of the potential inside the barrier. Both a thicker barrier and a shallower pocket are needed to explain the measured cross sections and these features are naturally explained by the M3Y + repulsion heavy-ion potential. If the actual potential pocket would be deeper than the potential used by us, e.g., the double-folding with density dependent M3Y interactions [24], then one would expect within the energy range between the barrier top and the threshold energy E_s the appearance of a resonant structure of the form encountered for lighter quasimolecular systems [33]. Apparently there is no indication of resonant structures in the excitation functions of ^{64}Ni + ^{64}Ni or in the other systems studied in Refs. [1–5].

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