Risk of Misinterpretation of Nearly Degenerate Pair Bands as Chiral Partners in Nuclei

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The experimental information on the observed nearly degenerate bands in the $N = 75$ isotones, in particular ^{134}Pr and ^{136}Pm, which are often considered as the best candidates for chiral bands, is critically analyzed. Most properties of the bands, in particular, the recently measured branching ratios and lifetimes, are in clear disagreement with the interpretation of the two bands as chiral bands. For $I = 14$–18 in ^{134}Pr, where the observed energies are almost degenerate, we have obtained a value of 2.0(4) for the ratio of the transition quadrupole moments of the two bands, which implies a considerable difference in the nuclear shape associated with the two bands. The insufficiency of the near-degeneracy criterion to trace nuclear chirality is emphasized.

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Spontaneous formation of handedness or chirality is a subject of general interest in molecular physics, in the characterization of elementary particles, and in crystallography. Chirality in triaxial nuclei is a topic of current interest in nuclear structure, being intensively investigated both theoretically and experimentally. It is characterized by the presence of three angular-momentum vectors, which are noncoplanar [1,2]. The observation of two almost degenerate $\Delta I = 1$ rotational bands with the same parity, of which the first example was the doublet bands in ^{134}Pr [3], has been so far taken as a sign of chiral bands. We note that the electromagnetic transition probabilities carry more stringent information on the intrinsic structure, including the chiral geometry, than excitation energies [4,5]. In ideal chiral pair bands all corresponding properties such as energies, spin alignments, shapes, electromagnetic transition probabilities, etc. must be identical or, in practice, very similar. The observation of the identity is the necessary condition for claiming chiral bands. As will be shown in the following analysis of the transition probabilities, the arguments in favor of the presence of chirality in atomic nuclei used so far in the literature are clearly insufficient.

In order to realize the chiral geometry, an appropriate amount of rotational angular momentum is needed to establish the third of the three orthogonal angular-momentum vectors building the geometry. However, at high rotational frequency the gradual alignments of the odd particles will destroy the chiral geometry and/or the crossing with bands involving a large alignment of two more nucleons (S bands) will make the chiral bands nonyrast. Thus, the possible chiral pair bands are expected to be yrast only in a limited spin range.

The nearly degenerate $\Delta I = 1$ rotational bands have been observed in many odd-odd nuclei in the $A = 130$ mass region, of which the main intrinsic configuration is considered to be $\pi h_{11/2} \otimes \nu h_{11/2}$. Examining the level schemes observed in the region of $I < 20$, we find two kinds of near degeneracy: in nuclei such as $^{134}_{59}\mathrm{Pr}_{75}$ the two bands have different alignments and cross with each other already in the region of $I < 20$, while in those such as $^{132}_{57}\mathrm{La}_{75}$ [6] and $^{136}_{61}\mathrm{Pm}_{75}$ [7], the two bands have almost the same alignments and are nearly parallel to each other.

The level scheme of ^{134}Pr was studied in detail up to high spins [8], and only in this nucleus the two nearly degenerate bands were observed at spins higher than the crossing with the S band [3,8]. Above the crossing, the two bands continue to remain nearly degenerate, and this was interpreted as a good indication that the chiral regime was reached. However, as discussed later, this degeneracy extended over the spin range $13 < I < 19$ may be induced by several band crossings.

To our knowledge, in only a few nuclei the electromagnetic transition probabilities of the nearly degenerate bands were measured: ^{134}Pr [8,9], ^{128}Cs [10], and ^{132}La [6]. The reported $B(E2, I \rightarrow I-2)_{\mathrm{in}}$ values for the two bands are clearly different in all three nuclei and, therefore, a condition necessary for identifying chiral pair bands is not fulfilled. In the present work we critically analyze the observed properties of the nearly degenerate pair bands to see whether or not they fulfill the necessary conditions for chirality.

In Fig. 1(a) the excitation energies of bands 1 and 2 of ^{134}Pr are plotted as a function of spin. We observe that: (i) band 1 is crossed by band 2 between $I = 15$ and 16; (ii) band 1 is crossed by a $4qp$-band 1 at $I = 19$; (iii) band 2 is crossed by a $4qp$-band 1 at $I = 21$; (iv) band 2 is crossed by a $4qp$-band 2 at $I = 23$ or higher. In Fig. 1(b) the excitation energy is plotted relative to a rigid rotor, which reveals the crossing between the bands even better and shows details which are not easily seen in Fig. 1(a), like the splitting at high spin in band 2. It is now easy to

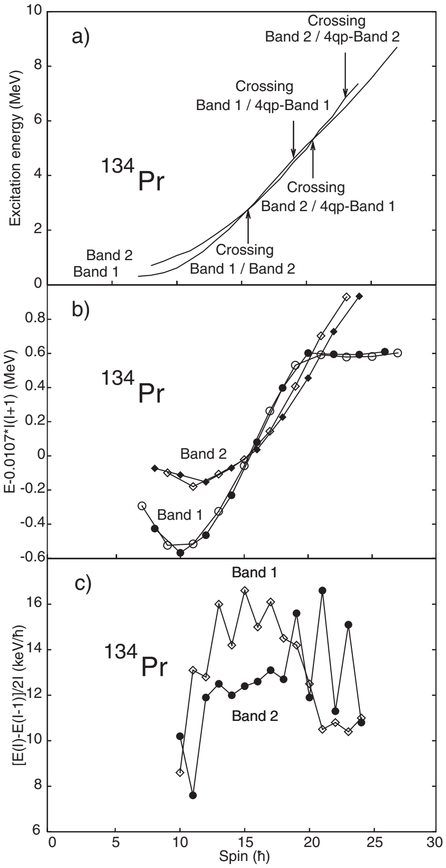

FIG. 1. Data on the two bands in ^{134}Pr: (a) excitation energies vs spin; (b) excitation energies relative to a rigid-rotor reference; (c) illustration of energy signature staggering.

understand the reason why the two bands of ^{134}Pr remain nearly degenerate in a certain spin range: because band 1 after the crossing with band 2 is crossed by a $4qp$ band which, having a larger alignment, approaches band 2 and crosses it again at a higher spin. The overall result of these crossings is that the observed energy difference between the yrast and yrare levels for $I = 14$–22 is less than 200 keV. It is therefore obvious that the observed near degeneracy of the bands in ^{134}Pr for $13 < I < 19$ cannot be regarded as a fingerprint of the chiral regime.

In Fig. 1(c) the energy signature staggering is illustrated as a function of spin. At low spins the signature staggering in band 1 is larger than that in band 2. Above spin 18 the signature staggering of band 2 becomes large, while that of band 1 becomes very small. If two bands are chiral partners, the signature staggering should be equal.

The observed energy difference between the two 15^+ and the two 16^+ states is only 36 and 44 keV, respectively, implying an interaction strength $|V| < 18$ keV. Such a small $|V|$ value indicates a large difference in the structure of the two bands, as, for example, a large shape difference or their chiral character or a difference in some quantum numbers. Interaction strengths of this size have been extracted in many cases from the crossing between bands based on different shapes, like, for example, between superdeformed and normal-deformed bands in the decay-out region for $A \sim 130$ nuclei.

The band crossing picture is easily seen also from Fig. 2, in which particle alignments are plotted as a function of rotational frequency. The alignment difference between the two bands is $\sim 2\hbar$ in the degenerate region of $14 < I < 18$, namely $0.35 < \hbar\omega < 0.5$ MeV. About $2\hbar$ is certainly too large for the interpretation as chiral pair bands. On the other hand, $\Delta i_x \sim 2\hbar$ is usually too small to be consistent with such a small interaction strength as $|V| < 18$ keV, if the shape is nearly the same and the particle configurations are similar.

Some branching ratios and lifetimes were recently measured in ^{134}Pr [9,11]. The equality is drastically broken in the reported measured values, though the experimental errors are not small. Furthermore, the observed nonzero values of $B(E2, I \to I\text{-}2)_{\mathrm{out}}$ for $I \gg 1$ demonstrate a band interaction which should be vanishingly small between ideal chiral pair bands [4]. Measured $B(E2, I \to I\text{-}2)_{\mathrm{in}}$ values are available for both bands only around the crossing region. Thus, in order to extract the unperturbed (shape-dependent) in-band $E2$ strength one has to take the band mixing into account. With the available information on branching ratios together with the observed level spacings in the crossing region it is possible to extract values of the interaction strength as well as the ratios of quadrupole moments of the two intrinsic configurations by imposing some simple assumptions: we neglect off-diagonal $E2$ matrix elements and any spin dependence in the interaction over the short spin range in question.

Assuming the two-bands crossing, the wave functions for the states with spins I are generally written as [12]

$$|y\rangle = \alpha_I|1\rangle + \sqrt{1 - \alpha_I^2}|2\rangle \tag{1}$$

$$|ny\rangle = \sqrt{1 - \alpha_I^2}|1\rangle - \alpha_I|2\rangle, \tag{2}$$

where $|1\rangle$ and $|2\rangle$ denote the wave functions of the original bands 1 and 2, respectively, while $|y\rangle$ and $|ny\rangle$ denote the observed yrast and nonyrast states, respectively. The relation between the admixed amplitudes α_I, interaction strength $|V|$, and level spacing ΔE_I can be expressed as

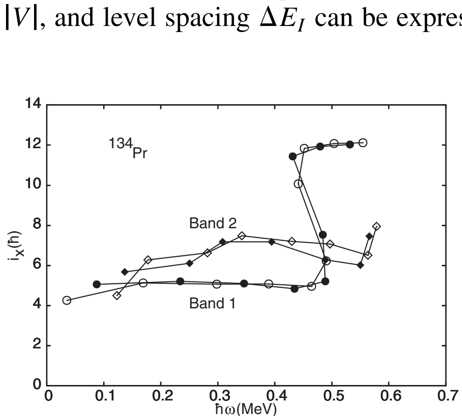

FIG. 2. Particle alignments for the two bands in ^{134}Pr. Filled (open) symbols denote $\alpha = 0(1)$. The used Harris parameters are $J_0 = 15\hbar^2$ MeV^{-1} and $J_1 = 30\hbar^4$ MeV^{-3}.

$$\frac{2V}{\Delta E_I} = \sin[2\sin^{-1}(\alpha_I)]. \quad (3)$$

The resulting ratios for the transition probabilities between the states with spins I and $I-2$ read:

$$\frac{B[E2, I^{ny} \to (I-2)^{y}]}{B[E2, I^{ny} \to (I-2)^{ny}]} = \left(\frac{Q_{0,1}\alpha_{I-2}\sqrt{1-\alpha_I^2} - Q_{0,2}\alpha_I\sqrt{1-\alpha_{I-2}^2}}{Q_{0,1}\sqrt{1-\alpha_{I-2}^2}\sqrt{1-\alpha_I^2} + Q_{0,2}\alpha_I\alpha_{I-2}}\right)^2 \quad (4)$$

$$\frac{B[E2, I^{y} \to (I-2)^{ny}]}{B[E2, I^{y} \to (I-2)^{y}]} = \left(\frac{Q_{0,1}\alpha_{I}\sqrt{1-\alpha_{I-2}^2} - Q_{0,2}\alpha_{I-2}\sqrt{1-\alpha_{I}^2}}{Q_{0,1}\alpha_I\alpha_{I-2} + Q_{0,2}\sqrt{1-\alpha_I^2}\sqrt{1-\alpha_{I-2}^2}}\right)^2, \quad (5)$$

where $Q_{0,1}$ and $Q_{0,2}$ express the $E2$ transition moments of bands 1 and 2, respectively. It is noted that the ratio (4) becomes equal to (5) in the limit of either $Q_{0,1}/Q_{0,2} = 1$ (namely the nuclear shapes associated with bands 1 and 2 are equal) or $\alpha_I = 0$ (namely, the yrast I state is pure $|2\rangle$). In contrast to what is expected for chiral bands, the two experimental ratios for the decays of the 17^{ny} and 17^{y} states and of the 18^{ny} and 18^{y} states in ^{134}Pr are clearly different, as shown in Table I.

Having measured values of ΔE_I, ΔE_{I-2}, $B[E2, I^{ny} \to (I-2)^{y}]/B(E2)_{\mathrm{in}}^{ny}$ and $B[E2, I^{y} \to (I-2)^{ny}]/B(E2)_{\mathrm{in}}^{y}$, one obtains α_I, α_{I-2}, V, and $Q_{0,1}/Q_{0,2}$ from Eqs. (3)–(5). Considering the relatively large ambiguity in the measured $B(E2)_{\mathrm{out}}/B(E2)_{\mathrm{in}}$ ratios, the possible ranges of V and $Q_{0,1}/Q_{0,2}$ allowed by the experimental data are determined. We find $V \sim 14$ and ~ 17 keV for the odd and even spin sequences, respectively, with the same range of $Q_{0,1}/Q_{0,2}$ between 1.6 and 2.4. In Table I the data are compared to values calculated from these extracted ratios. The value of $Q_{0,1}/Q_{0,2}$ which we obtained clearly indicates that the shapes associated with the nearly degenerate bands in ^{134}Pr are very different and, thus, they cannot be interpreted as chiral bands. The pronounced difference [9] in the measured values of $B(E2)_{\mathrm{in}}$ is in qualitative agreement with this result.

TABLE I. Comparison of experimental and calculated values of branching ratios, $\frac{B(E2, I\to I-2)_{\mathrm{out}}}{B(E2, I\to I-2)_{\mathrm{in}}}$, in the crossing region of the pair band in ^{134}Pr, using $Q_{0,1}/Q_{0,2} = 2$. Values of $|V|$ and admixed amplitudes are also listed. Experimental data are taken from Ref. [11].

I	$E_{\gamma,\mathrm{out}}$ (keV)	$E_{\gamma,\mathrm{in}}$ (keV)	$B(E2)_{\mathrm{out}}/B(E2)_{\mathrm{in}}$ Experiment	Calculation	$\lvert V\rvert$ (keV)	α_I
17^{ny}	991	1027	0.3(1)	0.33	14	0.118
17^{y}	909	873	0.6(1)	0.70	14	···
15^{y}	671	892	0(1)	0.034	14	0.894
15^{ny}	928	707	1.1(5)	0.78	14	···
18^{ny}	1113	1069	0.22(9)	0.17	17	0.099
18^{y}	896	940	<0.08	0.062	17	···
16^{ny}	813	976	0(1)	0.12	17	0.427
16^{y}	933	770	1.3(3)	1.36	17	···

For reference we show in Fig. 3 the potential energy surfaces of ^{134}Pr calculated with the ultimate cranking (UC) code [13]. For $I = 15$ and $\pi = +$, in addition to the (marked) global minimum at $(\varepsilon, \gamma) = (0.18, -33.7°)$ assigned to band 1, another minimum is developing in the γ-soft potential at $(\varepsilon, \gamma) \sim (0.21, +17°)$ which we may assign to band 2. Noting that $B(E2, I \to I-2) \propto [\delta\cos(\gamma + 30°)]^2 \simeq [\varepsilon\cos(\gamma + 30°)]^2$ for a given quadrupole deformed shape (ε, γ) and $I \gg 1$, we obtain $Q_{0,1}/Q_{0,2} \simeq 1.25$ for these shapes. Comparing this value of 1.25 with the value 2.0(4), which we have obtained in the analysis presented above, the actual shape difference between bands 1 and 2 seems to be much larger than what the UC calculations predict. Other calculations, like the Woods-Saxon Total Routhian Surfaces reported in Fig. 5 of Ref. [3], show instead a shape difference of the two minima which gives $Q_{0,1}/Q_{0,2} \simeq 1.7$.

The rotational frequencies at which the corresponding $4qp$ bands cross bands 1 and 2 are also different, being significantly larger in band 2 ($\hbar\omega \sim 0.55$ MeV) than in band 1 ($\hbar\omega \sim 0.47$ MeV). The different crossing frequencies in the two bands are in disagreement with the simple

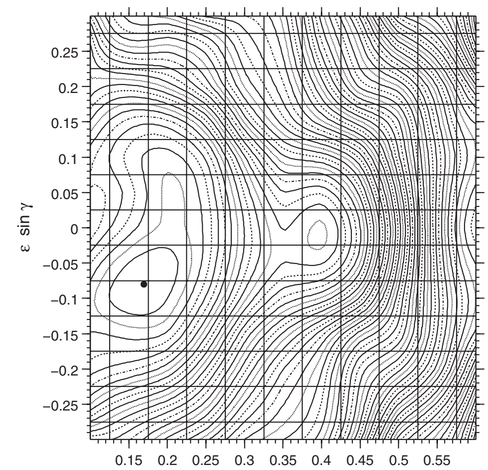

FIG. 3. Potential energy surface calculated by UC for the $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration with $(\alpha_\pi, \alpha_\nu) = (-1/2, -1/2)$ in ^{134}Pr at $I = 15\hbar$. The dot marks the global minimum. Contour lines are separated by 0.2 MeV.

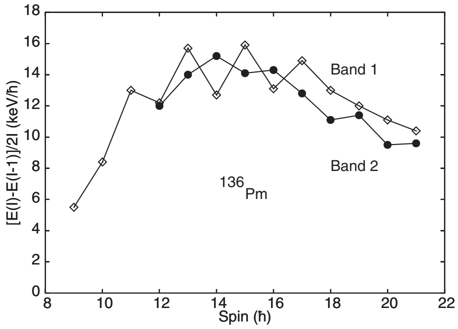

FIG. 4. Illustration of energy signature staggering as a function of spin for the two bands in ^{136}Pm.

chiral picture. On the other hand, the shape coexistence description can induce a dependence of the crossing frequency on both γ and β deformation for a given configuration. In the $N = 75$ isotones, the crossing observed in the yrast band may be assigned to the alignment of the second and third protons of the $h_{11/2}$ shell, the so-called BC crossing, and the crossing frequency is expected to be ~0.4–0.45 MeV.

The behavior of the two bands of ^{136}Pm is somewhat different from that of ^{134}Pr. The bands are only seen up to the crossing with the corresponding $4qp$ bands, which occurs at ~0.44 and ~0.39 MeV in bands 1 and 2, respectively, as shown in Fig. 3 of Ref. [7]. The crossing frequency in band 1 is larger than in band 2, in contrast to the case of ^{134}Pr.

The signature staggering of the two bands of ^{136}Pm shown in Fig. 4 is also different from that of ^{134}Pr. The signature staggering of band 1 at low spins has the same phase as in ^{134}Pr. The signature staggering in band 2, instead, is opposite to that in band 1, and this is different from what is observed in ^{134}Pr.

The presently available data on the two bands in ^{136}Pm are not in favor of their possible chiral character. Measurements of electromagnetic transition probabilities are necessary in order to test whether the shapes associated with the two bands are different as found in ^{134}Pr.

In conclusion, by having critically analyzed the observed properties of so-called chiral bands in ^{134}Pr and ^{136}Pm, we find that they do not agree with what is expected for chiral bands, in spite of the near degeneracy of the levels with $13 < I < 19$. In particular, we have obtained a value of 2.0(4) for the ratio of the $E2$ transition moments, $Q_{0,1}$ and $Q_{0,2}$, of the two bands in ^{134}Pr. We note that these quantities used in our analysis are not based on the assumption of a rigid shape. Thus, there must be a significant difference in the shapes associated with the two bands and it is therefore questionable if and to what extent a reminiscence of the chiral geometry is present in the ^{134}Pr data. Though chiral twin bands are theoretically expected to appear under specific circumstances, the present type of critical analysis should be applied also to other cases of many available publications in which the chiral interpretation is claimed.

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